OESTERREICHISCHE NATIONALBANK
EUROSYSTEM

WORKING PAPER 201

The Return on Social Security with Increasing Longevity

Markus Knell

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Publisher and editor *Oesterreichische Nationalbank*
Otto-Wagner-Platz 3, 1090 Vienna, Austria
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oenb.info@oenb.at
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Design *Communications and Publications Division*

DVR 0031577

ISSN 2310-5321 (Print)
ISSN 2310-533X (Online)

The Return on Social Security with Increasing Longevity

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May 2015

Abstract

In this paper I study the impact of increasing longevity on pay-as-you-go pension systems. First, I show that increasing longevity increases their internal rate of return. The size of the effect differs for different policy regimes. It is higher for the case where the retirement age is increased in order to keep the system in balance than for the case where the necessary adjustment is achieved by reducing pension benefits. Second, I study optimally chosen retirement decisions and I show that the socially optimal policy involves a shorter working life than the private optimum. The social optimum can be implemented by the use of a PAYG system that combines an actuarial and a flat pension.

Keywords: Pension System, Demographic Change, Increasing Life Expectancy, Retirement Decision
JEL-Classification: H55; J1; J18; D630

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Non-Technical Summary

The population of a country can grow for two reasons: an increase in the average cohort size (which depends on fertility and migration) and an increase in life expectancy (which depends on mortality). The literature has so far mainly concentrated on the first aspect of population growth. It has been shown, e.g., that ihe internal rate of return of a PAYG pension system is positively affected by a higher birth rate. In particular, the internal rate of return σ is given by the sum of the growth rate of wages g and the cohort size n, i.e. $\sigma = g + n$. To give an example, assume that the contribution rate is 25%, people start to work at the age of 20, work until the age of 60 and die at the age of 80. It can be calculated that for constant cohort sizes ($n = 0$) a balanced PAYG system can provide in this case a replacement rate of 50%. If cohort sizes increase by 1% each year ($n = 0.01$) then this sustainanable replacement rate increases to 67.8%, for $n = 0.02$ to 93%.

This leaves the question open what would happen in the reverse case where cohort sizes are constant and longevity increases? In fact, this seems to be the more realistic demographic pattern if one looks at the development over the recent decades. I assume that the longevity increase is linear and given by $\omega^c(t) = \omega^c(0) + \gamma \cdot t$. Intuitively, one could guess that an ageing population has a negative impact on the budget of a PAYG system and thus on its rate of return. In the paper I show that this conjecture is false and that the impact is positive. This is even true for the extreme case where the retirement age stays constant which might be surprising at first sight. The reason for this positive effect is that in order to keep the pension system in balance it is sufficient to base adjustments on period-specific instead of the larger cohort-specific demographic measures. For the cohort that dies at age 80 (i.e. that has a cohort longevity of 80) the average replacement rate would be 50% if lonegvity is constant. If longevity increases, however, then the older cohort that have been alive during the retirement period of this cohort die at younger ages (say at 79, 78, ...) thereby allowing the pension system to pay out higher replacement rates. The total internal rate of return can then be approximated by $\sigma = g + n + \frac{\gamma}{\omega^c(t)}$. I show that the size of the last effect $\frac{\gamma}{\omega^c(t)}$ is non-trivial and about half of the size of the effect of increasing cohort sizes n.

In the second part of the paper I expand the analysis in order to investigate whether for increasing longevity a PAYG system can be regarded as a Pareto improvement vis-á-vis the laissez-faire situation. I show that under a set of simplifying assumptions the optimal retirement age is proportional to longevity. In a next step I look at the social optimum in a stationary situation where the social planner maximizes per period utilities. The social optimum turns out to involve a shorter working life than the laissez-faire allocation. Finally, I show that this social optimum can be implemented by a pension policy that combines Bismarckian and Beveridgean elements, i.e. a pillar where pension benefits are related to the individual retirement age and a pillar that promises a flat pension payment.

1 Introduction

Demographic developments are an important factor when thinking about the economic role of social security systems. This was first demonstrated by Samuelson (1958) in the framework of a consumption-loan economy for which he had shown that there exist two equilibrium interest rates: an "autarkic rate" (determined by technology and individual preferences) and a "biological interest rate" that is equal to the population growth rate. This biological interest rate has interesting properties. First, it corresponds to the internal rate of return of a pay-as-you-go (PAYG) pension system as has been shown by Samuelson (1958) and later elaborated by Aaron (1966) and Cass & Yaari (1966). Second, the equilibrium associated with this pension system will Pareto dominate the autarkic equilibrium as long as the population growth rate exceeds the market interest rate.

The main focus of this pioneering and the following literature has been laid upon one specific demographic development that causes the population to grow: increasing birth rates. There exists, however, a second biological force that has mostly been neglected in these considerations and that can also have a non-negligible impact on population growth and the internal rate of return of PAYG pension systems: increases in longevity.

In this paper I focus on the impact of increasing longevity on social security systems where I will deal with two crucial questions. First, how does increasing longevity affect the internal rate of return of PAYG pension systems and does it matter which type of system is established? Second, does the presence of increasing longevity allow for Pareto improving interventions and which type of PAYG system could be used to implement a "golden rule allocation"?

In the first part of the paper I assume that the policy maker can perfectly control individual retirement behavior which therefore can be treated as a policy choice variable. I then derive approximate expressions for the internal rate of return (IRR) of the system when both demographic developments — changing fertility *and* changing mortality patterns — are present. To this end I use a continuous time model and I assume that cohort sizes grow exponentially while longevity increases in a linear fashion. I abstract from uncertainty and assume that all members of a generation reach the cohort-specific maximum age such that one can interchangeably refer to their longevity or their life expectancy.[1]. I show that the IRR depends on the design of the pension system and in particular on the parameters that are changed in order to keep the system financially balanced in the presence of demographic shifts. In order to trace out the possibilities I deal with two

[1]For a set-up with uncertain death see e.g. d'Albis et al. (2012) and Bruce & Turnovsky (2013)

polar cases. In the first policy regime the complete adjustment is done by varying the retirement age such that the dependency ratio is always held constant. In the second policy regime the retirement age is held constant and the adjustment is done by reducing pension benefits.

I find that for both policies the approximate solution for the IRR depends not only on the growth rate of wages and of the cohort size but also on increases in longevity. For the policy with a constant retirement age the IRR is just given by the sum of the growth rate of wages and the population growth rate which corresponds to the result in the original literature. The difference, however, is that population growth is now affected by increases both in the cohort size and in longevity. A second difference is that the size of the IRR varies with the design of the PAYG system. In particular, I show that it is higher for the policy that adjusts the retirement age.

The main reason for the positive contribution of increasing longevity is that in order to keep the pension system in balance it is sufficient to base adjustments on period life expectancies instead of the larger cohort life expectancies. For the policy with a constant retirement age this means that the necessary reductions in the pension benefits can be smaller than would be otherwise. For the policy with an increasing retirement age this property manifests itself in the fact that only a fraction of each additional year of longevity has to be spend on the labor market in order to keep the system balanced. This allows for a larger retirement span and thereby increases the IRR. For this adjustment policy there exists, however, an additional channel that increases the rate of return. In particular, the rise in the retirement age causes a constant increase in the contribution base. This has the same effect as the introduction of a PAYG system and thus entails "windfall profits" for the generation that is the beneficiary of these extra revenues. As a consequence, the contribution of increasing longevity is approximately twice as large for the policy with increasing retirement age than it is for the one with a constant age. In both cases the size of this effect is non-trivial. For reasonable parameter values the contribution of increases in longevity to the total IRR of a PAYG pension system is between one third and two thirds of a percentage point.

In the second part of the paper I expand the analysis in order to investigate whether a PAYG system can be regarded as a Pareto improvement vis-á-vis the laissez-faire situation. To this end I use a set-up with exogenous factor prices in which individuals choose their consumption profile and the retirement age in an optimal manner. I show that under a set of simplifying assumptions (in particular that the time discount rate and the exogenously given interest rate are equal to zero) the optimal retirement age is proportional to longevity which is equivalent to one of the two policies studied in the first part of the paper. In a next step I look at the golden rule allocation, i.e. at the social optimum in a stationary situation where the social planner maximizes per period utilities. The social optimum turns out to involve a shorter working life than the laissez-faire allocation. Finally, I show that this social optimum can be implemented by a pension policy that combines Bismarckian and Beveridgean elements, i.e. a pillar where pension benefits are related to the individual retirement age and a pillar that promises a flat pension payment.

The paper is organized as follows. In the next section I lay out the continuous time model of the pension system. In section 3 I present the two policy regimes for the assumption of exogenously given retirement ages and I derive approximate solutions for the IRR for these constellations. In section 4 I look at individually and socially optimal policies. Section 5 concludes.

2 Model

I work with a **deterministic model in continuous time**. In every instant of time t a generation is born that has size $N(t)$ and a life span of $\omega^c(t)$ years. I assume that all members of a generation reach this maximum attainable age such that $\omega^c(t)$ is at the same time the measure of cohort longevity and of cohort life expectancy.

Each member of generation t works for $R^c(t)$ periods, earns a wage $W(t+a)$ during each of these working periods ($a \in [0, R^c(t)]$) and receives a pension benefit $P(t+a)$ in each period of retirement ($a \in [R^c(t), \omega^c(t)]$). While working, individuals pay contributions to the PAYG pension system at rate $\tau(t)$. The (relative) **pension level** is defined as:

$$q(t) = \frac{P(t)}{W(t)}. \tag{1}$$

Wages and the **cohort size** are assumed to grow in an exponential manner at rates g and n, respectively:

$$W(t) = W(0)e^{gt}, \tag{2}$$

$$N(t) = N(0)e^{nt}. \tag{3}$$

Cohort longevity, on the other hand, is assumed to increase linearly over time:

$$\omega^c(t) = \omega^c(0) + \gamma{\cdot}t, \tag{4}$$

where $0 \leq \gamma < 1$. The assumption about the linear increase in $\omega^c(t)$ is in line with empirical results. Lee (2003), e.g., refers to a number of studies that have found a linear trend in life expectancy for a large number of industrial countries. The empirical estimates suggest a value for γ between 0.15 and 0.25.[2]

For the following derivations it is necessary to distinguish between the viewpoint of generation t (i.e. the one born in t) and the outlook of the pension system in period t. In particular, $\omega^p(t)$ stands for **period longevity** (or period life expectancy), i.e. the oldest age *observed* in period t. It can be calculated from $\omega^c(t - \omega^p(t)) = \omega^p(t)$. Solving the equation $\omega^c(0) + \gamma(t - \omega^p(t)) = \omega^p(t)$ for $\omega^p(t)$ leads to:

$$\omega^p(t) = \frac{1}{1+\gamma}\omega^c(t). \tag{5}$$

In a similar fashion, I also introduce the variable $R^p(t)$ that denotes the "period retirement age", i.e. the number of working years of the generation that retires in period t that will in general differ from $R^c(t)$, the number of working years of the generation born in t.

In the following I will always assume that if generation t works in some period then all generations that are younger will work as well. This allows to express the following **aggregate values** without the use of "indicator variables". The size of the active population $L(t)$, the retired population $B(t)$ and the total population $T(t)$ are then given by:

$$L(t) = \int_0^{R^p(t)} N(t-a)\,\mathrm{d}a, \tag{6}$$

$$B(t) = \int_{R^p(t)}^{\omega^p(t)} N(t-a)\,\mathrm{d}a, \tag{7}$$

$$T(t) = \int_0^{\omega^p(t)} N(t-a)\,\mathrm{d}a. \tag{8}$$

Given assumption (3) the **dependency ratio** $z(t) = \frac{B(t)}{L(t)}$ can be calculated as:

$$\begin{aligned} z(t) &= \frac{e^{-nR^p(t)} - e^{-n\omega^p(t)}}{1 - e^{-nR^p(t)}}, \text{ for } n \neq 0, \\ z(t) &= \frac{\omega^p(t) - R^p(t)}{R^p(t)}, \text{ for } n = 0. \end{aligned} \tag{9}$$

[2]Equation (4) assumes that life expectancy increases without bound. Since this is a controversial assumption I briefly discuss the case where life expectancy is assumed to reach an upper limit in appendix C.

For later reference, note that equation (8) can be used to calculate the population growth rate in this economy. As shown in appendix A it can be approximated as:

$$g^T(t) \equiv \frac{\dot{T}(t)}{T(t)} \approx n + \frac{\gamma}{\omega^c(t)}. \tag{10}$$

As is apparent from equation (10) population growth decreases over time and in the limit, as $t \to \infty$ one has that $g^T(t) \to n$. The reason for this is that longevity increases only linearly and thus the percentage growth rate in longevity is given by $\frac{\gamma}{\omega^c(t)}$ which approaches zero as mortality rates approach zero. This is not the case for cohort growth which is assumed to proceed at an exponential (Malthusian) rate. If one had assumed that instead of (3) the cohort size also increases linearly with $N(t) = N(0) + n \cdot t$ then one would get a parallel result for the cohort growth rate and $g^T(t) \approx \frac{n}{N(t)} + \frac{\gamma}{\omega^c(t)}$. On the other hand, one could also assume that longevity grows in an exponential fashion, i.e. $\omega^c(t) = \omega^c(0)e^{\gamma t}$. This is of course a completely unrealistic assumption that is just made for illustrative purposes. In this case one would get a population growth rate of $g^T(t) \approx n + \gamma$.

It is assumed throughout the paper that the **budget of the social security system** is always balanced, i.e. that $\tau(t)W(t)L(t) = P(t)B(t)$ or:

$$\tau(t) = q(t)z(t). \tag{11}$$

The **internal rate of return** (IRR) for cohort t is defined as the rate $\sigma(t)$ for which the total of discounted benefits received by the cohort over all pension periods $a \in [R^c(t), \omega^c(t)]$ equals the total of discounted contributions for all working periods $a \in [0, R^c(t)]$. Using the definition of $q(t)$ in (1) and the assumption about constant wage growth in (2) one can write this implicit definition of $\sigma(t)$ as:

$$\int_0^{R^c(t)} \tau(t+a)e^{(g-\sigma(t))a}\,\mathrm{d}a = \int_{R^c(t)}^{\omega^c(t)} q(t+a)e^{(g-\sigma(t))a}\,\mathrm{d}a. \tag{12}$$

For calculating the IRR in the face of demographic trends one needs to specify how the retirement age and the social security system react in order to keep the budget in balance. I will only focus on **adjustment policies** with a fixed contribution rate, i.e. $\tau(t) = \hat{\tau}$. Increases in the contribution rate expand the “size” of the PAYG system. This expansion would add — so to say — a new supplement to the existing pension scheme thereby creating the usual windfall gains for the introductory generations. This, however, would render comparisons between different policies difficult and I will therefore abstract from this possibility. This leaves changes in the retirement age $R^c(t)$ and in pension benefits $P(t)$ (or corresponding pension levels $q(t)$) as the two adjustment parameters.

In section 3 I will treat the development of the retirement age as given which amounts to the assumption that the system can perfectly control the retirement decisions of the insured population. In order to map the field of possible retirement behavior I will deal with two polar cases. In the first case ("policy A") I assume that the retirement age is flexible and it adjusts in way such as to hold the dependency ratio and the budget of the system constant. In the second regime ("policy B") the retirement age is assumed to be constant while the pension level is adjusted such as to keep the pension system in balance.

In section 4, on the other hand, I assume that individuals choose their retirement age in an optimal manner. I will discuss how the individual optimum differs from the social optimum and how a PAYG system can be designed to implement this first-best allocation. It will turn out that under specific assumptions the resulting system corresponds to an instance of policy A.

3 The internal rate of return for two adjustment policies

3.1 Policy A: Increasing retirement age

In this policy regime the retirement age is adjusted in such a way as to keep the dependency ratio constant at some level $\hat{z}$. This means that $R^p(t)$ is chosen such that $z(t) = \hat{z}, \forall t$. This is a natural benchmark since it corresponds to a situation where both the contribution rate and the pension level are held constant at $\hat{\tau}$ and $\hat{q}$, respectively (cf. (11)). As will be discussed in section 4.3 this policy is related to the Swedish notional defined contribution (NDC) system (cf. Palmer 2012, Holzmann & Palmer 2012) where the development of life expectancy is taken into account when calculating the pension payment. In order to prevent a decline in annual pension benefits each cohort has to prolong its working life and there exists a cohort-specific "benchmark retirement age" that has to be reached in order to guarantee a constant pension level $\hat{q}$.

One can use (9) together with the policy assumption $z(t) = \hat{z}$ to derive the required path for the retirement age. The solution for this "benchmark retirement age" is stated in the following proposition.[3]

Proposition 1 *In order to stabilize the dependency ratio at $z(t) = \hat{z}$ the retirement age has to be determined according to the following rule:*

$$R^p(t) = \frac{\omega^p(t)}{1+\hat{z}} \text{ for } n = 0, \tag{13}$$

$$R^p(t) = \omega^p(t) + \frac{1}{n} \ln\left[\frac{1+\hat{z}}{1+e^{n\omega^p(t)}\hat{z}}\right] \approx \frac{\omega^p(t)}{1+\hat{z}}\left[1 - \frac{n\hat{z}\omega^p(t)}{2\,(1+\hat{z})}\right] \text{ for small } n \neq 0. \tag{14}$$

This result is quite intuitive as can be best seen by looking at a stationary situation with $n = 0$, $R^p = 45$, $\omega^p = 60$, $\hat{\tau} = 0.2$ and $\hat{q} = 0.6$. The system is in balance and the dependency ratio is given by $\hat{z} = \frac{1}{3}$.[4] Now assume that period longevity ω^p increases to 64. In order to keep the dependency ratio constant expression (13) shows that this increase in longevity by 4 years does not require an increase in the retirement age of the same magnitude but only from age 45 to age 48. The gain in longevity is shared between working and retirement in the same proportion (i.e., 3:1) that could also be observed before.

Expression (14) shows that in the case of a growing population ($n > 0$) the increase in $R^p(t)$ that stabilizes $z(t)$ is smaller than in the case with $n = 0$ ($\frac{\partial R^p(t)}{\partial n} < 0$). An increasing population ceteris paribus decreases the dependency ratio and thus (partly) counteracts the effects of population aging. By the same token a shrinking population size ($n < 0$) necessitates an increase in retirement age that is more than proportional to the increase in longevity.

Given the fact that for policy A the contribution rate and the pension level are constant over time one can write the implicit definition of the IRR in equation (12) as: $\hat{\tau}\int_0^{R^c(t)} e^{(g-\sigma(t))a}\,\mathrm{d}a = \hat{q}\int_{R^c(t)}^{\omega^c(t)} e^{(g-\sigma(t))a}\,\mathrm{d}a$. For the calculation of the IRR one thus needs an expression for the retirement age of the generation born in period t, i.e. $R^c(t)$. This variable is implicitly defined by $R^p(t + R^c(t)) = R^c(t)$.[5]

[3] All proofs are collected in the appendix.

[4] Note that this example is based on a stylized life-cycle model where individuals start to work at the age of 20 and retire, after 45 years of work, at the age of 65. From then on they receive 15 years of pension payments ($\hat{z} = \frac{15}{45} = \frac{1}{3}$) and die at the age of 80. Thus the starting value of $\omega^c(0) = 60$ corresponds to a "de facto" life expectancy of 80 years.

[5] In order to see this note that generation t will retire in period $t + R^c(t)$. The retirement age *observed* in this future period $t + R^c(t)$ will then be equal to $R^c(t)$, i.e. $R^p(t + R^c(t)) = R^c(t)$.

Proposition 2 *The benchmark retirement age stated in proposition 1 implies a retirement age for the generation born in period t that is given by:*

$$R^c(t) = \frac{\omega^c(t)}{1+\hat{z}(1+\gamma)} \text{ for } n = 0, \tag{15}$$

$$R^c(t) \approx \frac{\omega^c(t)}{1+\hat{z}(1+\gamma)}\left[1 - \frac{n\hat{z}(1+\hat{z})(1+\gamma)\omega^c(t)}{2\left(1+\hat{z}(1+\gamma)\right)^2}\right] \text{ for small } n \neq 0. \tag{16}$$

The expression for $R^c(t)$ in proposition 2 can be used in (12) to derive an approximation of the IRR as specified in the following proposition.

Proposition 3 *Under the assumptions of an exponentially growing cohort size (equ. (3)), a linearly increasing longevity (equ. (4)) and a retirement age that holds the dependency ratio constant (equ. (16)), the internal rate of return of the PAYG pension system can be approximated as:*

$$\sigma(t) \approx g + n + \gamma\frac{2}{\omega^c(t)}. \tag{17}$$

I will discuss equation (17) after having derived the parallel result for policy B.

3.2 Policy B: Decreasing pension levels

Under this policy regime it is assumed that the retirement age is held constant ($R^p(t) = \hat{R}$) while the pension level $q(t)$ is adjusted to keep the system in balance. In particular, this requires to set:

$$q(t) = \hat{q}\frac{\hat{z}}{z(t)}. \tag{18}$$

The relative pension level $q(t)$ thus has to be decreased if $z(t)$ gets larger than $\hat{z}$. This can happen if the size of the average cohort decreases and/or if longevity increases. The use of (18) leads to a constantly balanced pension system irrespective of the determination of the retirement age.[6]

For the assumptions $R^p(t) = \hat{R}$ and (18) the implicit definition of the IRR in equation (12) can be written as $\hat{\tau}\int_0^{\hat{R}} e^{(g-\sigma(t))a}\,\mathrm{d}a = \hat{q}\int_{\hat{R}}^{\omega^c(t)} \frac{\hat{z}}{z(t+a)} e^{(g-\sigma(t))a}\,\mathrm{d}a$. Using linearizations one can derive an approximated solution for the IRR for the case of policy B.

Proposition 4 *Under the assumptions of an exponentially growing cohort size (equ. (3)), a linearly increasing longevity (equ. (4)), a constant retirement age and the adjustment*

[6] In order to see this, simply insert (18) together with $\tau(t) = \hat{\tau}$ into (11). This leads to $\hat{\tau} = \hat{q}\frac{\hat{z}}{z(t)}z(t) = \hat{q}\hat{z}$. This is fulfilled since (per assumption) the values of $\hat{\tau}$, $\hat{q}$ and $\hat{z}$ are chosen such that $\hat{\tau} = \hat{q}\hat{z}$.

of the pension level according to (18), the internal rate of return of the PAYG pension system can be approximated as:

$$\sigma(t) \approx g + n + \gamma \frac{1}{\omega^c(t)}. \tag{19}$$

3.3 Main properties of policies A and B

Propositions 3 and 4 contain a number of interesting results and the main properties are (qualitatively) identical for policies A and B.

The first implication of equations (17) and (19) is that the **IRR increases with the growth rate of wages** g ($\frac{\partial \sigma(t)}{\partial g} > 0$). This is obvious and follows from the fact that increasing wages will also raise future pension benefits.

Second, the **IRR increases in the cohort growth rate** n ($\frac{\partial \sigma(t)}{\partial n} > 0$). Population growth makes a PAYG system more attractive, since this increases the internal return of the system. In fact, for constant longevity ($\gamma = 0$) equations (17) and (19) reduce to $\sigma(t) = g + n$, i.e. the internal rate of return of the PAYG system is simply given by the growth rate of the wage bill. This is the well-known formulation of the "biological interest rate" for which Samuelson (1958) has given the following "common-sense explanation": "In a growing population [...] retired men are outnumbered by workers more than in the ratio of the work span to the retirement span. With more workers to support them, the aged live better than in the stationary state—the excess being positive interest on their savings" (Samuelson 1958, p.473).

Third, equations (17) and (19) imply that the **IRR also increases in** γ. In particular, noting that $\omega^c(t) = \omega^c(0) + \gamma t$ it can be calculated that $\frac{\partial \sigma(t)}{\partial \gamma} = \frac{2\omega^c(0)}{\omega^c(t)^2} > 0$ for policy A and $\frac{\partial \sigma(t)}{\partial \gamma} = \frac{\omega^c(0)}{\omega^c(t)^2} > 0$ for policy B. Increasing longevity thus also contributes to the "biological interest rate" of PAYG pension schemes, although this effect is not as well-known as the one due to growing cohort sizes. In fact if one compares these expression to the population growth rate (10) one sees that for policy B the IRR is just the sum of wage growth g and the population growth rate $n + \frac{\gamma}{\omega^c(t)}$. This indicates that the impact of the second biological force (the decreasing mortality rates) on the IRR is analogous to the impact of the more frequently studied first force (the increasing cohort sizes). The reason why the impact of increasing longevity on the IRR decreases over time has already been discussed in section 2. I have shown there that it has to do with the fact that longevity increases only linearly which diminishes the percentage growth rate for later generations while cohorts are assumed to grow exponentially.

In order to get a feeling for the **size of the longevity-related part of the biological interest rate** assume that $\hat{z} = \frac{1}{3}$, $\gamma = 0.2$ and $\omega^c(0) = 60$ (cf. footnote 4). For policy A the part of the IRR that is due to the increases in longevity is given by $\gamma\frac{2}{60} = 0.00667$. This is smaller than the contribution of productivity growth which is often assumed to be between $g = 0.015$ and $g = 0.02$. It is also about one third smaller than typical assumptions about the population growth rate[7] but certainly non-negligible. For higher life spans the magnitude decreases but even for $\omega^c(100) = 80$ it is still given by $\gamma\frac{2}{80} = 0.005$ which is half the size of the benchmark population growth rate given by $n = 0.01$.

3.4 Intuition behind the results

In order to give the intuition behind the effect of increasing longevity on the biological interest rate I want to focus on the case with $g = 0$ and $n = 0$, i.e. $W(t) = W$ and $N(t) = N$. The total flow of contributions over the working life for cohort t is given by:

$$TC(t) = \hat{\tau} W R^c(t). \tag{20}$$

Total benefits, on the other hand, can be written as:

$$TB(t) = \int_{R^c(t)}^{\omega^c(t)} q(t+a)W \, \mathrm{d}a = \hat{\tau} W \int_{R^c(t)}^{\omega^c(t)} \frac{1}{z(t+a)} \, \mathrm{d}a, \tag{21}$$

where I use (11). The ratio of total benefits to total contributions then comes out as $\frac{TB(t)}{TC(t)} = \frac{\int_{R^c(t)}^{\omega^c(t)} \frac{1}{z(t+a)} \mathrm{d}a}{R^c(t)}$.[8] For constant longevity and constant retirement age this fraction reduces to $\frac{TB(t)}{TC(t)} = \frac{\int_R^\omega \frac{1}{\hat{z}} \mathrm{d}a}{R} = \frac{\omega - R}{R\hat{z}} = \frac{\omega - R}{R\frac{\omega - R}{R}} = 1$. For a stationary demographic structure total benefits just correspond to total contributions (under the assumption of $g = 0$ and $n = 0$).

For increasing longevity and policy B, however, total contributions are given by $TC(t) = \hat{\tau} W \hat{R}$, while total benefits are:

$$TB(t) = \hat{\tau} W \int_{\hat{R}}^{\omega^c(t)} \frac{1}{z(t+a)} \, \mathrm{d}a = \hat{\tau} W \int_{\hat{R}}^{\omega^c(t)} \frac{\hat{R}}{\omega^p(t+a) - \hat{R}} \, \mathrm{d}a.$$

The cohort has contributed $\hat{R}$ periods to the system and receives a pension payment for

[7] The world population, e.g., has currently been estimated to grow at an annual rate of about 1.1%.

[8] This corresponds to another frequently used measure of intergenerational distribution, the present value ratio (cf. Geanakoplos et al. 1999, Fenge & Werding 2003).

$(\omega^c(t) - \hat{R})$ periods. At first sight one might thus suspect that a balanced pension system will be able to provide an annual pension payment of $\frac{\hat{\tau} W \hat{R}}{\omega^c(t) - \hat{R}}$ for $(\omega^c(t) - \hat{R})$ periods. This, however, is more modest than what the social security system can afford. In fact, in each of the retirement periods $(t + a)$ where $a \in [\hat{R}, \omega^c(t)]$ the system takes the total revenues $\hat{\tau} W \hat{R}$ and distributes them equally among the $(\omega^p(t + a) - \hat{R})$ cohorts of retired workers thereby granting each a pension payment of $\frac{\hat{\tau} W \hat{R}}{\omega^p(t+a) - \hat{R}}$. This annuity payment, however, is larger than $\frac{\hat{\tau} W \hat{R}}{\omega^c(t) - \hat{R}}$ since $\omega^p(t + a) < \omega^c(t)$ for $a \in [\hat{R}, \omega^c(t))$ and only for the very last period $\omega^p(t + \omega^c(t)) = \omega^c(t)$. One can use (5) to write $\omega^p(t + a) = \frac{\omega^c(t) + \gamma a}{1 + \gamma}$ and to calculate that $\frac{TB(t)}{TC(t)} = \frac{(1+\gamma)\ln(1+\gamma)}{\gamma} \approx 1 + \frac{\gamma}{2}$. In other words, the PAYG system provides a total stream of pensions that is $(1 + \frac{\gamma}{2})$ times larger than the total stream of contributions. In order to transform this total return into an annual measure one can ask what constant rate of return ς would amount to this total return if the returns would accrue in a continuous fashion. This is given by $\frac{\int_0^{\omega^c(t)} e^{\varsigma a} \, \mathrm{d}a}{\omega^c(t)} = \frac{e^{\varsigma \omega^c(t)} - 1}{\varsigma \omega^c(t)} \approx 1 + \frac{\varsigma \omega^c(t)}{2}$. Equating this expression to $1 + \frac{\gamma}{2}$ implies that $\varsigma = \frac{\gamma}{\omega^c(t)}$ which is exactly the internal rate of return in (19) of proposition 4.

For policy A with an increasing retirement age one can follow a parallel logic. In this case $TC(t) = \hat{\tau} W R^c(t) = \hat{\tau} W \frac{\omega^c(t)}{1 + \hat{z}(1+\gamma)}$, while total benefits are $TB(t) = \hat{\tau} W \int_{R^c(t)}^{\omega^c(t)} \frac{1}{z(t+a)} \, \mathrm{d}a$. Since policy A is designed in a way such that $z(t) = \hat{z}, \forall t$ the latter expression can be written as:

$$TB(t) = \hat{\tau} W (\omega^c(t) - R^c(t)) \frac{1}{\hat{z}} = \frac{\hat{\tau} W \omega^c(t) \frac{\hat{z}(1+\gamma)}{1+\hat{z}(1+\gamma)}}{\hat{z}} = \frac{\hat{\tau} W \omega^c(t)(1 + \gamma)}{1 + \hat{z}(1 + \gamma)}.$$

Therefore one can conclude that the total rate of return of the PAYG system is given by $\frac{TB(t)}{TC(t)} = (1 + \gamma)$ which corresponds to an annual rate of return of $\varsigma = \frac{2\gamma}{\omega^c(t)}$ which is again equal to the internal rate of return in (17) of proposition 3. The reason why the internal rate of return is higher under policy A than under policy B stems from the fact that in this case the contribution base increases every period due to the fact that the retirement age is constantly extended. These extra contributions can be distributed to the currently retired population thus giving rise to "introductory gains". This process repeats itself from period to period and thus every cohort can expect to receive similar "windfall profits" once they are retired. The increase in the contribution base and the associated "introductory gains" are only present for policy A and this leads—as the formulas show—to an IRR that is approximately twice as high than for policy B.

The logic behind the biological rate of interest due to increases in longevity is thus completely parallel to the biological rate of interest due to fertility growth. Again, the "aged live better than in the stationary state" because a constantly increasing longevity enables the social security system to pay out annuities that are "generously priced" (based on period instead of cohort life expectancies). For policy A there exists the additional feature that a constantly postponed retirement age leads to a constantly increasing number of workers that finance the longer retirement span. Both processes result in an "excess" that provides a positive interest on pension savings.

4 Optimal behavior

So far I have assumed an exogenously given (or a perfectly controlled) retirement age and I have focused on the internal rate of return to describe the distributional consequences of different assumptions and corresponding adjustment policies. It is important to note, however, that the IRR is not a welfare measure and that differences in its magnitude should not be confused with statements about the advantage of one system over the other. Since the adjustment policies involve different amounts of lifetime work and lifetime leisure and since they are associated with a different temporal structure of contributions and benefit payments they are not directly comparable. Whether an individual will prefer one policy or the other will depend on the exact specification of his or her intertemporal utility function. This is the topic of the present section. The introduction of individual utility functions allows me to extend the analysis into two directions. First, one can discuss optimal behavior and in particular optimally chosen retirement behavior. It will be interesting to see, e.g., whether and under which conditions the optimal choices will coincide with the exogenously given retirement age of policies A and B in section 3. Second, one can also compare the results of individual optimization with the choice of a social planner. The assumption of increasing longevity, however, introduces some difficulties for the study of socially optimal policies that are related to the inherent non-stationarity of the problem and to the selection of an appropriate criterion for evaluating intergenerational consumption patterns.[9] An exhaustive treatment of these difficult topics is beyond the scope of this paper and I will mostly focus on a specific set of assumptions that allows for closed-form solutions and for an intuitive discussion of the issues involved.

[9]On the difficulties raised by population ethics see Ponthiére (2003) and Blackorby et al. (2005).

4.1 Individual optimum

I start with the individual optimization problem that can be found in the related literature (cf. Sheshinski 1978, Crawford & Lilien 1981, Bloom et al. 2007, Kalemli-Ozcan & Weil 2010, Bagchi 2015). Agents are assumed to maximize their lifetime utility by choosing how much to consume in each period and how long to work. I ignore the effects of any eventual bequest motives, of the family structure, of all possible sources of uncertainty and — for the moment — from the existence of a public pension system. The intertemporal utility function for the representative member of generation t is given by:

$$\mathbb{U}(t) = \int_0^{\omega^c(t)} e^{-\delta a} U(C(t,a))\,\mathrm{d}a - \int_0^{R^c(t)} e^{-\delta a} V(\omega^c(t), a)\,\mathrm{d}a, \tag{22}$$

where δ is the rate of time preference, $C(t,a)$ the level of consumption of cohort t at age a and $V(\omega^c(t), a)$ is the disutility of labor schedule that might also depend on longevity $\omega^c(t)$. In the following I will use the simple specification $V(\omega^c(t), a) = \upsilon$ (as is also used, e.g., by Sheshinski (1978) and Kalemli-Ozcan & Weil (2010)).[10] It is assumed that labor supply is fixed at 1 before retirement (at the age of $R^c(t)$) and zero afterwards. Furthermore, as in section 2, I again assume that retirement is a one-time decisions and people do not return to the labor market once they have joined the ranks of the pensioners.

Lifetime utility (22) is maximized subject to the budget constraint:

$$\frac{dA(t,a)}{da} = \chi(t,a)W(t+a) + rA(t,a) - C(t,a), \tag{23}$$

where $A(t,a)$ are the assets of generation t at age a, r is the (exogenously given) interest rate and $\chi(t,a)$ is an indicator variable with the value $\chi(t,a) = 1$ for $a \in [0, R^c(t)]$ and $\chi(t,a) = 0$ for $a \in [R^c(t), \omega^c(t)]$. Agents choose their consumption paths and their retirement age $R^c(t)$ subject to the conditions that $C(t,a) > 0$ and $A(t,a) \geq 0$ (no borrowing).

In appendix D I study the problem in this general form and I specify the first-order conditions for the consumption path and the retirement age. In the following, however, I want to focus on a benchmark case that is often used in the related literature (e.g.

[10] Alternatively, one could also assume (cf. Bloom et al. 2007) that the health status improves proportionally with longevity (see appendix D). This assumption leads to qualitatively identical results as the assumption of $V(\omega^c(t), a) = \upsilon$. Both variants imply an optimal policy that corresponds to policy A in section 3. It would be straightforward to define a disutility of labor schedule for which policy B is optimal. For example $V(\omega^c(t), a) = 0$ for $a \in [0, \hat{R}]$ and $V(\omega^c(t), a) = \infty$ for $a \in (\hat{R}, \omega^c(t)]$. For policy B, however, there does not exist a stationary equilibrium and I will therefore focus on the case of policy A.

Sheshinski 1978, Crawford & Lilien 1981, Cremer et al. 2004, Kalemli-Ozcan & Weil 2010). I abstract from the growth of cohort size ($n = 0$) and I assume that the wage level is constant ($W(t) = W, \forall t$) and that both the interest rate and the discount rate are zero, i.e. $g = r = \delta = 0$. In this case the first-order condition for consumption implies that each generation has a flat consumption profile (i.e. $C(t, a) = C(t)$, $\forall a$). One can then write (22) as:

$$\mathbb{U}(t) = \omega^c(t)U(C(t)) - \upsilon R^c(t). \tag{24}$$

Since I abstract for the moment from public transfers, the lifetime budget constraint is given by:

$$\omega^c(t)C(t) = R^c(t)W. \tag{25}$$

This implies that $C(t) = \frac{R^c(t)W}{\omega^c(t)}$ and thus the first-order condition concerning the retirement age can be written as:[11]

$$U'(C(t))W = \upsilon. \tag{26}$$

Each generation will work until the costs of the additional period of work is equal to the benefit of this effort. Equation (26) implicitly defines the optimal retirement age. If one assumes log utility of consumption ($U(x) = \ln(x)$) one gets that:

$$R^c(t) = \frac{1}{\upsilon}\omega^c(t) = \mu^*\omega^c(t), \tag{27}$$

where $\mu^* \equiv \frac{1}{\upsilon}$. This is exactly analogous to policy A in section 3 (for $n = 0$) where retirement $R^c(t)$ has been assumed to be proportional to longevity $\omega^c(t)$. Equation (27) thus shows that policy A corresponds in fact to an individually optimal strategy under a set of specific assumptions concerning technology and individual preferences.[12] From (25) it follows that consumption is given by $C(t) = C^* \equiv \mu^* W$. Furthermore, from (27) and the relation $R^c(t - R^p(t)) = R^p(t)$ one can conclude that $R^p(t) = \frac{\mu^*}{1+\gamma\mu^*}\omega^c(t)$. Inserting the solutions for $C(t)$ and $R^c(t)$ into (24) gives an expression for lifetime utility of generation t: $\mathbb{U}(t) = \omega^c(t)(\ln(\mu^* W) - \upsilon\mu^*)$.

For further reference I also want to look at individual savings that are associated with

[11] I assume here that there exists an interior optimum. This is the case for $U'(W)W < \upsilon$. Otherwise, the optimal retirement age is $R^c(t) = \omega^c(t)$.

[12] The strict proportionality is due to the assumption of log utility. For a general utility function one can use the implicit function theorem on (26) to derive that $\frac{dR^c(t)}{d\omega^c(t)} = \frac{R^c(t)}{\omega^c(t)}$, i.e. the relation between the retirement age and longevity is always positive. This might not be the case if the age of death is uncertain as has been shown by Kalemli-Ozcan & Weil (2010) and d'Albis et al. (2012). For a calibrated study see Chen & Lau (2014) and for a model including human capital investments Zhang & Zhang (2009).

this policy. In particular, it follows that during working life the savings flow is given by $S^W(t,a) = W - C = W(1-\mu^*)$ for $a \in [0, R^c(t)]$ which leads to a wealth stock at the moment of retirement given by $R^c(t)W(1-\mu^*)$. This fund is then slowly depleted during retirement at a dissaving flow of $S^P(t,a) = -C = -\mu^* W$ for $a \in [R^c(t), \omega^c(t)]$. Individual (i.e. cohort-specific) saving over time is necessarily zero. This can be seen by noting that: $\int_0^{\omega^c(t)} S(t,a)\,\mathrm{d}a = \int_0^{R^c(t)} S^W(t,a)\,\mathrm{d}a + \int_{R^c(t)}^{\omega^c(t)} S^P(t,a)\,\mathrm{d}a$ which simplifies to $R^c(t)W(1-\mu^*) + (\omega^c(t) - R^c(t))(-\mu^* W) = \omega^c(t)W(\mu^*(1-\mu^*) - \mu^*(1-\mu^*)) = 0$. In a certain period of time, however, aggregate savings $S(t)$ is *not* zero.[13] In order to see this start from the definition $S(t) = \int_0^{\omega^p(t)} N(t-a)S(t-a,a)\,\mathrm{d}a$. For a constant cohort size ($n = 0$ or $N(t) = N$) this leads to:

$$S(t) = N\left(R^p(t)W(1-\mu^*) + (\omega^p(t) - R^p(t))(-\mu^* W)\right) = \frac{\gamma\mu^*(1-\mu^*)\omega^c(t)WN}{(1+\gamma\mu^*)(1+\gamma)}.$$

For $\gamma = 0$ aggregate savings is zero, while it is positive for increasing longevity. This is parallel to the standard life-cycle model of consumption where savings is also understood as a pure storage technology and where it can be shown that for growing cohort sizes (the second important demographic development) the stock of savings is also positive (Modigliani 1986).

4.2 Social optimum

The individual optimum can now be contrasted with the socially optimal, first-best allocation that would be chosen by a social planner that can observe all relevant variables. To this end, one needs to specify a social welfare function. As stated above, this involves a number of intricate questions that have to do with welfare economics in general and with population ethics in particular. For example, one has to make assumptions about the form of the social welfare function (utilitarian, egalitarian etc.) and also about the relevant concept of utility. In our context the latter issue is of particular interest. De la Croix & Ponthiére (2010), e.g., argue that in the presence of increasing longevity it is no longer clear whether a social planner should maximize total utility of the succeeding generations (which they term the "complete view of welfare") or rather their respective per period levels. "A social planner may want to maximize the level of welfare *per period lived* [...] It is not obvious that such an intensity view of welfare can be *a priori* regarded as more or less plausible than the complete view" (p.235). In the following I will also take

[13]This has also been shown in a setting with uncertain survival by Sheshinski (2006).

the intensity view of welfare and assume that this is the criterion that guides the choices of the social planner. While for the complete view it would not be possible to refer to steady state utility since longevity is not converging to a constant value the intensity view allows for a meaningful discussion of stationary utility and of a golden rule allocation (i.e. the allocation that yields the highest per period utility).[14]

I am thus looking for a situation with $C(t) = C$ and $R^c(t) = \mu\omega^c(t)$ that gives the highest intensity of utility $\frac{\mathbb{U}(t)}{\omega^c(t)} = \ln(C) - \upsilon\mu$. The social planner maximizes this expression subject to the aggregate resource constraint $R^p(t)W = \omega^p(t)C + S(t)$. Since in this economy savings are completely unproductive and are basically only stored goods the social planner will choose a level of $S(t) = 0$. This implies that the golden rule consumption level is given by:

$$C^{SP} = \frac{R^p(t)}{\omega^p(t)}W = \frac{\mu(1+\gamma)}{1+\gamma\mu}W, \tag{28}$$

where I use the relations $R^p(t) = \frac{\mu\omega^c(t)}{1+\gamma\mu}$ and $\omega^p(t) = \frac{\omega^c(t)}{1+\gamma}$. The first-best retirement age (or rather working span μ) is then given by the maximization of $\ln\left(\frac{\mu(1+\gamma)}{1+\gamma\mu}W\right) - \upsilon\mu$ with respect to μ. This leads to a quadratic equation with a positive root given by:

$$\mu^{SP} \equiv \frac{\sqrt{\frac{4\gamma+\upsilon}{\upsilon}} - 1}{2\gamma} \approx \frac{1}{\upsilon} - \frac{\gamma}{\upsilon^2}. \tag{29}$$

For $\gamma = 0$ it follows that the social optimum coincides with the private optimum and $\mu^{SP} = \mu^*$ and $C^{SP} = C^*$. In this case there exists no scope to improve on the laissez-faire allocation. For increasing longevity ($\gamma > 0$), on the other hand, the social planner will choose a shorter working life ($\mu^{SP} < \mu^*$) and a lower per period consumption level ($C^{SP} < C^*$). Note, however, that even if the social planner would choose a level of $\mu^{SP} = \mu^*$ the per period utility would be larger than in the private optimum. This follows from the fact that $C^{SP} = \frac{\mu^*(1+\gamma)}{1+\gamma\mu^*}W > \mu^*W = C^*$ while the disutility of labor would be the same as in the private optimum (since $\mu^{SP} = \mu^*$).

4.3 Implementation of the social optimum

The next question is how the socially optimal allocation could be implemented in a decentralized manner. In this section I will show that it would be possible to do so by

[14] Note, however, that I am using a small open economy framework with exogenously given factor prices while golden rule allocations are typically analyzed in the context of general equilibrium models.

installing a PAYG pension system that combines a Bismarckian NDC (notional defined contribution) system with a Beveridgean flat minimum pension. The NDC system is characterized by a fixed contribution rate τ while the pension is calculated as the total of all contributions divided by remaining life expectancy at the moment of retirement (see Palmer 2012). In Knell (2012) I show that in the case of increasing life expectancy a NDC system can use period life expectancy in order to be compatible with a balanced budget. In the present case this means that the standard annual NDC pension for cohort t is given by $P^{NDC}(t) = \frac{\tau W R^c(t)}{\omega^p(t+R^c(t))-R^c(t)}$. The denominator can also be written as $\omega^p(t+R^c(t))-R^c(t) = \frac{\omega^c(t)-R^c(t)}{1+\gamma}$. Furthermore, one can again conjecture that the optimal retirement age will be proportional to the life span, i.e. $R^c(t) = \mu\omega^c(t)$. The NDC pension can thus be written as $P^{NDC}(t) = \frac{\tau W\mu(1+\gamma)}{1-\mu}$.[15] In addition to this NDC pension, however, there also exists a minimum pension that is proportional to the wage level and that is paid unconditionally in every pension period, i.e. $P^M(t) = mW$. In order to finance these outlays for the flat pension the amount of the NDC pension is reduced by a factor κ. Taken together, the pension payment for cohort t is thus given by:

$$P(t) = P^{NDC}(t) + P^M(t) = \frac{\kappa\tau W\mu(1+\gamma)}{1-\mu} + mW, \tag{30}$$

which is constant across generations ($P(t) = P$). Using the individual budget constraint one can thus write the cohort-specific consumption level as:

$$\begin{aligned} C(t) &= \frac{R^c(t)(1-\tau)W + (\omega^c(t)-R^c(t))\left(\frac{\kappa\tau W R^c(t)(1+\gamma)}{\omega^c(t)-R^c(t)} + mW\right)}{\omega^c(t)} \\ &= W\left(\mu(1-\tau) + (1-\mu)\left(\frac{\kappa\tau\mu(1+\gamma)}{1-\mu} + m\right)\right), \end{aligned} \tag{31}$$

which is again constant across generations ($C(t) = C$). The individual maximizes $\mathbb{U}(t) = \omega^c(t)\ln C - \upsilon R^c(t) = \omega^c(t)\left(\ln C - \upsilon\mu\right)$ with respect to μ. This comes out as:

$$\mu = \frac{1}{\upsilon} - \frac{m}{1-m-(1-\kappa(1+\gamma))\tau}, \tag{32}$$

[15] In order to see that a pure NDC system (without a flat pension) designed along these lines is in balance one has to note that total revenues are given by $\tau W R^p(t) = \tau W\frac{\mu\omega^c(t)}{1+\gamma\mu}$, while total expenditures come out as $(\omega^p(t) - R^p(t))\frac{\tau W\mu(1+\gamma)}{1-\mu}$ which can be simplified to $\frac{(1-\mu)\omega^c(t)}{(1+\gamma\mu)(1+\gamma)}\frac{\tau W\mu(1+\gamma)}{1-\mu} = \tau W\frac{\mu\omega^c(t)}{1+\gamma\mu}$. The use of cohort life expectancy to calculate the pension annuity would, on the other hand, lead to a permanent surplus since then $P^{NDC}(t) = \frac{\tau W\mu}{1-\mu}$.

which confirms the conjecture that the optimal retirement age is proportional to longevity and can be written as $R^c(t) = \mu\omega^c(t)$. This result shows that even in the presence of a mixed pension system, policy A (as studied in section 3) can be regarded as an individually optimal policy under the given set of assumptions.

Note that for $m = 0$ equation (32) implies that $\mu = \frac{1}{\upsilon}$. In the absence of a flat pension a NDC system would therefore give rise to the same individually optimal retirement age $\mu = \mu^*$ that would also be chosen in a situation without a pension system (cf. equation (27)) which is longer than the social optimum given in (29). The introduction of a flat pension $P^M(t) = mW$, however, shortens the optimal length of working life ($\frac{\partial\mu}{\partial m} < 0$) and can be used by the social planner to steer the economy towards the first best solution.

In particular, the social planner wants to implement a retirement period μ^{SP} (given by (29)) and a constant consumption level (given by (28)), together with a balanced pension system. The balanced pension budget requires $\tau W R^p(t) = P(\omega^P(t) - R^p(t))$ which implies $\tau\mu = \left(\frac{\kappa\tau\mu(1+\gamma)}{1-\mu} + m\right)\frac{1-\mu}{1+\gamma}$. A constant consumption profile, on the other hand, requires $P = (1-\tau)W$ which implies $\frac{\kappa\tau\mu(1+\gamma)}{1-\mu} + m = 1 - \tau$. The latter two equations together with (32) can be used to express the pension parameters τ, m and κ as a function of the working span μ (a choice parameter) and the exogenously given parameters υ and γ. This comes out as:

$$\begin{aligned} \tau &= \frac{1-\mu}{1+\gamma\mu}, \\ m &= \frac{(1+\gamma)\mu(1-\mu\upsilon)}{1+\gamma\mu}, \\ \kappa &= \mu\upsilon. \end{aligned}$$

This can be illustrated by using the values $\gamma = 0.2$ and $\upsilon = 4/3$. If the social planner wants to implement $\mu = \mu^*$, i.e. a working span that corresponds to the private optimum without pensions, one gets that $\mu = 0.75$, $\tau = 0.22$, $m = 0$ and $\kappa = 1$. The consumption level in this case is given by $C = 0.78W$ which is larger than the laissez-faire optimum where it is given by $C = \mu W = 0.75W$. Note that in the case of constant longevity a flat consumption profile would be associated with $\tau = 0.25$. The fact of increasing longevity thus allows a reduction of the contribution rate by three percentage points.

If the social planner implements the golden rule allocation given by (29) then this leads to $\mu = 0.66$, $\tau = 0.3$, $m = 0.08$ and $\kappa = 0.88$. Compared to the case with $\mu = \mu^*$ the social planner thus implements a shorter working life. The cohort with a life span of 80, e.g., will retire at the age of 60 instead of 65. This is accompanied by a higher contribution rate (30% instead of 22%) and a flat pension that amounts to 8% of gross income W. The consumption level is now lower than before and given by $C = 0.7W$, although per period utility is of course higher for the golden rule allocation.

Summing up, this section has established three main results. First, under certain assumptions concerning individual preferences it can be shown that a situation where the retirement increases in proportion to increases in longevity is individually optimal. Second, the individual optimum does not correspond to the social optimum. Third, the social optimum can be implemented by using a PAYG pension scheme that combines a NDC system with a flat minimum pension.

In section 3 of the paper I have abstracted from these issues related to optimal policy and I have studied the consequences of increasing longevity on the internal rate of return for specific constellations of pension schemes and exogenously given retirement ages. I have looked there at an arbitrary pension system with some contribution rate $\hat{\tau}$ while in this section I have derived the optimal contribution rate given by $\tau^{SP} = \frac{1-\mu^{SP}}{1+\gamma\mu^{SP}}$. The fact that even for $g = n = 0$ both policies A and B involved a positive IRR is just a mirror image of the results of this section. In the absence of a PAYG pension system and in the presence of a storage technology with an interest $r = 0$ the rate of return of private savings is just zero. The positive IRR of the public pension system thus allows for Pareto improving intergenerational transfers as has been elaborated above for the case of proportionally increasing retirement ages (policy A). Section 3 has shown, however, that even for the extreme case of a constant retirement age (policy B) the IRR due to increasing longevity is positive which indicates that it allows for improvements as compared to the laissez-faire situation.

5 Conclusions

In this paper I have studied the impact of increasing longevity on the rate of return and the role of PAYG pensions. In the first part I have presented an approximation for the IRR under different assumptions about the retirement age and the adjustment policy. In the case of the first policy the retirement age is changed in a way such that the dependency ratio is held constant. The paper has derived explicit solutions for the determination of the generation-specific reference retirement age that is needed in order to hold the dependency ratio constant. It has been shown that an increase in longevity requires a less than 1:1 increase in the retirement age in order to keep the pension system in balance. As an alternative policy regime I have also looked at the case where the retirement age is held constant and the pension level is adjusted in order to keep the pension system in balance. The approximate solutions for the IRR have a similar form and similar properties for both policy regimes. In particular, under both assumptions the IRR increases in the growth rate of wages g, in the growth rate of the cohort size n and in the speed γ with which longevity increases. I have also shown, however, that the impact of increasing longevity on the IRR is larger for the adjustment policy that involves an increase in the retirement age. This is due to the fact that increases in the retirement age broaden the contribution base which allows to distribute additional "windfall profits" among the retired population. It is important to note, however, that even for the case of a constant retirement age (where the extra "introductory gains" are absent) the contribution of increasing longevity to the internal rate of return is non-negligible.

In the second part of the paper I have introduced a model where individual make optimal consumption and retirement decisions. It turns out that under specific assumptions the optimal retirement age is proportional to individual longevity thus corresponding to policy A that has been studied in the first part. In the social optimum, the proportion of the total life span that is spend in the labor market is, however, shorter than in the private optimum. Finally, I have also demonstrated that this golden rule allocation can be implemented by a specific pension policy that combines a Bismarckian pillar (where pension benefits are conditioned on the retirement age) and a Beveridgean pillar (which is an unconditional pension payment). The optimal policy crucially depends on the assumption about the disutility of labor. In this paper I have chosen a particularly simple specification that is compatible with a stationary situation. For future research it would be interesting to look at socially optimal policies and their possible implementation in PAYG systems for different assumptions concerning this crucial factor.

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Appendices

A Derivations and Proofs

A.1 Population growth

One can insert $\omega^p(t) = \frac{\omega^c(t)}{1+\gamma} = \frac{\omega^c(0)+\gamma t}{1+\gamma}$ and $N(t-a) = N(t)e^{-an}$ into (8). The population growth rate $g^T(t)$ can be calculated as:

$$g^T(t) = \frac{\dot{T}(t)}{T(t)} = n + n\frac{\gamma}{1+\gamma}\frac{1}{e^{n\frac{\omega^c(t)}{1+\gamma}} - 1}.$$

Using the approximation that $e^x \approx 1 + x$ one can rewrite this as:

$$\frac{\dot{T}(t)}{T(t)} \approx n + \frac{\gamma}{\omega^c(t)}.$$

For a linear increase in the cohort growth rate $N(t) = N(0) + n \cdot t$ one gets that:

$$g^T(t) = n\left(\frac{1}{N(t)} + \frac{\gamma}{1+\gamma}\frac{1}{e^{n\frac{\omega^c(t)}{1+\gamma}} - 1}\right) \approx \frac{n}{N(t)} + \frac{\gamma}{\omega^c(t)}.$$

For an exponential increase in longevity, i.e. $\omega^c(t) = \omega^c(0)e^{\gamma t}$ one gets:

$$g^T(t) = n + n\frac{\Pi(\gamma\omega^c(t))}{e^{\frac{n\Pi(\gamma\omega^c(t))}{\gamma} - 1}(1 + \Pi(\gamma\omega^c(t)))},$$

where $\Pi(z)$ is the product log defined as the principal solution for x in $z = xe^x$. One can approximate the growth rate in this case as $g^T(t) \approx n + \gamma$.

A.2 Proposition 1

For $n = 0$ a stabilization of the dependency ratio at $z(t) = \hat{z}$ requires to set $\frac{\omega^p(t) - R^p(t)}{R^p(t)} = \hat{z}$. Solving this expression for $R^p(t)$ gives equation (13).

Similarly, (14) follows from the solution to $\frac{e^{-nR^p(t)} - e^{-n\omega^p(t)}}{1 - e^{-nR^p(t)}} = \hat{z}$. The Taylor approximation (used in the second part of equation (14)) is around $n = 0$.

A.3 Proposition 2

The starting point for the proof of the case where $n = 0$ is the (identity) relation between period and cohort retirement age given by $R^p(t + R^c(t)) = R^c(t)$. The left-hand side of this expression can be calculated as $R^p(t + R^c(t)) = \frac{\omega^p(t+R^c(t))}{1+\hat{z}} = \frac{\omega^c(t)+\gamma R^c(t)}{(1+\hat{z})(1+\gamma)}$, where I use (13) for the first equality and (5) for the second. Equating this with $R^c(t)$ and solving for $R^c(t)$ gives equation (15).

For the case with $n \neq 0$ one can use equation (14) to write the expression $R^p(t+R^c(t))$ as:

$$R^p(t + R^c(t)) = \frac{\omega^c(t) + \gamma R^c(t)}{1+\gamma} + \frac{1}{n} \ln \left[\frac{1+\hat{z}}{1 + e^{n\frac{\omega^c(t)+\gamma R^c(t)}{1+\gamma}} \hat{z}} \right].$$

I take a linear approximation (around $n = 0$) of this expression, set it equal to $R^c(t)$ and solve for $R^c(t)$. This (rather lenghty) solution for $R^c(t)$ is again linearized around $n = 0$ which leads to the expression in equation (16).

A.4 Proposition 3

For the proof of proposition 3 I use the guess-and-verify approach. In particular, I start with the definition of the IRR in equation (12). For policy A the dependency ratio is constant at $z(t) = \hat{z}$. Since the contribution rate is also constant at $\hat{\tau}$ it follows that also the pension level is constant at $\hat{q}$. Therefore equation (12) can be written as:$\hat{\tau} \int_0^{R^c(t)} e^{(g-\sigma(t))a}\, \mathrm{d}a = \hat{q} \int_{R^c(t)}^{\omega^c(t)} e^{(g-\sigma(t))a}\, \mathrm{d}a$. One can then insert the guessed solution $\sigma(t) = g + n + \gamma\frac{2}{\omega^c(t)}$ (see equation (17)) into both sides of this equation. The left-hand side (the present value of contributions (PVC)) comes out as:

$$PVC = \frac{\hat{\tau}\omega^c(t)\left(1 - e^{-R^c(t)(n+\frac{2\gamma}{\omega^c(t)})}\right)}{2\gamma + n\omega^c(t)},$$

while the right-hand side (the present value of benefits (PVB)) simplifies to:

$$PVB = \frac{\frac{\hat{\tau}}{\hat{z}}\omega^c(t)\left(e^{-R^c(t)(n+\frac{2\gamma}{\omega^c(t)})} - e^{-2\gamma-n\omega^c(t)}\right)}{2\gamma + n\omega^c(t)},$$

where I have used the equation $\hat{q} = \frac{\hat{\tau}}{\hat{z}}$. In the next step I insert

$$R^c(t) \approx \frac{\omega^c(t)}{1+\hat{z}(1+\gamma)} \left[1 - \frac{n\hat{z}(1+\hat{z})(1+\gamma)\omega^c(t)}{2\left(1+\hat{z}(1+\gamma)\right)^2}\right] \tag{33}$$

from equation (16) into PVC and PVB and approximate these expressions around $n = 0$ and $\gamma = 0$. One gets that the approximations for the present value of contributions and benefits are equal and given by $PVC \approx PVB \approx \frac{\hat{\tau}\omega^c(t)(2-2\gamma-n\omega^c(t))}{2(1+\hat{z})}$. This verifies the initial conjecture that $\sigma(t) \approx g + n + \gamma\frac{2}{\omega^c(t)}$.

It has to be noted, however, that the approximation is rather crude since γ is typically not small (as stated in section 2 empirical estimates are arounbd $\gamma = 0.2$). In appendix B I therefore compare the exact values for the IRR with the approximations given in equations (17) and (19). As can be seen there, the approximations are not completely accurate but they provide the right order of magnitude.

A.5 Proposition 4

I use again the guess-and-verify approach to proof proposition 4 following similar steps as in appendix A.4. For policy B the retirement age (and the contribution rate) is constant while the pension level is changed according to (18). The IRR in equation (12) can be thus be written as:$\hat{\tau}\int_0^{\hat{R}} e^{(g-\sigma(t))a}\,\mathrm{d}a = \hat{q}\int_{\hat{R}}^{\omega^c(t)} \frac{\hat{z}}{z(t+a)} e^{(g-\sigma(t))a}\,\mathrm{d}a$. One can then insert the guessed solution (19), i.e. $\sigma(t) = g + n + \gamma\frac{1}{\omega^c(t)}$ into both sides of this equation. The left-hand side (the PVC) comes out as:

$$PVC = \frac{\hat{\tau}\omega^c(t)\left(1 - e^{-\hat{R}(n+\frac{\gamma}{\omega^c(t)})}\right)}{\gamma + n\omega^c(t)}.$$

In order to get a closed-form solution for the right-hand side of the equation I use the approximation that $e^{(g-\sigma(t))a} \approx 1 + (g - \sigma(t))a$. The resulting expression is lengthy and is omitted here (but is available upon request).

In the final step I approximate the expressions for PVC and PVB around $n = 0$ and $\gamma = 0$. These approximations are again equal and given by $PVC \approx PVB \approx \frac{\hat{\tau}\hat{R}\left((2-n\hat{R})\omega^c(t)-\gamma\hat{R}\right)}{2\omega^c(t)}$. This verifies the initial conjecture that $\sigma(t) \approx g + n + \gamma\frac{1}{\omega^c(t)}$.

B Approximate versus exact solutions

In the course of deriving equations (17) and (19) a number of linear approximations has been used (see appendix A). In order to check the quality of these approximations I have used numerical methods to directly solve for $\sigma(t)$ in equation (12) for both policy

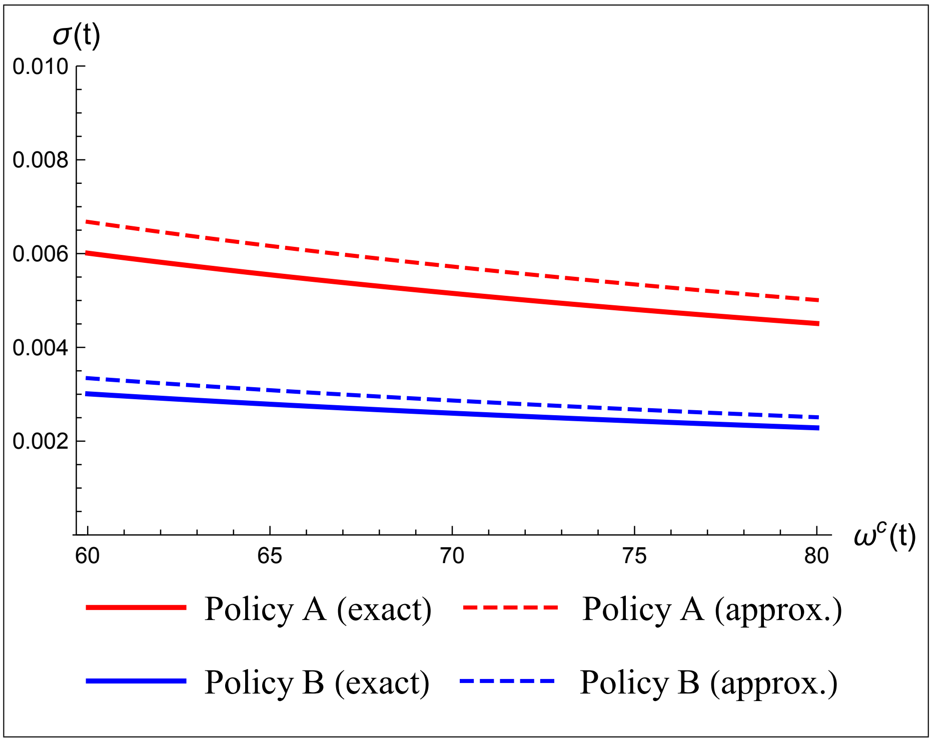

Figure 1: The figure shows the internal rate of return for policy A (changes in the retirement age) and policy B (changes in the pension level). The solid lines refer to the exact (numerical) solutions for $\sigma(t)$ while the dashed lines correspond to the approximations given by (17) and (19), respectively. The parameters were chosen as follows: $\hat{z} = 1/3$, $n = 0$, $g = 0$, $\gamma = 0.2$ and $\omega^c(0) = 60$.

regimes.[16] These exact solutions for the IRR are illustrated together with their approximate counterparts in Figure 1 for all generations with a life expectancy between 80 and 100.[17]

Comparing the values one can note that the approximations in equations (17) and (19) are not perfect, in particular in as far as the starting values for $\omega^c(0) = 60$ are concerned. Nevertheless, the approximations provide the right order of magnitude and they also track quite accurately the slope of the IRR over time. Different numerical examples have confirmed this conclusion.[18]

I have also looked at a third policy that is a mixture of policies A and B. In particular,

[16]In doing so I have also used (for policy A) the exact values for $R^c(t)$ (based on $R^p(t) = \omega^p(t) + \frac{1}{n}\ln\left[\frac{1+\hat{z}}{1+e^{n\omega^p(t)}\hat{z}}\right]$ in equation (14)).

[17]Note again that a value of $\omega^c(t) = 60$ corresponds to a life expectancy of 80 and a value of $\omega^c(t) = 80$ to a life expectancy of 100.

[18] Only for extreme cases one can observe some change of patterns. In contrast to the approximated expressions (17) and (19) it comes out, e.g., that for large values of n and $\omega^c(t)$ the exact internal rates of return might *decrease* in γ. For reasonable parameter values, however, this is not likely to happen and for decreasing cohort size ($n < 0$) it can be precluded altogether.

this policy stipulates an increase in the retirement age as would be stabilizing for the case where $n = 0$ (i.e. $R^p(t) = \frac{\omega^p(t)}{1+\hat{z}}$) while achieving the rest of the adjustment needs by relying on changes in the pension level according to (18). This can be viewed as a policy that uses increases in the retirement age to counter increases in longevity and changes in pension benefits to deal with fluctuations in cohort sizes. For constant cohort sizes ($n = 0$) this policy obviously coincides with policy A. For $n \neq 0$ approximations similar to the ones used for (17) lead to an IRR that is identical to the one of policy A, i.e. $\sigma(t) \approx g + n + \gamma \frac{2}{\omega^c(t)}$. If one calculates exact numerical solution, however, one can observe that for $n > 0$ the IRR lies above the value for policy A while for $n < 0$ the opposite is the case. This is due to the fact that for $n > 0$ ($n < 0$) the increase in $R^p(t)$ given by (14) is smaller (larger) than $R^p(t) = \frac{\omega^p(t)}{1+\hat{z}}$ and therefore also the "introductory gains" are smaller (larger) than for policy A. Summing up one can state that the impact of increasing longevity is largest for the pension policy that involves the fastest extension of the contribution base.

C Longevity has an upper limit

It is possible to doubt whether the assumption about a constantly increasing longevity (cf. (4)) is reasonable. Taken literally it seems inconceivable to assume that life expectancy or the maximum age increases without bound. On the other hand, even though one would not believe that such development can go on forever, the history of the last decades and the forecast over the next 50 years is nevertheless best described by the assumption of a constant linear increase.[19] In fact, a similar controversy has already been raised by the publication of Samuelson's original article where the population growth rate has been the focus of the discussion. In particular, Lerner has criticized the assumption of a constantly growing population as a "mirage" and a "chain letter swindle" , arguing that the biological interest rate will collapse once the growth will come to an end (Lerner 1959, p.523f.). While a thorough treatment of this issue is beyond the scope of this paper, I want to briefly sketch in this appendix the result for the case where one assumes that longevity reaches a maximum age ω^{max} in some period $\hat{t}$. In particular this means that $\omega^c(t) = \omega^c(0) + \gamma \cdot t$ for $t < \hat{t}$ and $\omega^c(t) = \omega^c(\hat{t}) = \omega^{max}$ for $t \geq \hat{t}$.

In order to study the impact of this alternative life expectancy development one has to first consider what it implies for the pattern of period life expectancy. For $t < \hat{t}$ it still

[19] On this discussion see, e.g., Oeppen & Vaupel (2002), Carnes et al. (2003) and the literature cited therein.

holds (cf. (5)) that $\omega^p(t) = \frac{\omega^c(t)}{1+\gamma}$. For $\hat{t} \leq t < \hat{t} + \omega^{max}$ period life expectancy continues to grow at speed $\frac{\gamma}{1+\gamma}$, i.e. $\omega^p(t) = \omega^p(\hat{t}) + \frac{\gamma}{1+\gamma}(t - \hat{t})$. Finally, for $t \geq \hat{t} + \omega^{max}$ one has that period life expectancy is constant at $\omega^p(t) = \omega^{max}$.

Under policy A (and for the case where $n = 0$) until the stop in longevity people will set their retirement age according to (15), i.e. (written in terms of $\omega^p(t)$): $R^c(t) = \frac{(1+\gamma)\omega^p(t)}{1+\hat{z}(1+\gamma)}$. From period $t = \hat{t}+\omega^{max}$ onwards period longevity does not increase anymore and it settles at ω^{max}. The retirement age in this period is given by: $R^p(\hat{t} + \omega^{max}) = \frac{\omega^p(\hat{t}+\omega^{max})}{1+\hat{z}}$ and from this one can calculate that the generation that retires in this moment is generation $\hat{t}+\omega^{max} - R^p(\hat{t}+\omega^{max}) = \hat{t} + \frac{\hat{z}}{1+\hat{z}}\omega^{max}$. The cohort-specific retirement ages are thus given by: $R^c(t) = \frac{\omega^p(t)(1+\gamma)}{1+\hat{z}(1+\gamma)}$ for $t < \hat{t} + \frac{\hat{z}}{1+\hat{z}}\omega^{max}$, and $R^c(t) = \frac{\omega^{max}}{1+\hat{z}}$ for larger t.

From these values one can derive approximate value for the internal rate of return $\sigma(t)$ as:

$$\begin{array}{lclll} \sigma(t) & \approx & g + \gamma\frac{2}{\omega^c(t)} & \text{for} & t \leq \hat{t}, \\ \sigma(t) & \approx & g + \gamma\frac{2}{\omega^{max}}\left(1 - (t-\hat{t})\frac{1+\hat{z}}{\hat{z}\omega^{max}}\right) & \text{for} & \hat{t} < t \leq \hat{t} + \frac{\hat{z}}{1+\hat{z}}\omega^{max}, \\ \sigma(t) & = & g & \text{for} & t > \hat{t} + \frac{\hat{z}}{1+\hat{z}}\omega^{max}. \end{array}$$

In other words, if longevity reaches an upper limit then the internal rate of return will decrease from the higher level $\sigma(t) \approx g + \gamma\frac{2}{\omega^c(t)}$ (see equation (17)) to the normal steady state level ($\sigma(t) = g$) in a smooth fashion. The longevity-related part of the biological interest will have a positive effect on all generations up to generation $\hat{t} + \frac{\hat{z}}{1+\hat{z}}\omega^{max}$ without involving below-normal rates of return for any other generation.

D Optimal behavior

In order to derive the optimality conditions for the standard retirement model I follow the set-up presented in Bloom et al. (2007). As stated in section 4 the intertemporal utility function for the representative member of generation t is given by:

$$\mathbb{U}(t) = \int_0^{\omega^c(t)} e^{-\delta a} \left[U(C(t,a)) - \chi(t,a)V(\omega^c(t),a)\right] \mathrm{d}a, \tag{34}$$

where δ is the rate of time preference, $C(t,a)$ the level of consumption of cohort t at age a, $\chi(t,a)$ is an indicator variable with $\chi(t,a) = 1$ for $a \in [0, R^c(t)]$ and $\chi(t,a) = 0$ for $a \in [R^c(t), \omega^c(t)]$ and $V(\omega^c(t),a)$ is the disutility of labor schedule that might also depend on longevity $\omega^c(t)$. Bloom et al. (2007) assume that at each age the health status improves proportionally with longevity. In other words, disutility of labor $V(\omega^c(t),a)$ is assumed to be homogeneous of degree 0, i.e. $V(\alpha\omega^c(t), \alpha a) = V(\omega^c(t), a)$ for $\alpha > 0$. For later derivations they assume a simple explicit form that possesses this property:

$$V(\omega^c(t), a) = \upsilon e^{\psi \frac{a}{\omega^c(t)}}. \tag{35}$$

In the paper I have focused on the case where $\psi = 0$ and $V(\omega^c(t), a) = \upsilon$. The budget constraint is given by (23) in the text and repeated here for convenience:

$$\frac{dA(t,a)}{da} = \chi(t,a)W(t+a) + rA(t,a) - C(t,a), \tag{36}$$

where $A(t,a)$ are the assets of generation t at age a and r is the (exogenously given) interest rate.

Individuals must take decisions about their consumption path and their retirement age (i.e. their control variables are C and χ). Bloom et al. (2007) state the Hamiltonian for this problem and they show that the first-order conditions can be summarized as:

$$\dot{C}(t,a) = \frac{dC(t,a)}{da} = (r-\delta)\frac{U'(C(t,a))}{-U''(C(t,a))} \tag{37}$$

$$\chi(t,a) = 1 \Longleftrightarrow U'(C(t,a))W(t+a) \geq V(\omega^c(t), a). \tag{38}$$

If one assumes that agent have constant relative risk aversion with $U(C(t,a)) = \frac{C(t,a)^{1-\beta}}{1-\beta}$ for $\beta \geq 0$ and $U(C(t,a)) = \ln(C(t,a))$ for $\beta = 1$ the first condition (37) can be written as:

$$\frac{\dot{C}(t,a)}{C(t,a)} = \frac{r-\delta}{\beta}. \tag{39}$$

This implies that:

$$C(t,a) = C(t,0)e^{\frac{r-\delta}{\beta}a}.$$

The initial level of consumption can be calculated from the intertemporal budget constraint:

$$\begin{aligned}
\int_0^{\omega^c(t)} e^{-ra} C(t,a)\,\mathrm{da} &= \int_0^{R^c(t)} e^{-ra} W(t+a)\,\mathrm{da} \Longrightarrow \\
\int_0^{\omega^c(t)} e^{-ra} C(t,0) e^{\frac{r-\delta}{\beta}a}\,\mathrm{da} &= W(t)\int_0^{R^c(t)} e^{-ra} e^{ga}\,\mathrm{da} \Longrightarrow \\
C(t,0)\frac{\beta\left(1-e^{-\frac{\omega^c(t)(\delta-(1-\beta)r)}{\beta}}\right)}{\delta-(1-\beta)r} &= W(t)\frac{-e^{-(r-g)R^c(t)}+1}{r-g}.
\end{aligned} \tag{40}$$

where $W(t+a) = W(t)e^{ga}$.

The second first-order condition (38) states that individuals will work at age a if the utility gain from consumption purchased by the wage exceeds the disutility of work. The retirement age is thus given by $V(\omega^c(t), R^c(t)) = U'(C(t, R^c(t)))W(t + R^c(t))$. Using the functional form (35) this can be written as:

$$\upsilon e^{\psi \frac{R^c(t)}{\omega^c(t)}} = W(t)e^{R^c(t)g} \left(C(t,0)e^{\frac{r-\delta}{\beta} R^c(t)} \right)^{-\beta}. \tag{41}$$

Equations (40) and (41) are two equations in two unknowns (the retirement age $R^c(t)$ and the initial consumption level $C(t, 0)$). The solutions to these equations together with the equation $C(t, a) = C(t, 0)e^{\frac{r-\delta}{\beta} a}$ determines the path of consumption and the optimal retirement age. Bloom et al. (2007) derive approximate solutions for specific parameters ($\beta = 2$ and $\beta = 1$).

One can also look at the special case with $\delta = r = g = 0$ and $\psi = 0$. In this case it holds that $C(t, a) = C(t), \forall a$ and the two conditions reduce to $C(t) = W \frac{R^c(t)}{\omega^c(t)}$ and $\upsilon = WC^{-\beta}$. This can be solved as $R^c(t) = \omega^c(t)(W)^{\frac{1-\beta}{\beta}} (\frac{1}{\upsilon})^{\frac{1}{\beta}}$. One can see that in general changes in wages will have an income and a substitution effect for the retirement age. A higher wage increases the incentive to prolong the working period (the substitution effect) while at the same time also raising lifetime income which increases the demand for earlier retirement (the income effect). For the log utility function ($\beta = 1$) these two effects cancel and the retirement choice is independent of the wage level. This has been assumed in section 4 of the paper.

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May 12, 2014	Saleem A. Bahaj	191	Systemic Sovereign Risk: Macroeconomic Implications in the Euro Area
May 16, 2014	John Bagnall, David Bounie, Kim P. Huynh, Anneke Kosse, Tobias Schmidt, Scott Schuh and Helmut Stix	192	Consumer Cash Usage: A Cross-Country Comparison with Payment Diary Survey Data
May 19, 2014	Konstantins Benkovskis Julia Wörz	193	“Made in China” - How Does it Affect Measures of Competitiveness?
June 25, 2014	Burkhard Raunig, Johann Scharler and Friedrich Sindermann	194	Do Banks Lend Less in Uncertain Times?
July 28, 2014	Martin Feldkircher and Florian Huber	195	The International Transmission of U.S. Structural Shocks – Evidence from Global Vector Autoregressions
September 16, 2014	Kim P. Huynh, Philipp Schmidt-Dengler, Helmut Stix	196	The Role of Card Acceptance in the Transaction; Demand for Money
October 10, 2014	Martin Brown, Helmut Stix	197	The Euroization of Bank Deposits in Eastern Europe
October 17, 2014	Ludmila Fadejeva, Martin Feldkircher, Thomas Reininger	198	Spillovers from Euro Area and U.S. Credit and Demand Shocks: Comparing Emerging Europe on the Basis of a GVAR Model
December 18, 2014	Esther Segalla	199	Shock Transmission through International Banks: Austria
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May 19, 2015	Markus Knell	201	The Return on Social Security with Increasing Longevity

Call for Entries:
Visiting Research Program

The Oesterreichische Nationalbank (OeNB) invites applications from external researchers for participation in a Visiting Research Program established by the OeNB's Economic Analysis and Research Department. The purpose of this program is to enhance cooperation with members of academic and research institutions (preferably post-doc) who work in the fields of macroeconomics, international economics or financial economics and/or with a regional focus on Central, Eastern and Southeastern Europe.

The OeNB offers a stimulating and professional research environment in close proximity to the policymaking process. Visiting researchers are expected to collaborate with the OeNB's research staff on a prespecified topic and to participate actively in the department's internal seminars and other research activities. They are provided with accommodation on demand and have, as a rule, access to the department's data and computer resources and to research assistance. Their research output will be published in one of the department's publication outlets or as an OeNB Working Paper. Research visits should ideally last between 3 and 6 months, but timing is flexible.

Applications (in English) should include

- a curriculum vitae,
- a research proposal that motivates and clearly describes the envisaged research project,
- an indication of the period envisaged for the research stay, and
- information on previous scientific work.

Applications for 2015 should be e-mailed to *eva.gehringer-wasserbauer@oenb.at* by November 1, 2015.

Applicants will be notified of the jury's decision by mid-December. The following round of applications will close on May 1, 2016.