Modeling Dependent Credit Risks for Application to Off-Site Banking Supervision

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During the past five years the Oesterreichische Nationalbank (OeNB), together with the Austrian Financial Market Authority (FMA) and university experts, has developed and implemented several modern tools for the purposes of off-site banking analysis and supervision. One of these tools is the Value-at-Risk (VaR) model, which allows for the standardized quantification of every single bank's economic capital. Within this portfolio model framework, a total VaR is calculated as an aggregation of credit, market and operational VaR, assuming perfect correlation between the risk categories. The methodology for measuring the credit risk of a bank's portfolio is currently based on the standard CreditRisk+ model, an actuarial model for aggregating risks in a credit portfolio with a single risk factor.

In 2005 the OeNB and the Vienna University of Technology launched a research project with the aim of developing an extended version of the credit risk model that is able to account better for portfolio diversification effects. As the background risk factors in the standard CreditRisk+ model have to be orthogonal, resemblance to real-world industrial sectors or other macroeconomic factors, which often appear to be strongly correlated, is not possible. This paper gives an overview of our approach to modeling correlations among systematic risk factors. Other extensions of the model, like the ability to calculate a single obligor's risk contribution and the incorporation of stochastic loss given default, are touched upon.

1 Introduction

As on-site audits take a long time and require substantial amounts of resources, and as they cannot be carried out very frequently because there are so many banks in Austria, off-site analysis plays a major role in the supervision process. Therefore, the Oesterreichische Nationalbank (OeNB), together with the Austrian Financial Market Authority (FMA) and university experts, launched several projects in the recent years with the aim of developing modern tools for sound single-bank risk quantification. These models use supervisory reporting data and allow for the identification of possible bank problems in a standardized way. The timely anticipation of risk potentials and imminent bank problems is an essential prerequisite for maintaining the country's financial stability.

One of these new off-site analysis tools is a portfolio model which makes it possible to estimate every single bank's economic capital; it is set to cover the total losses over a one-year horizon with a certain probability. Both Value-at-Risk (VaR) and Expected Shortfall (ES) have become the most common measures for quantifying economic capital. They not only make it possible to quantify risks in the individual risk categories, but also to handle them in an aggregated manner: the total VaR is calculated as an aggregation of credit, market and operational VaR, assuming perfect correlation between the risk catego-

ries. The comparison of the total possible loss for the next year at a certain confidence level with all available capital reserves enables us to draw conclusions on the risk-bearing capacity of every single bank in Austria (see OeNB and FMA, 2004).

To calculate the credit VaR and to assess credit risk, we currently use a model based on the standard CreditRisk[+] model, an actuarial model for aggregating risks in a credit portfolio, with a single risk factor. In this framework, dependence between obligors arises implicitly due to a single systematic risk factor, which drives the obligors' probabilities of default. The conventional CreditRisk[+] approach allows for more than one common factor; these factors have to be statistically independent. The orthogonality of the background risk factors hinders any resemblance to real-world macroeconomic factors or industry sectors, which often appear to be strongly correlated.

In 2005 the OeNB and the Vienna University of Technology launched a research project[2] with the aim of developing an extended version of the credit risk model that is able to account better for portfolio diversification effects. As the background risk factors in the standard CreditRisk[+] model have to be orthogonal, resemblance to real-world industrial sectors or other macroeconomic factors, which often appear to be strongly correlated, is not possible. This paper gives an overview of our approach to the incorporation of correlations among systematic risk factors. Other extensions of the model, like the ability to calculate a single obligor's risk

contribution and the incorporation of stochastic loss given default, are touched upon.

1.1 An Overview of the Model

The consideration of risk factor dependencies represents the main extension of the new approach in comparison to the standard CreditRisk[+] model and the numerically stable implementation currently in use. We use two multivariate factor distributions, which incorporate factor correlations. The moment-generating functions of both distributions have a closed analytical form, which fit into the framework of the standard model and can be handled using the recursion algorithms similar to the ones developed for the previous implementation. The parameters of the new distributions then have to be fitted to the covariance matrix of the risk factors.

First, we consider a multivariate gamma distribution of the following form: the dependence between the sectors results from the common dependency on one (hidden) background variable; that is why this model is called the "hidden gamma" model. As in Giese (1996), constraints on the model's correlation parameters impose a very heterogeneous structure on the default correlation between obligors in different sectors and render the calibration to a target covariance matrix very difficult – there will only be rare cases where the assumed covariance matrix provides a good approximation for the true covariance matrix to be modeled. The hidden gamma distribution can provide a good fit only if

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[2] *The project was headed by Professor Uwe Schmock, Institute of Mathematical Methods in Economics, Research Unit Financial and Actuarial Mathematics, Vienna University of Technology.*

factors with high variances are also significantly more strongly correlated than other factors.

The second multivariate factor distribution we consider is the compound gamma distribution. It has a more convenient covariance structure. In this case the gamma distributions themselves are mixture distributions, where the factor variables are independently gamma distributed, conditional on a positive gamma-distributed random variable T (in this case, the shape parameters of the factors' gamma distributions, not the sector variables themselves, are uniformly scaled by T). The compound gamma model produces less artificial correlation structures than the one produced by the hidden gamma distribution.

The parametric characteristics of these two distributions allow the recursion algorithm already used for the single factor model to be used. The model calibration, on the other hand, is not straightforward. Fitting the model to an externally given covariance structure is difficult, and because of the parameter restrictions, dependencies cannot always be matched sufficiently well.

The second important extension of the model is its possibility to calculate individual obligors' risk contributions. For further portfolio analysis, the expected shortfall of a certain level can be decomposed into risk contributions by single obligors and risk sectors, in a way that detects the impact on the portfolio risk by subportfolios (see Tasche, 2004, and Schmock, 2006).

Finally, the standard framework is extended to allow for stochastic loss given default. When collateral is used, the risk becomes twofold: First, there is uncertainty with respect to the ability to access collateral and to the costs required to sell it. Second, there is uncertainty with respect to the market value and liquidity of collateral. Therefore, the use of collateral to mitigate credit risk causes additional loss given default risk, which can be accounted for by a stochastic loss given default rate. Currently, binomial and some empirical distributions can be handled.

1.2 Related Research

Bürgisser et al. (1999) suggest an approach to calibrating the single-factor sector variance in a way that accounts for sector dependencies. In Giese (1996), this model is compared to the hidden gamma and the compound gamma models: The compound gamma model represents a reasonable trade-off in comparison to the other models mentioned, displaying consistently fatter tails than the standard CreditRisk$^+$ model and the single factor model of Bürgisser et al. Compared to the hidden gamma model, the compound gamma model smoothes the heterogeneity of the original covariance matrix and because of this produces less fat tails than the hidden gamma model and in most cases the best fit to the empirical covariance structure.

A recent approach to a general framework for calibrating dependent credit risk models is described in Gusso (2003). It comprises two urn models for the joint probability of defaults of dependent credit risks and introduces an estimation approach based on the expectation-maximization algorithm.

1.3 Applications to Off-Site Banking Analysis and Supervision

First, we use the new model to calculate the credit loss distribution of every single bank in Austria, but also of a benchmark portfolio consisting of all banks' loan exposures, on a quarterly basis in a standardized way starting with the first quarter of 2007. Our aim is to assess the credit risk of individual banks and to gain some insights about the credit risk situation of the banking system as a whole.

Second, credit loss at a certain confidence level is aggregated together with market loss and the loss from operational risk at the same confidence level in order to derive the total loss of a single credit institution, assuming perfect correlation between the three risk categories. The total loss is then compared to the bank's capacity to cover losses quantified by its available capital reserves of different quality. Probabilities of different levels of financial distress are calculated, for example the probability of losses exceeding the level which was provided for or the probability that components of balance sheet equity (excess equity) need to be used to cover the losses, with the bank continuing to exist, and so on (see OeNB and FMA, 2004). Additionally, the main risk contributors can be identified for every single bank.

Third, various scenario analyses could also be performed to deliver information about possible threats to the soundness of banks.

The model is implemented in Java and can be run via a standard user interface. The risk factor variances and the parameters driving the factor correlation can be estimated using Matlab.

All consolidated findings about possible bank problems are disclosed quarterly in a series of standard reports which contain detailed information on our risk measures.

2 Data

The main sources of data used are supervisory data from the monthly balance sheet reports (monthly reports) to the OeNB and the database of the OeNB's Major Loans Register. In addition, we use default frequency data in industry groups from the Austrian rating agency Kreditschutzverband (KSV).

Credit and financial institutions are obliged to report major loans to the OeNB monthly. This reporting obligation exists if credit lines granted to or utilized by a borrower exceed EUR 350,000. The Major Loans Register covers about 80% of the total loan volume of Austrian banks, but its level of individual coverage may be very low, especially in the case of small banks.

In addition to balance sheet data, monthly reports contain a fairly extensive assortment of other data that are required for supervisory purposes, including capital adequacy figures as well as figures for loans and for deposits with various maturity buckets.

The data provided by monthly reports and the Major Loans Register provide us with detailed information on the banks' loan portfolios. This database contains all loans exceeding a volume of EUR 350,000 on an obligor-by-obligor basis; an approximation of the volume under this threshold is made on the basis of a report that is part of the monthly report and that provides the number of loans to domestic nonbanks for different volume buckets. No comparable sta-

tistics are available for nondomestic loans. However, one can assume that most of cross-border lending exceeds the threshold of EUR 350,000, and hence the risk associated with non-domestic loans below this threshold appears negligible.

As risk factors are set to be the industry sectors in our framework, we assign each loan to an industry sector. The definition of these industry sectors is based on the NACE classification[3] of the debtors and is not hard-coded in the implementation. Currently, for our test purposes we have defined four risk sectors on the basis of the NACE code: basic industries; production, trading and other services; public services; and a residual sector. Since only loans above the threshold volume are reported to the Major Loans Register, we assign the loans below this threshold to the residual sector. Nondomestic loans have to be assigned to the residual sector because of a lack of information about the respective industry affiliation.

The probability of default of an individual loan depends on the rating which the bank assigns to the respective customer and the default frequency of the industry sector the customer belongs to. The bank's rating is reported to the Major Loans Register and is mapped onto a master scale within the OeNB, which makes it possible to assign a probability of default to each loan. The default frequency data are from the KSV. The KSV database provides us with time series of insolvencies and the total number of firms in most NACE branches at a quarterly frequency. This allows us to calculate a time se-

ries of historically observed default frequencies for our industry sectors. To construct insolvency statistics for the residual sector, for which no reliable information on the number of insolvencies and the total number of firms is available, we take averages from the data that are available.

3 Description of the Model

The CreditRisk[+] model is an actuarial model for aggregating risks in a credit portfolio with only little data about the obligors, a situation quite common for regulators. It was introduced by Credit Suisse First Boston and it is broadly accepted as a portfolio credit risk model. The main input data needed are expected default probabilities and exposures. Furthermore, one or more independent risk factors are introduced in the basic model. These risk factors scale the expected default probabilities randomly and should reflect changes in obligors' creditworthiness. The distribution of the risk factors is assumed to be a gamma distribution with expectation one and variance σ_k varying over risk factors 1 to K. The risk of a single obligor can depend on more than only one risk factor, which is represented by weights measuring the risk affiliation of obligors to specific risk factors. In order to facilitate calculations and to make analytical expressions possible, the distribution of the number of defaults conditional on the realization of the risk factor (i.e. for fixed risk factors) is approximated by a Poisson distribution. The distribution of the number of defaults, finally, turns out to be the convolution of K negative binomial distributions

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[3] NACE: Nomenclature statistique des Activités économiques dans la Communauté Européenne — Statistical classification of economic activities in the European Community.

arising from the mixing of the conditional Poisson distribution with the gamma distribution of the risk factors. Furthermore, the classical model only allows for fixed losses given default, which are rounded to multiples of a common loss unit in order to reduce the number of calculations needed for the evaluation. Finally, all the data can be aggregated in a probability-generating function of the total loss, out of which a recursive, numerically stable algorithm calculates the distribution of the total loss (see Haaf et al., 2004).

3.1 The Single Factor Model Currently in Use

The model currently in use for the off-site analysis of banks assumes that all stochastic changes of the default rates are driven by a single risk factor. This implies that economic booms and recessions affect all obligors equally. Mathematically, these model assumptions lead to relatively high covariances that probably overestimate the true dependencies of defaults. Losses given default are assumed to be deterministic (a fixed ratio of the exposure).

3.2 The Extended K-Factor Model

Various extensions of the model have been considered. A first step was to implement a model featuring various risk factors. Risk affiliations can be set such that obligors depend on various risk factors. This way of modeling results in diversification effects that lower measures of extreme events, such as VaR. In practice, groups of industry sectors are modeled to depend on one risk factor for each group. This leads to zero covariance between the groups and relatively high covariance within groups. Modeling a certain degree of depen-

dence between groups requires models of dependent risk factors.

3.2.1 Modeling of Dependencies

Dependencies between risk factors are modeled in two ways, as proposed in an article by Giese (1996).

In the hidden gamma model, the risk factors depend on a common random variable, which we will also call a risk factor. A certain degree of dependence can be introduced in this way. The possibilities of changing the magnitude of dependence and the flexibility of the dependence structure are limited.

An additional random variable that scales the distributional parameters of the risk factors is introduced into the compound gamma model. The variance of the random variable induces a covariance of the risk factors and thereby dependencies between the defaults. It is clear that this single additional parameter, which can only take values in a certain range, will not be sufficient to model an arbitrary covariance structure.

Nevertheless, it is possible to model various degrees of dependence of risk factors between the extremes represented by the single factor model and the K-factor model with independence.

3.2.2 Stochastic Loss Given Default

Assuming independence between losses given default and all other random variables (including risk factors) makes it possible to extend the model in such a way that stochastic losses given default could be integrated. Different loss given default distributions depending on the risk affiliations can be modeled and thereby help to improve the model. Furthermore, using stochastic loss given default allows for stochastic round-

ing, a method significantly reducing the errors arising when rounding the individual exposures to the multiples of a common loss unit.

3.2.3 Risk Contributions

The Expected Shortfall (ES) risk measure was considered in addition to Value-at-Risk (VaR); see Acerbi and Tasche, 2002). Algorithms were implemented that precisely calculated contributions to ES by obligors or subportfolios. A detailed analysis of the portfolio composition based on a coherent risk measure is now possible.

4 Calibration [4]

We use data from the Austrian rating agency KSV for the calibration. These data consist of defaults reported quarterly and grouped by industry sectors. In a first step, we calculate default numbers on a yearly basis. Then we assign all domestic loans to three generic risk sectors. For nondomestic loans and all other loans without industry sector information, we introduce a residual sector. As no reliable information on the number of insolvencies and the total number of firms for the residual sector is available, we are not able to calibrate its variance and assume it to be equal to the highest of the other three sector variances.

The defaults of each industry sector are assumed to depend on one risk factor. In the calibration process, the parameter of the resulting distribution (negative binomial or Poisson) is estimated for each of the three industry sectors. In a first approach, these risk factors and thereby the defaults in different industry sectors are assumed to be independent.

4.1 Theoretical Background

4.1.1 Negative Binomial Distribution vs. Poisson Distribution

A negative binomial distribution with an expected value modeled to depend on the total number of companies registered in a given period is fitted to the data of yearly defaults. In this model the expected number of defaults in period i is given by $E[N_i]=\lambda T_i$. The variance of defaults in year i is given by $V[N_i]=\lambda T_i(1+\sigma^2\lambda T_i)$. For σ^2 equal to zero this corresponds to a Poisson distribution with expectation and variance given by $E[N_i]=V[N_i]=\lambda T_i$.

Each risk sector is analyzed in order to decide whether to use a Poisson or a negative binomial distribution. This is done by a test for overdispersion that uses the likelihood function.

4.1.2 The Likelihood Function

The likelihood function is defined as the product of the respective probability function evaluated at the realizations of the random variable. In the Poisson case this would be

$$L(N_1,...,N_n;\lambda) = \prod_{i=1}^{n} \frac{(\lambda T_i)^{N_i}}{N_i!}e^{-\lambda T_i}.$$

Under the assumption of independence of defaults in different periods, this results in the joint probability of the observed values given a Poisson distribution with the parameter λT_i in year i.

For the negative binomial distribution this corresponds to

$$L(N_1,...,N_n;\lambda,\sigma^2) =$$

$$= \prod_{i=1}^{n} \frac{\Gamma(N_i+\frac{1}{\sigma^2})}{N_i!\Gamma(\frac{1}{\sigma^2})}\left(\frac{1}{1+\lambda T_i \sigma^2}\right)^{\frac{1}{\sigma^2}}\left(\frac{\lambda T_i \sigma^2}{1+\lambda T_i \sigma^2}\right)^{N_i}.$$

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[4] *Because of space constraints it was not possible to describe the calibration approach in more detail. Please contact the authors for additional information about the technical details.*

It is obvious that the likelihood function of the Poisson distribution is easier to handle than the one of the negative binomial distribution. The parameters λ and σ^2 are chosen such that the likelihood of the given data is maximized.

4.1.3 Test of Overdispersion

A distribution whose variance exceeds its expectation is called overdispersed. The classical example of a distribution which is not overdispersed is the Poisson distribution where $E[N_i]=V[N_i]=\lambda T_i$, and an example of overdispersion is the negative binomial distribution where $V[N_i]=\lambda T_i(1+\sigma^2\lambda T_i)$, which is strictly greater than $E[N_i]$ if σ^2 is strictly greater than zero.

One test for overdispersion is the likelihood ratio test. The statistic is defined as

$$lr = -2\log\left(\frac{L^+}{L}\right),$$

where L^+ is the likelihood of the data under the hypothesis of Poisson distribution (using the parameter λ that maximizes this expression) and L is the likelihood under the hypothesis of negative binomial distribution (with the parameters maximizing the likelihood). The null hypothesis is the Poisson distribution, and it is rejected with a given level of significance α if the calculated value of lr exceeds the $1-\alpha$ quantile of the chi-square distribution with one degree of freedom. With this test we are able to detect whether the defaults in the sectors defined are significantly overdispersed.

On the other hand, the result of such a test can be reported using p values, where p is the probability of an occurrence of a more significant result (here: an unusual result under the null hypothesis). This is $p=P[\chi^2 > lr]$, where χ^2 has chi-square distribution with one degree of freedom. Small values of p support the alternative hypothesis (namely a negative binomial distribution), whereas relatively big values (greater than 10%) support the null hypothesis of a Poisson distribution.

4.1.4 Point Estimates

We use parameters that maximize the likelihood function as a point estimator for the parameters. These parameters are called maximum likelihood estimators (MLEs). First proper moment estimators are calculated using methods described in Mack (2002). This task is more complicated in a situation with varying numbers of registered firms than in a situation with a fixed number of firms. The likelihood is maximized numerically using these moment estimators as starting values.
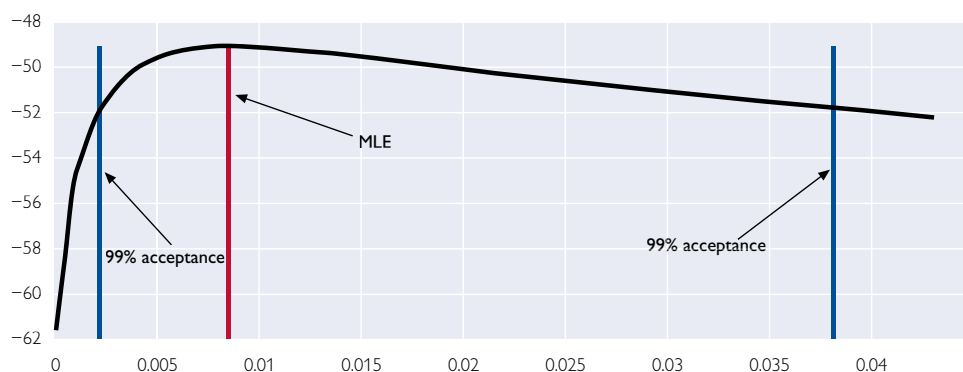
4.1.5 Confidence Intervals Based on the Normal Distribution

An MLE has the desirable property of being asymptotically unbiased, and the asymptotic distribution is a normal distribution. The variance of the arising normal distribution is the inverse of the Fisher information. This quantity is usually hard to calculate, but as described in Panjer and Willmot (1992), we use the observed information to calculate the asymptotic variance. An interval with a given level of confidence can then be calculated. As the sample is very small, the intervals are relatively wide and the lower interval bounds of nonnegative parameters even are calculated to be negative. The results have to be interpreted with caution, as the normality assumption holds asymptotically but not necessarily in a small sample.

Chart 1

Acceptance Interval for Maximum Likelihood Estimators (MLEs)

Source: Own calculations.

4.1.6 Confidence Intervals Based on the Likelihood Ratio Statistic

Another procedure to calculate intervals in which the true parameter is supposed to lie with a given probability is to choose the interval ends such that a likelihood ratio test with a given significance is still accepted. We numerically calculate values for σ^2 lower (σ_l^2) and higher (σ_u^2) than the MLE such that a likelihood ratio test with level of significance p is still accepted under the null hypothesis which postulates that σ_l^2 resp. σ_u^2 is the true parameter against the alternative of the MLE. For a confidence level of $\gamma=1-\alpha$ the significance of the test is chosen to be $p=\frac{\alpha}{2}$ corresponding to interval ends such that the true parameter lies within with probability γ (e.g. 99%). These intervals have the satisfactory properties of avoiding negative values, and they are asymmetric around the MLE corresponding to the speed at which the likelihood decreases.

Chart 1 plots the logarithm of the likelihood of a sample of defaults for varying values of σ^2. The estimator for which the log likelihood attains its maximum is indicated, and the values passing a likelihood ratio test of level of significance of 0.5% with the greatest distance to this MLE are indicated as well.

4.2 Effects of Different Calibrations

As the compound gamma distribution can be said to have a more convenient covariance structure then the hidden gamma distribution, and to save space, we present the results of our investigations only for the case of the compound gamma model.

We use a test portfolio consisting of all nondefaulted loans of the Major Loans Register as of December 2005. Every obligor is assigned to only one of four risk sectors. These four risk sectors (basic industries; production, trading and other services; public services; and a residual sector), as already mentioned before, are defined on the basis of the NACE code. The master scale probability of default is taken as the obligor's probability of default. A loss given default of 0% is assumed for the loan fraction covered by collateral and of 100% for the remaining loan fraction.

Table 1 presents effects of different VaR and ES calibrations for different confidence levels. First, the single factor model results are quoted for

Table 1

Effects of Different Parameter Calibrations on Loss

Model	Risk factor variances	Correlation parameter σ^2	ES 95%	ES 99%	ES 99.9%	VaR 95%	VaR 99%	VaR 99.9%
One factor	0.015		4,590	5,085	5,753	4,270	4,787	5,471
	0.250		7,703	9,569	12,070	6,498	8,447	11,013
Four factors	MLE	0.002	4,332	4,783	5,414	4,051	4,503	5,149
Variation of sector variance	confidence level 99%	0.002	4,436	4,899	5,540	4,142	4,615	5,269
	acceptance level 99%	0.002	4,496	4,970	5,618	4,193	4,682	5,343
	all equal to 0.25	0.002	5,994	7,055	8,452	5,301	6,422	7,866
Four factors	all equal to 0.25	0.000	5,979	7,037	8,432	5,289	6,406	7,846
Variation of correlation coefficient	all equal to 0.25	0.002	5,994	7,055	8,452	5,301	6,422	7,866
	all equal to 0.25	0.019	6,099	7,186	8,607	5,387	6,541	8,013
	all equal to 0.25	0.250	7,703	9,569	12,070	6,498	8,447	11,013

Source: Own calculations.

Note: ES stands for Expected Shortfall, VaR stands for Value-at-Risk; MLE stands for maximum likelihood estimator.
  Compound gamma model used in the four-factor case. Loss in billions.
  MLE for the four sector variances (0.0086, 0.0047, 0.0023, 0.0086).
  99% confidence level for MLE: (0.0270, 0.0134, 0.0150, 0.0270), and 99% acceptance level for MLE: (0.0381, 0.0186, 0.0188, 0.0381).

two different sector variances. Subsequently, figures for a four-factor compound gamma model are given first for different sector variances and then for different correlation parameters.

Until recently a lack of data prevented us from validating our VaR model. We tried to avoid underestimating correlation risk and used a relatively high risk factor variance for the single risk factor of 0.25.

New data have helped us develop a better and more flexible model, which can be validated (as will be de-
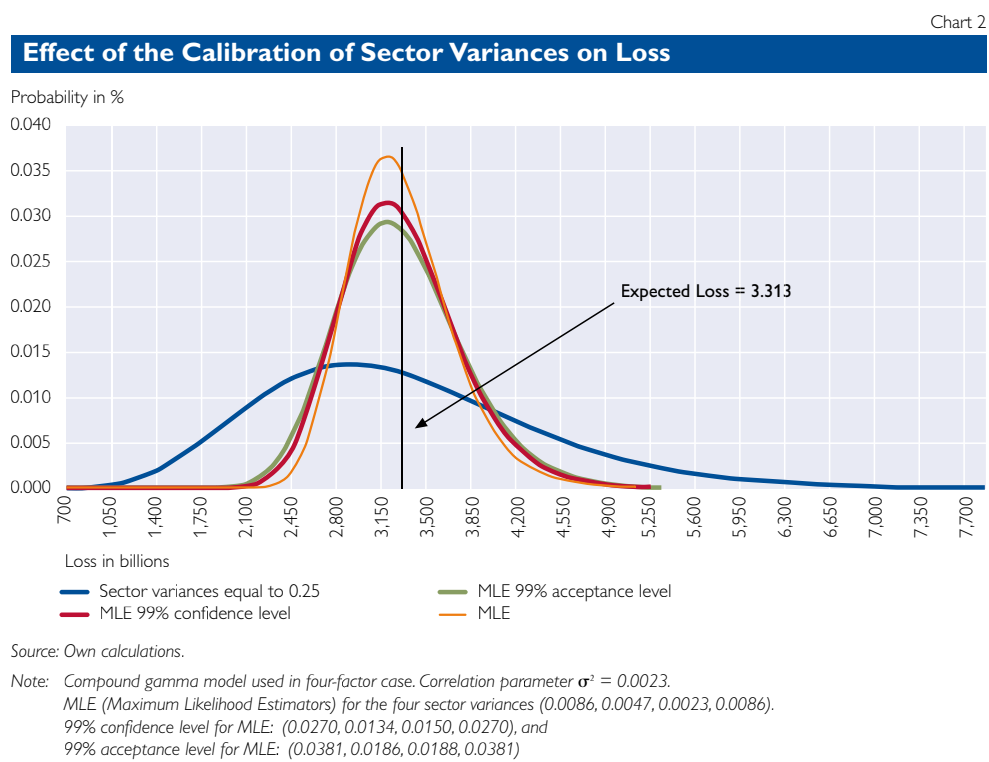
Chart 2

Effect of the Calibration of Sector Variances on Loss

Probability in %

Expected Loss = 3.313

Loss in billions

— Sector variances equal to 0.25
— MLE 99% confidence level
— MLE 99% acceptance level
— MLE

Source: Own calculations.

Note: Compound gamma model used in four-factor case. Correlation parameter σ^2 = 0.0023.
  MLE (Maximum Likelihood Estimators) for the four sector variances (0.0086, 0.0047, 0.0023, 0.0086).
  99% confidence level for MLE (0.0270, 0.0134, 0.0150, 0.0270)) and
  99% acceptance level for MLE (0.0381, 0.0186, 0.0188, 0.0381)

Chart 3

Effect of the Calibration of Correlation Parameters on Loss

Probability in %

Expected Loss = 3.313

Loss in billions

— Correlation parameter = 0.25 — Correlation parameter = 0.0186
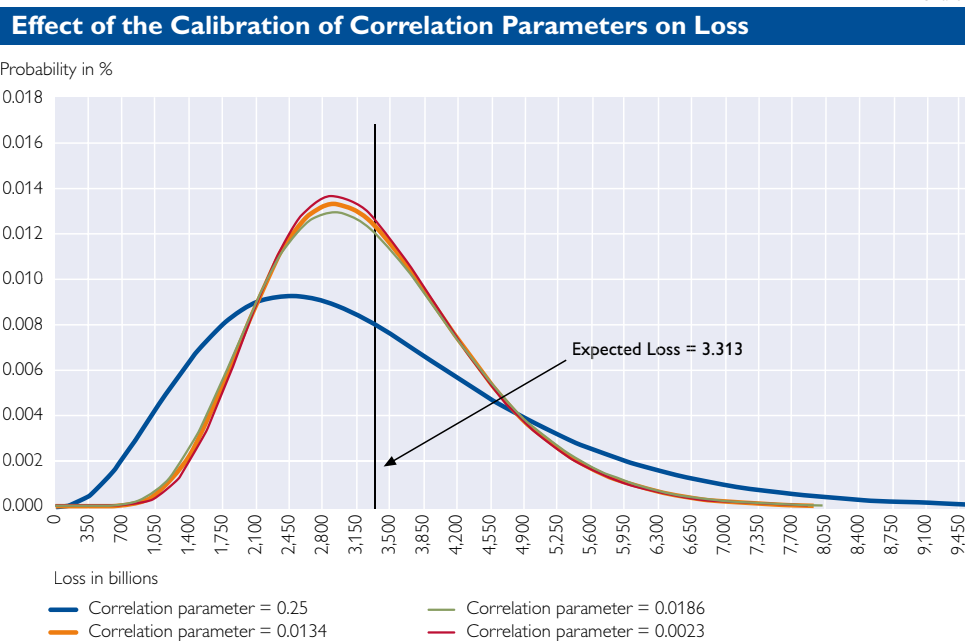— Correlation parameter = 0.0134 — Correlation parameter = 0.0023

Source: Own calculations.

Note: Compound gamma model used in calculations (correlation parameter σ^2). All sector variances equal to 0.25.

scribed in section 5). This will allow us to choose a less conservative and hence a more realistic parameter calibration, which will deliver more precise results.

Clearly, higher risk factor variances lead to higher losses. Higher correlations do so as well, but in a less pronounced way. The single factor model and the four-factor model with equal factor variances and a maximum possible correlation produce the same results. This may be surprising at first glance, but these models are mathematically equivalent.

Chart 2 shows the histograms of credit loss in billions with various calibration methods for the variances of the risk factors. A 99% confidence interval resulting from the normal distribution was calculated, as was a 99% acceptance interval from a likelihood ratio statistic.

Chart 3 presents the loss probability functions for different values of the covariance parameter σ^2. Higher values for σ^2 obviously lead to heavier-tailed loss distribution.

5 Validation

For validating purposes we use the power curve and the accuracy ratio (AR) concept. The reliability of the AR method is not always guaranteed, as our validation sample contains only a relatively small number of defaults. Nevertheless, the calculated AR gives an indication of the model's power and is a measure of how well it discriminates between defaulting (bad) and nondefaulting (good) banks (see Engelmann et al., 2003). The AR combines the discriminatory power of the model for every possible cutoff rate in one number, which varies between 0 (for a random model) and 1 (for a perfect model).

For our testing purposes, we calculated the ratio of a single bank's credit VaR at the 95% confidence level and related it to the bank's capacity to cover losses as measured by

Chart 4

Validation Results

Defaults in %

Number in %

— Random model   — Case 1   — Case 2   — Optimal model
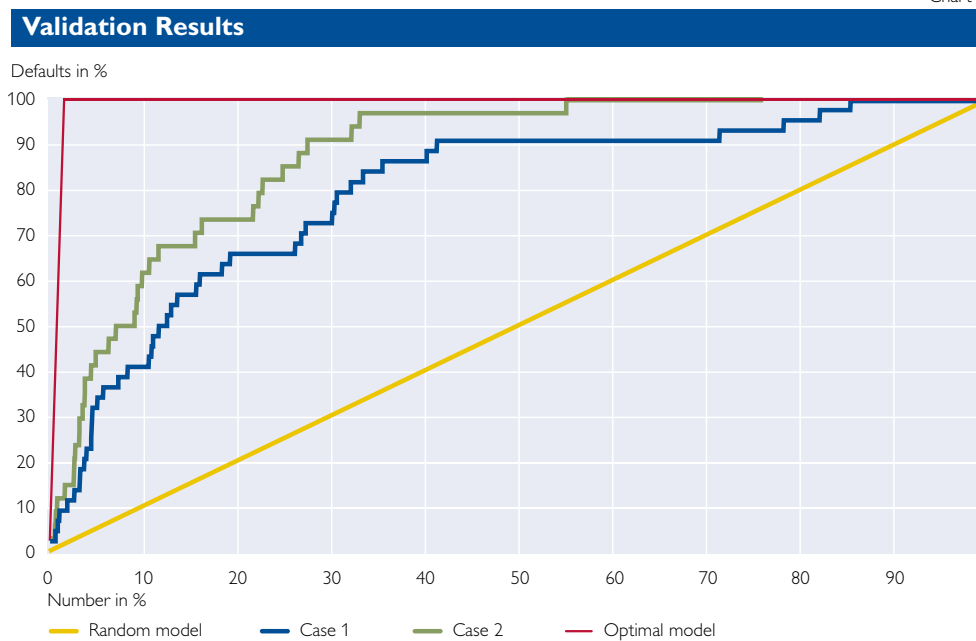
Source: Own calculations.

its available capital reserves and excess equity. We did this for all banks and all quarters of 2003 (no data are available to calculate results further back). Then we compared our results with the list of banks which actually experienced problems during 2004.

Chart 4 presents the cumulative accuracy profiles for two test cases. The first case describes the discriminatory power of the model when all banks are taken into account. In this case, an AR of 60.5% could be achieved. The second case relates to credit institutions for which more than 50% of the total loan volume is covered by the Major Loans Register. The test reveals that in this second case, significant power improvements could be achieved. An AR of 69.6% was even obtained.

The reason for the better performance in the second case is the fact that very little data are available for small-scale loans which do not have to be reported to the major loan register. The lack of essential information about the individual loan amount, the probability of default and collateral value makes it impossible for us to gain insights into the potential riskiness of portfolios consisting mainly of such loans.

Our validation results are a good example that more detailed data on portfolio exposures will contribute to the improvement of the model performance. But starting with 2008, banks will be obliged to report more detailed data on loans below the Major Loans Register threshold and will provide more detailed data on obligors' collateral type. This additional information is expected to increase our model's discriminatory power even more.

6 Conclusion

We have been able to calculate the loss distribution for a more realistic factor distribution than that of a single factor model while remaining within the analytical framework. The possibility of calculating a single obli-

gor's risk contributions allows us to obtain important information about the main single risk drivers. Incorporating stochastic loss given default rates into the model accounts for the uncertainty caused by handling collaterals. All these results can be obtained with a relatively low computational effort within several minutes, even for very large portfolios.

Further investigation should be made to extend the framework to a general approach in a way that allows for less constrained and hence economically more adequate modeling of risk factor dependencies, and also for more flexible calibration methods. The economic implications of different dependency estimations should be studied in more detail, and more reliable validation results are needed. In an even more realistic model, we would want to consider dependencies also among recovery rate categories and among severity and default risk.

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