An Empirical Analysis of the Network Structure of the Austrian Interbank Market[1]

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We provide an empirical analysis of the network structure of the Austrian interbank market based on a unique data set of the Oesterreichische Nationalbank (OeNB). The analysis relies on the idea that an interbank market can be interpreted as a network where the banks form the nodes and the claims and liabilities between them define the edges of the network. This approach allows us to apply results from general network theory, which is widely applied in other scientific disciplines – mainly in physics. Specifically, we use different measures from this network theory to investigate the empirical network structure of the Austrian banking system. We focus on the question of how this structure affects the stability of the network (the banking system) with respect to the elimination of a node in the network (the default of a single bank). Regarding the network structure, we find that there are very few banks with many interbank linkages whereas there are many with only a few links. This feature of networks has been repeatedly found to be conducive to the robustness of the network against the random breakdown of links (the default of single institutions due to external shocks). In addition, the interbank network shows a community structure that exactly mirrors the regional and sectoral organization of the current Austrian banking system. Moreover, the banking network has typical structural features found in numerous other complex real world networks: a low clustering coefficient and a relatively short average shortest path length. These empirical findings are in marked contrast to network structures that have been assumed in the theoretical economic and econo-physics literature.

Introduction

Safeguarding the stability of the financial system is one of the core tasks of central banks. They are therefore mainly concerned with problems of *systemic risk,* i.e. the risk of a large-scale breakdown of financial intermediation. Systemic risk is a key issue in banking and has two main components: The exposure of banks to common risk factors and the danger of *domino effects* of insolvencies. These domino effects play an important role in the banking system because banks are linked by a complex system of mutual credit relations. In such a system the insolvency of one institution can affect the financial positions of others and in a chain reaction increase financial distress in the banking system as a whole. From an abstract viewpoint, the system of mutual credit relations between financial institutions can be viewed as a *network* where banks form the nodes of the network and their interbank relations form financial links which are the network's edges. From a financial stability point of view, it is interesting to understand how the structure of this interbank network affects the financial stability properties of the banking system as a whole. This paper takes a first step in this direction by uncovering the empirical structure of the Austrian interbank network as far as it can be reconstructed from the data reported to the Austrian central bank, the Oesterreichische Nationalbank (OeNB).

In our analysis we can draw on a rich set of results from other disciplines. Especially the physics community has largely contributed to the empirical analysis and to a functional understanding of the structure of complex real world networks in general (for an overview see Dorogovtsev and Mendes, 2003). One of the most important contributions to recent network theory seems to be the inter-

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pretation of network parameters with respect to the stability, robustness and efficiency of an underlying system (e.g. Albert et al., 2000). Clearly, these insights are relevant for the issues of financial stability and the network structure of mutual credit relations in the interbank market.

The network of mutual credit relations between financial institutions is considered to play a key role in the risk of contagious defaults. In the theoretical economic literature on contagion, some authors (e.g. Allen and Gale, 2000; Freixas et al., 2000; or Thurner et al., 2003) suggest network topologies that may be interesting to look at. Allen and Gale (2000) suggested studying a complete graph of mutual liabilities. The properties of a banking system with this structure are then compared to properties of systems with non-complete networks. In Freixas et al. (2000), a circular graph is contrasted with a complete graph. In Thurner et al. (2003), a much richer set of different network structures is studied. Yet, surprisingly little is known about the *actual* empirical network structure (technically also referred to as the *network topology*) of mutual credit relations between financial institutions. To our best knowledge the network topology of interbank markets has so far not been studied empirically.

In this paper we take a first step to fill this gap by analyzing a unique data set of the OeNB. Our main finding is that the network structure of the Austrian interbank market has a power law in the degree distribution. This means that there are very few banks with many interbank linkages whereas there are many with only a few links. This feature of networks has been repeatedly found to be conducive to the stability of the network against the random breakdown of links. In the present context, this means that – given the actually observed structure of interbank claims and liabilities – the banking system is relatively robust with respect to domino effects caused by the breakdown of single credit institutions which could ultimately lead to the collapse of the entire financial system. We furthermore find evidence of other features of the network – such as low clustering and the short average length of links between institutions – that confirm the general structural features of the interbank network found in the data. Finally, another important message of this work is that the rather large classes of potential networks can be narrowed to empirically relevant structures for the future modeling of interbank relations.

The Austrian Interbank Network

The interbank network is characterized by the liability (or exposure) matrix L. The entries L_{ij} are the liabilities bank i has vis-à-vis bank j. We use the convention of writing liabilities in the rows of L. If the matrix is read column-wise (transposed matrix L^T) we see the claims or interbank assets banks hold with each other. It must be noted that L is a square matrix but not necessarily symmetric. The diagonal of L is zero, i.e. no bank self-interaction exists. In the following we are looking for the bilateral liability matrix L of all (about $N = 900$) Austrian banks, the central bank (OeNB) and an aggregated foreign banking sector. Our data consist of 10 L matrices, each representing liabilities for quarterly single month periods between the years 2000 and 2003. To obtain the Austrian interbank network from central bank data we draw on two major sources: we exploit structural features of the monthly balance sheet returns of Austrian banks and the Major Loans Register in combination with an estimation technique.

For historical reasons, the Austrian banking system is organized in sectors. The large majority of banks belong to one of seven sectors: savings banks (S), Raiffeisen credit cooperatives (R), Volksbank credit cooperatives (VB), joint stock banks (JS), state mortgage banks (SM), building and loan associations (BLA), and special purpose banks (SP). Banks have to break down their balance sheet reports on claims and liabilities vis-à-vis other banks according to the different banking sectors, the central bank and foreign banks. This practice of reporting on balance interbank positions breaks the liability matrix L down to blocks of sub-matrices for the individual sectors. The savings banks and the Volksbank sector are organized along a two-tier structure with a sectoral head institution. The Raiffeisen sector is organized along a three-tier structure, with a head institution for every federal province of Austria. The provincial head institutions are subsumed under a central institution, Raiffeisenzentralbank (RZB), which is at the top of the Raiffeisen structure. Banks with a head institution have to disclose their positions vis-à-vis the head institution, which gives additional information on L. Since many banks in the system hold interbank liabilities only vis-à-vis their head institutions, it is possible to exactly pin down many entries in the L matrix. In a next step, this information is combined with the data from the OeNB's Major Loans Register. This register contains all interbank loans above a threshold of EUR 350,000. This information provides us with a set of constraints (inequalities) and zero restrictions for individual entries L_{ij}. Up to this point one can obtain about 90% of the L matrix entries exactly.

For the rest we employ an estimation routine based on local entropy maximization, which has already been used to reconstruct unknown bilateral interbank exposures on the basis of aggregate information (Upper and Worms, 2002; and Blien et al., 1997). The procedure finds a matrix that fulfils all the known constraints and treats all other parts (unknown entries in L) as contributing equally to the known row and column sums. These sums are known since the total claims vis-à-vis other banks have to be reported to the central bank. The estimation problem can be set up as follows: Assume we have a total of K constraints. The column and row constraints take the form

$$\sum_{j=1}^{N} L_{ij} = b_i^r \,\forall i \quad \text{and} \quad \sum_{i=1}^{N} L_{ij} = b_j^c \,\forall j \tag{1}$$

with *r* denoting *row* and *c* denoting *column*. Constraints imposed by the knowledge about particular entries in L_{ij} are given by

$$b^l \leq L_{ij} \leq b^u \text{ for some } i, j. \tag{2}$$

The aim is to find the matrix L (among all the matrices fulfilling the constraints) that has the least discrepancy to some a-priori matrix U with respect to the (generalized) cross entropy measure

$$C(L,U) = \sum_{i=1}^{N} \sum_{j=1}^{N} L_{ij} \ln\left(\frac{L_{ij}}{U_{ij}}\right). \tag{3}$$

U is the matrix which contains all known exact liability entries. For those entries (bank pairs) ij on which we have no information from central bank data, we set $U_{ij} = 1$. We use the convention that $L_{ij} = 0$ whenever $U_{ij} = 0$ and define $0 \ln(\frac{0}{0})$ to be 0. This is a standard convex optimization problem, the necessary optimality conditions can be solved efficiently by an algorithm described in Fang et al. (1997) and Blien et al. (1997). As a result, we obtain a rather precise (see below) picture of the interbank relations at a particular point in time. Given L we plot the distribution of its entries in chart 1(b). The distribution of liabilities follows a power law for more than three decades with an exponent of -1.87, which is within a range well known from wealth- or firm-size distributions (Solomon and Levy, 2000; and Axtell, 2001).

Extracting the Network Topology from the Interbank Data

There are three possible approaches to describe the structure as a graph. The first approach is to look at the liability matrix as a *directed graph*. The vertices are all Austrian banks. The central bank, the OeNB, and the aggregate foreign banking sector are represented by a single vertex each. The set of all initial (starting) vertices is the set of banks with liabilities in the interbank market; the set of end vertices is the set of all banks that are claimants in the interbank market. Therefore, each bank that has liabilities vis-à-vis other banks in the network is considered an initial vertex in the directed liability graph. Each bank for which this liability constitutes a claim, i.e. each bank acting as a counterparty, is considered an end vertex in the directed liability graph. We call this representation the *liability adjacency matrix* and denote it by A^l (^{l} indicating liability). $A^l_{ij} = 1$ whenever a connection starts from row node i and leads to column node j, and $A^l_{ij} = 0$ otherwise. If we take the transpose of A^l we get the interbank asset matrix $A^a = (A^l)^T$. A second way to look at the graph is to ignore directions and regard any two banks as connected if they have either a liability or a claim vis-à-vis each other. This representation results in an undirected graph whose corresponding adjacency matrix $A_{ij} = 1$ whenever we observe an interbank liability or claim. Our third graph representation is to define an undirected but weighted adjacency matrix $A^w_{ij} = L_{ij} + L_{ji}$, which measures the gross interbank interaction, i.e. the total volume of liabilities and assets for each node. The decision on which representation to use depends on the questions addressed to the network. For statistical descriptions of the network structure, the matrices A, A^a, and A^l will be sufficient; to reconstruct the community structure from a graph, the weighted adjacency matrix A^w will be the more useful choice.

Functional Clusters

There exist various ways to find functional clusters within a given network. Many algorithms take into account local information around a given vertex, such as the number of nearest neighbors shared with other vertices and the number of paths to other vertices (see, e.g., Wasserman and Faust, 1994; or Ravasz et al., 2001). Recently a global algorithm was suggested which extends the concept of vertex betweenness (Freeman, 1977) to links (Girvan and Newman, 2001). This elegant algorithm outperforms most traditional approaches in terms of misspecifications of vertices to clusters; however it does not pro-

(a)

(b)

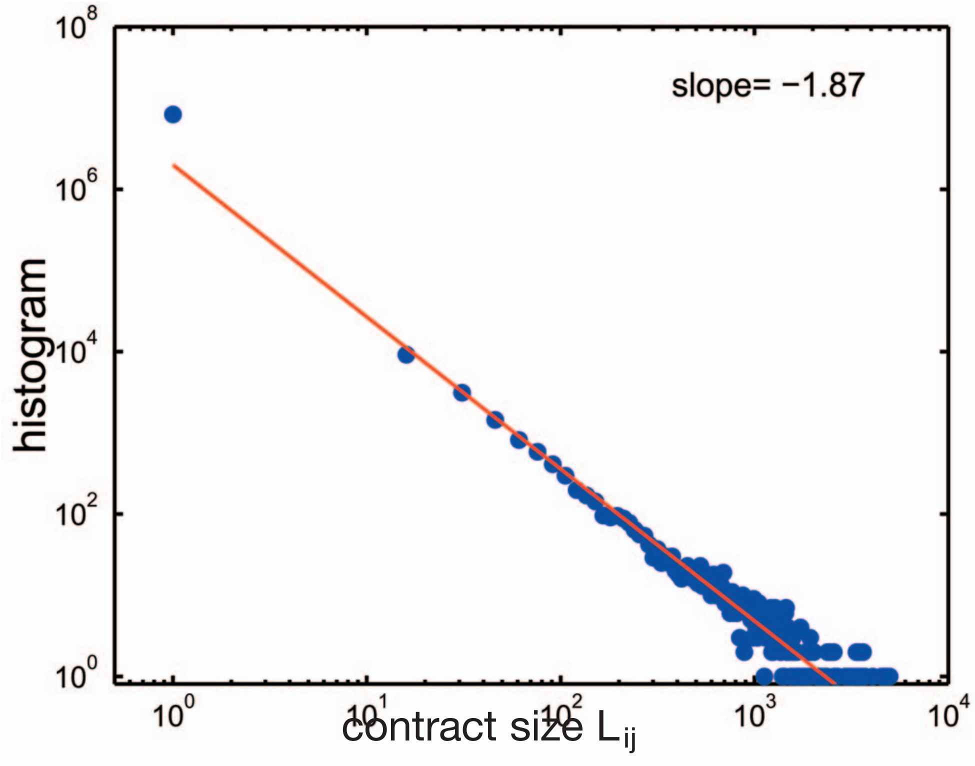


Chart 1: The Austrian Interbank Network and Histogram of Contract Size

***Note:** The banking network of Austria (a). Clusters are grouped (colored) according to regional and sectoral organization: R sector with its federal sub-structure: yellow RB, orange RSt, light orange RK, gray RV, dark green RT, black RN, light green RO, light yellow RS. VB sector dark grey, S sector orange-brown, other pink. Data are from the September 2002 L matrix, which is representative for all the other matrices. In (b) we show the contract size distribution within this network (histogram of all entries in L) which follows a power law with exponent -1.87. Data are aggregated from all 10 matrices.*

vide a measure for the differences of clusters. In Zhou (2003a) an algorithm was introduced which – while having at least the same performance rates as Girvan and Newman (2001) – provides such a measure, the so-called dissimilarity index. The algorithm is based on a distance definition presented in Zhou (2003b).

In analyzing our interbank network we apply the latter algorithm to the weighted adjacency matrix A^w. As the only preprocessing step we clip all entries in A^w above a level of EUR 300 million for numerical reasons, i.e. $A^w_{clip} = \min(A^w, 300\text{m})$. The community structure obtained in this way (chart 1a) can be compared with the actual community structure in the real world. Chart 2 shows the result for the community structure obtained from one representative data set.

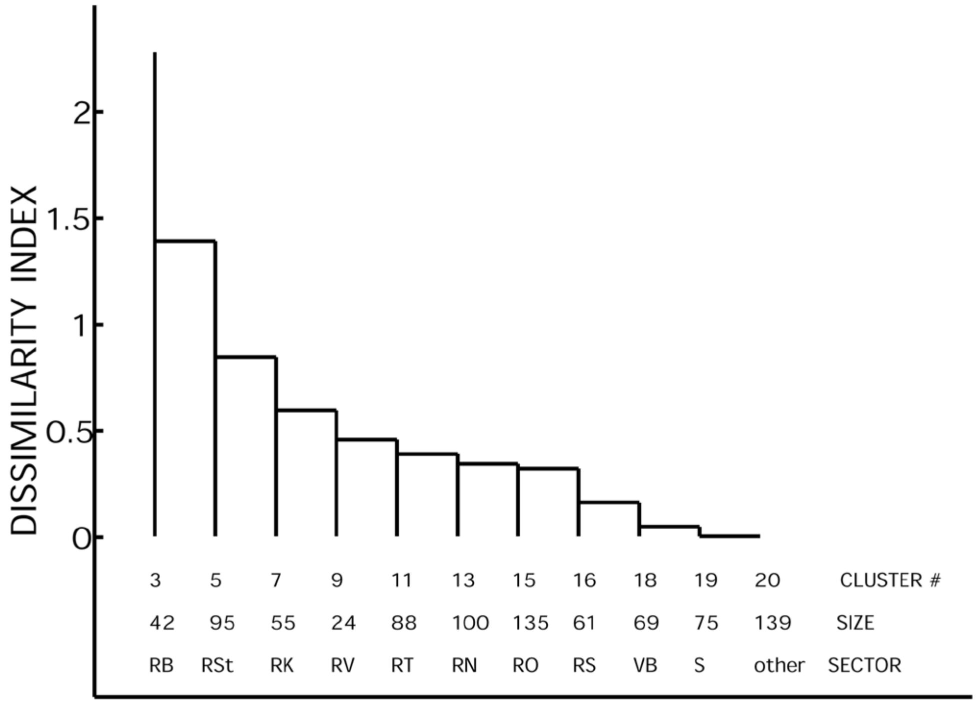


Chart 2: Dissimilarity Index of Different Banking Groups

Note: Community structure of the Austrian interbank market network from September 2002 data. The dissimilarity index is a measure of the "differentness" of the clusters.

The results from other datasets are practically identical. The algorithm identifies communities of banks which are organised along a two- or three-tier structure, i.e. the R, VB, and S sectors. For banks which are not structured in a hierarchical way, such as banks in the SP, JS, SM, BLA sectors, no strong community structure is expected. By the algorithm these banks are grouped together in a cluster called 'other'. The Raiffeisen sector, with its substructure in the federal provinces, is further grouped into clusters which are clearly identified as R banks within one of the eight federal provinces (B, St, K, V, T, N, O, S[3]). In chart 2 these clusters are marked as, e.g., 'RS', with 'R' indicating the Raiffeisen sector and 'S'

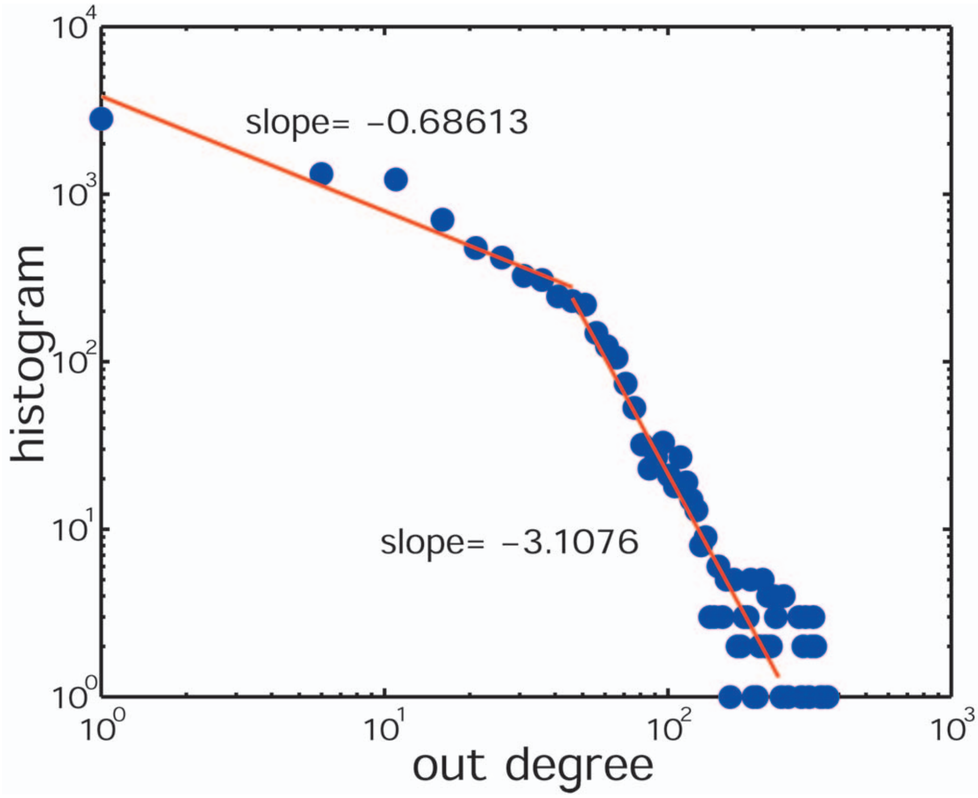
[3] *B for Burgenland, St for Styria, K for Carinthia, V for Vorarlberg, T for Tyrol, N for Lower Austria, O for Upper Austria, S for Salzburg.*

the province of Salzburg. Overall, there were 31 misspecifications into wrong clusters within the total $N = 883$ banks, which is a misspecification rate of 3.5 %. This result demonstrates the quality of the dissimilarity algorithm and – more importantly – the quality of the entropy approach to reconstruct matrix L.

Degree Distribution

Like many real world networks, the degree distribution of the interbank market follows a power law for all three representations A^l, A^a, and A. Charts 3 (a) and (b) show the out-degree (liabilities) and in-degree (assets) distribution of the vertices in the interbank liability network. Chart 3 (c) shows the degree distribution of the interbank connection graph A. In all three cases we find two regions which can be fitted by a power law. Accordingly, we fit one regression line to the small degree distribution and one to the obvious power tails of the data using an iteratively re-weighted least square algorithm. The power decay exponents γ_{tail} to the tails of the degree distributions are $\gamma_{tail}(A^l) = 3.11, \gamma_{tail}(A^a) = 1.73$ and $\gamma_{tail}(A) = 2.01$. The size of the out-degree exponent is within the range of several other complex networks, like, e.g., the collaboration networks of actors (3.1; Albert and Barabási, 2000), sexual contacts (3.4; Liljeros et al., 2001); exponents in the range of 2 are, for example, the Internet (2.1; Albert et al., 1999) or mathematicians' collaboration networks (2.1; Barabási et al., 2002), and examples for exponents of about 1.5 are e-mail networks (Ebel et al., 2002) and co-autorships (1.2; Newman, 2001). For the left part of the distribution (small degrees) we find $\gamma_{small}(A^l) = 0.69, \gamma_{small}(A^a) = 1.01$ and $\gamma_{small}(A) = 0.62$. These exponents are small compared to other real world networks. One example are food webs with a coefficient of 1.0. (see Montoya and Solé, 2000). We have checked that the distribution for the low degrees is almost entirely dominated by banks of the R sector. Typically in the R sector most small Raiffeisen banks have links to their federal provincial head institutions and very few contacts with other banks; this leads to a strong hierarchical structure, which is clearly visible in chart 1(a). This hierarchical structure is perfectly reflected by the small scaling exponents (Trusina et al., 2003).

(a)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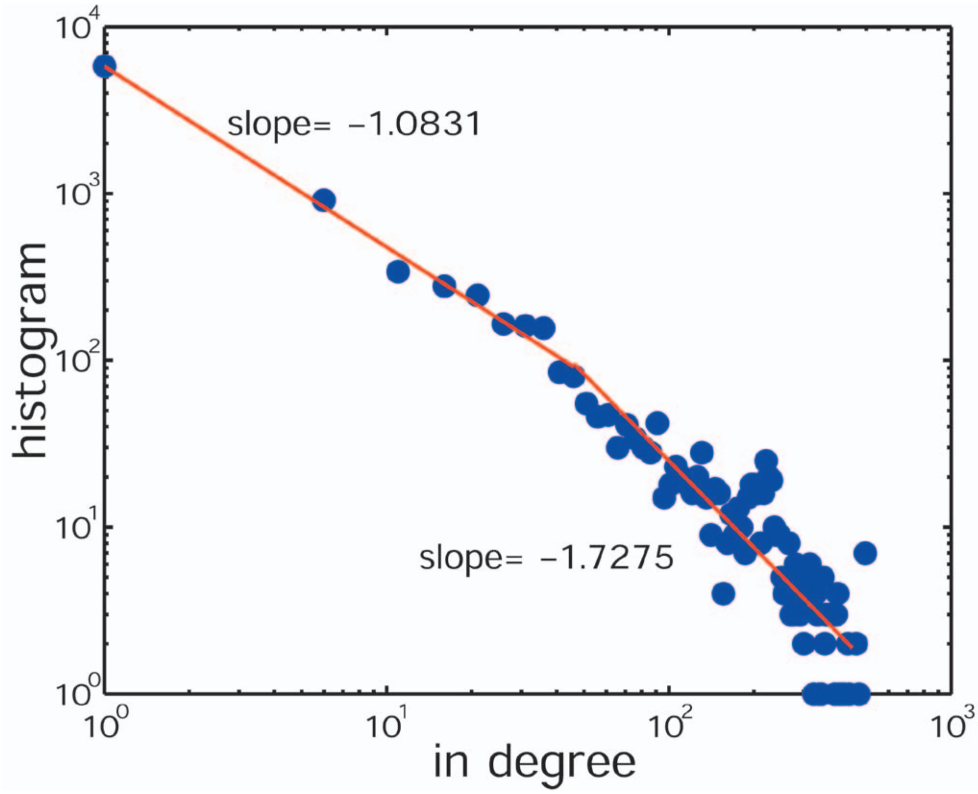
slope= −3.1076
histogram
out degree
10^0
10^1
10^2
10^3
10^4

(b)


slope= −1.0831
slope= −1.7275
histogram
in degree
10^0
10^1
10^2
10^3
10^4

(c)

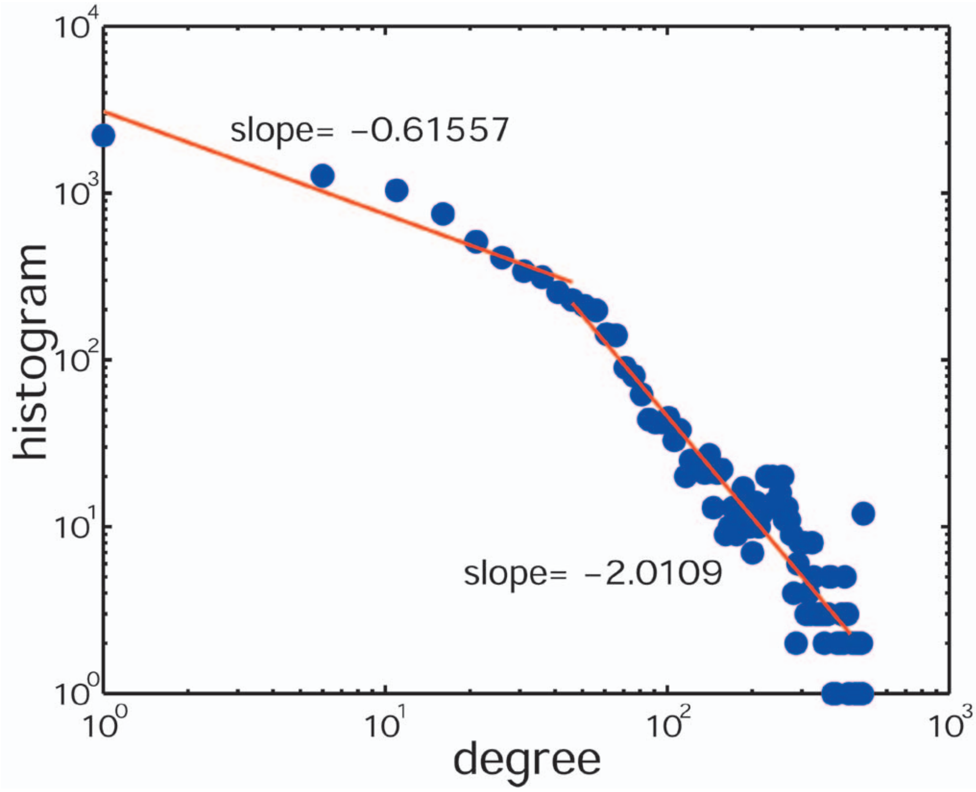


Chart 3: Degree Distribution

Note: Empirical out-degree (a) and in-degree (b) distribution of the interbank liability network. In (c) the degree distribution of the interbank connection network is shown. All the plots are histograms of aggregated data from all the 10 datasets.

Clustering Coefficient

To quantify clustering phenomena within the banking network, we use the so-called clustering coefficient C defined by

$$C = \frac{3 \times (number\ of\ triangles\ on\ graph)}{number\ of\ connected\ triples\ of\ vertices} \quad (4)$$

It provides the probability that two vertices that are connected to any given vertex are also connected with one another. A high clustering coefficient means that two banks that have interbank connections with a third bank have a greater probability to have interbank connections with one another than any two banks randomly chosen on the network. The clustering coefficient is well defined in undirected graphs only. We find the clustering coefficient of the connection network (A) to be $C = 0.12 \pm 0.01$ (mean and standard deviation over the 10 data sets), which is relatively small compared to other networks. In the context of the interbank market, a small C is a reasonable result. While banks may be interested in some diversification of interbank links, keeping a link is also costly. So if, for instance, two small banks have a link with their head institution there is no reason for them to additionally open a link between themselves.

Average Path Length

We calculate the average path length for the three networks A^l, A^a, A with the Dijkstra algorithm (Gibbons, 1985) and find an average path length of $\bar{\ell}(A^l) = \bar{\ell}(A^a) = 2.59 \pm 0.02$. Note the possibility that in a directed graph not all nodes can be reached and we restrict our statistics to the giant components of the directed graphs. The average path length in the (undirected) interbank connection network A is $\bar{\ell}(A) = 2.26 \pm 0.03$. From these results the Austrian interbank network looks like a very small world with about three degrees of separation. This result looks natural in the light of the community structure described earlier. The two- and three-tier organization with head institutions and sub-institutions apparently leads to short interbank distances via the upper tier of the banking system and thus to a low degree of separation.

Conclusions

Our analysis provides a first picture of an interbank network by studying a unique dataset for the Austrian interbank market. Even though the Austrian interbank market is small it is structurally very similar to the interbank system in many European countries, including the large economies of Germany, France and Italy. We show that the liability (contract) size distribution follows a power law. These results can be understood as being driven by the underlying size and wealth distributions of banks which show similar power exponents. We find that the interbank network shows – like many other realistic networks – power law dependencies in the degree distributions. We were able to show that different scaling exponents relate to different network structures in different banking sectors within the total network. The scaling exponents of the Raiffeisen credit cooperatives (R) are very low, due to the hierarchical structure of this sector, while the other banks have scaling exponents also found in other complex real world networks. Regardless of the size of the scaling exponent, the existence of a power law is a strong indication of a stable network with respect to random bank defaults or even intentional attack (Albert et al., 2000). The interbank network shows a low clustering coefficient, a result that mirrors the analysis of community structure which shows a clear network pattern, where banks would first have links with their head institutions, whereas these few head institutions have links between each other. A consequence of this structure is that the interbank network is a small world with a very low "degree of separation" between any two nodes in the system. A further important message of this paper is that our results allow excluding large classes of unrealistic types of networks for future modeling of interbank relations which have so far been used in the literature.

References

Albert, R. and A.-L. Barabási. 2000. Topology of Evolving Networks: Local Events and Universality. Phys. Rev. Lett. 85. 5234–5237.

Albert, R., H. Jeong and A.-L. Barabási. 1999. Diameter of the World Wide Web. Nature 401. 130.

Albert, R., H. Jeong and A.-L. Barabási. 2000. Error and attack tolerance of complex networks. Nature 406. 378–382.

Allen, F. and D. Gale. 2000. Financial Contagion. Journal of Political Economy 108, 1.

Axtell, R. L. 2001. Zipf Distribution of U.S. Firm Sizes. Science 293. 1818–1820.

Barabási, A.-L., H. Jeong, Z. Neda, E. Ravasz, A. Schubert and T. Vicsek. 2002. Scale Free Tolopogy of E-mail Networks. Physica A 311. 590.

Blien, U. and F. Graef. 1997. Entropy Optimizing Methods for the Estimation of Tables. In: Balderjahn, I., R. Mathar and M. Schader (eds.). Classification, Data Analysis, and Data Highways. Berlin: Springer Verlag.

Dorogovtsev, S. N. and J. F. F. Mendes. 2003. Evolution of Networks: From Biological Nets to the Internet and WWW. Oxford University Press.

Ebel, H., L. I. Mielsch, S. Bornholdt. 2002. Scale Free Tolopogy of E-mail Networks. Phys. Rev. E 66, 036103.

Fang, S. C., J. R. Rajasekra and J. Tsao. 1997. Entropy Optimization and Mathematical Programming. Kluwer Academic Publishers.

Freeman, L. C. 1977. A Set of Measures of Centrality Based Upon Betweenness. Sociometry 40, 35.

Freixas, X., L. Parigi and J. C. Rochet. 2000. Systemic Risk, Interbank Relations and Liquidity Provision by the Central Bank. Journal of Money, Credit and Banking 32.

Gibbons, A. 1985. Algorithmic Graph Theory. Cambridge University Press.

Girvan, M. and M. E. J. Newman. 2002. Community Structure in Social and Biological Networks. Proc. Natl. Acad. Sci. 99. 7821–7826.

Hartmann, P. and O. DeBandt. 2000. Systemic Risk: An Overview. European Central Bank Working Paper No. 35.

Liljeros, F., C. F. Edling, L. A. N. Amaral, H. E. Stanley and Y. Aberg. 2001. The Web of Human Sexual Contacts. Nature 411. 907.

Montoya, J. M. and R. V. Solé. 2000. Topological Properties of Food Webs. From Real Data to Community Assembly Models. Santa Fe Institute Working Papers. 00-10-059.

Newman, M. E. J. 2001. Who is the Best Connected Scientist? A Study of Scientific Coauthorship Networks, Scientific Collaboration Networks. Phys. Rev. E 64, 016131 and 016132.

Ravasz, E., A. L. Somera, D. A. Mongru, Z. N. Oltvai and A.-L. Barabási. 2001. Hierarchical Organization of Modularity in Metabolic Networks. Science 297. 1551–1555.

Sheldon, G. and M. Maurer. 1998. Interbank Lending and Systemic Risk. Swiss Journal of Economics and Statistics 134. 685.

Solomon, S. and M. Levy. 2000. Market Ecology, Pareto Wealth Distribution and Leptokurtic Returns in Microscopic Simulation of the LLS Stock Market Model. e-print: arXiv.
http://xxx.lanl.gov/abs/cond-mat/0005416.

Summer, M. 2003. Banking Regulation and Systemic Risk. Open Economies Review 1. 43.

Thurner, S., R. Hanel and S. Pichler. 2003. Risk Trading, Network Topology, and Banking Regulation. Quantitative Finance 3. 306–319.

Trusina, A., S. Maslov, P. Minnhagen and K. Sneppen. 2003. Hierarchy Measures in Complex Networks. e-print: arXiv: http://xxx.lanl.gov/abs/cond-mat/0308339.

Upper, C. and A. Worms. 2002. Estimating Bilateral Exposures in the German Interbank Market: Is there a Danger of Contagion? Deutsche Bundesbank, Discussion paper 09.

Wasserman, S. and K. Faust. 1994. Social Network Analysis: Methods and Applications. Cambridge University Press.

Watts, D. J. 1999. Small Worlds: The Dynamics of Networks between Order and Randomness. Princeton University Press.

Zhou, H. 2003a. Electronic States Around a Vortex Core in High-Tc Superconductors Based on the t-J model. Phys. Rev. E (in print) e-print: arXiv: http://xxx.lanl.gov/abs/cond-mat/0302030.

Zhou, H. 2003b. Distance, Dissimilarity Index, and Network Community Structure. e-print: arXiv.
http://xxx.lanl.gov/abs/physics /0302032.