Monetary Transmission and Asset-Liability Management by Financial Institutions in Transitional Economies – Implications for Czech Monetary Policy

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1 Introduction

Two links of the monetary policy transmission chain are hidden in the function of the financial sector: first, the transmission of central bank key rate decisions into the money market and to other fixed income interest rates and second, the transmission of money market rate movements into the rates of lending to nonbanks. The two additional questions often invoked in the monetary transmission context are the importance of autonomous factors in the quantity of money growth, and the money market rate's impact on the exchange rate.

As could be expected, the effects of changes in monetary policy on the interaction between the financial and the real sectors in the Czech economy are very poorly explained by macroeconomic models of the VAR type (see Kodera and Mandel, 1997; and Izák, 1998, for an empirical overview). Nor is the Neo-Keynesian "extended IS-LM" approach initiated by Bernanke and Blinder, 1988, very helpful, since its core assumption – the posited interest rate-driven demand for credit – refers to the factors lying outside the financial sector. Especially troubling for policymakers are the many observed periods when the credit conditions for the nonbanking public were reluctant to react logically to central bank key rate decisions. This reality calls for an extension of the presently existing models of the financial sector to enable them to account for the specific features of transitional economies. In particular, the models should accommodate such phenomena as autonomous behavior of the monetary aggregates, volatility at the long end of the yield curve, credit crunches, the behavior of the forward premium on the national currency in the forex market, etc. In a transitional economy like the Czech Republic, the need for a new modeling approach is especially strong. For one thing, the Czech financial markets are still far less transparent than those of the leading industrial countries, for which modern international finance theory has been created, or even those of other Visegrad countries. For another thing, the Czech post-reform development shows that the formal process of financial liberalization and capital market evolution conceals deep structural deficiencies. Even an attempt to understand the true reasons of many seemingly illogical processes in the Czech financial sector, let alone guide these processes in the sense of a gradual convergence to the industrial world standard, calls for a synthesis of many existing formal approaches. A synergy of financial market microstructure theory, stochastic finance and international macroeconomics would be desirable. The present paper constitutes a move in the direction of such a synthetic approach.

Juxtaposing the available evidence about the monetary transmission between the central bank, commercial banks and private customers with the existing views on optimizing investor behavior, one soon arrives at the need to focus on one distinctive motive of the behavior of financial intermediaries. That is, one models an agent who is acting in two financial market segments, the interbank money market and the market for private securities.

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At the same time, the financial firm invests funds it does not own, i.e. the deposits of the public, which means that it is subject to unpredicted infusion and withdrawal of those funds. Therefore, the flow of earnings must be managed in such a way as to absorb uncertain shocks to liquidity at all future dates. Accordingly, should the model elucidate the actions of the financial intermediary, it seems reasonable that the gaps between future incoming and outgoing payments (referred to as cash flow constraints below) be reflected by the set of restrictions entering its optimization problem. This is why the model that I introduce in the paper describes the behavior of a financial institution facing two types of constraints: the liquidity constraint and the cash flow constraint.

A convenient consequence of the abovementioned constraints is that the model generates a demand for securities which is not entirely determined by the standard risk-sharing reasons, but also reflects the current asset-liability management considerations. Due to this, monetary policy plays a role in the model regardless of whether its changes are expected or not, which is an ideal feature for the transmission mechanism applications that I am studying.

The model works with the concept of the financial system represented by a single firm, which is a competitive price taker in the asset markets. Modeling the behavior of the financial sector in this way might have become too complex had one dealt with matters like deposit market structure, optimal corporate liability design and other microfinance problems. However, the topic of this study – the relevance of asset-liability management decisions by financial firms for credit creation and monetary policy transmission – can, if necessary, be viewed from a strictly macroeconomic perspective. The fact that the results do not involve the exercise of market power by the firms in the financial services industry justifies the representative agent simplification.

The most important concept which arises in the solution of the model is that of so-called *shadow asset prices.* They are generated by the co-state variables of the individual optimization problem, provided that the available asset holdings are regarded as state variables. In the present discrete time and time-separable utility setting, I do not need to invoke the co-states explicitly to derive the first-order conditions of optimality. Therefore, in this paper, in a slight abuse of terminology, I call the shadow security price a variable equal to the ratio of its "true" shadow price to the shadow price of cash. The correct names of the variables can easily be regained by treating the same problem in the Lagrange multiplier form. In the present understanding, the shadow price has the property of clearing the market under the representative investor assumption and no outside supply or demand for the asset in question. Put differently, if the actual price is equal to the shadow price, the agents have no incentive to transact. As it turns out, the shadow prices satisfy the conventional no-arbitrage relationships regardless of the degree to which the actual prices of the same assets are distorted by the presence of transaction costs, information asymmetries and other microstructural effects. All of the named factors are summarized in the model under the concept of the *transaction function* of the corresponding market segment, making the portfolio reshuffling cost an increasing, strictly convex function of the transaction volume.

One of the principal consequences of the selected approach is the derivative notion of the *shadow interest rate* on a loan granted by the financial institution to a nonbank. One obtains this rate by first deriving the shadow price of said loan as if it were one of the regularly traded assets. The actual price of the loan contract must be equal to unity at the date of issue, meaning that one looks at a loan with unit principal. On the other hand, the already mentioned no-arbitrage relations between the shadow prices of traded assets allow the loan, viewed as a temporal sequence of the coupon and the principal payments, to be linked to the shadow prices of discount bonds traded in the money market. The result is the shadow rate – the coupon rate at which the financial institution is willing to grant the loan of the desired amount of unity. More generally, the shadow rate characterizes the lending conditions the borrower would have to observe if he abided by the preferred loan size and only negotiated the interest rate. Since, in aggregate, the lender-borrower bargaining should result in a joint determination of *both the loan amount and the lending rate,* the shadow rate very seldom coincides with the actual one. Instead, it provides a measure of the current credit conditions: Its growth indicates a contraction, its fall a relaxation of the credit channel of monetary policy transmission. (The main, or money channel, condition corresponds to the general level of money market rates.)

The selected inferences of the model have the objective of covering the four already named aspects of the monetary transmission in an open economy, namely the propagation of central bank rate changes along the zero coupon yield curve, the impact of money market rates on the lending rates to nonbanks, controllability of monetary aggregates and the exchange rate effects of the key rate changes. All four aspects are analyzed by comparing the inferences of the model with the available Czech data.

By way of international comparison, I had a valuable opportunity to collect some data on the Austrian banking sector and take a look at them in the context of the findings for the Czech Republic.

The paper is structured as follows. In section 2, I define the model of an optimizing financial intermediary (subsections 2.1 to 2.3), the method of solving the optimization problem for optimal investment/lending policies involving the concept of shadow asset prices (subsection 2.4) and discuss the equilibrium (subsection 2.5). Section 3 gives some applications of the model, including the term structure of interest rates (subsection 3.1), shadow and actual pricing of private sector debt contracts (subsection 3.2) and the uncovered return parity condition on the exchange rate (subsection 3.3). Next, selected phenomena of the Czech financial markets and monetary transmission are related to distinctive elements of the model. In this context, I discuss the propagation of the central bank key rate shocks along the yield curve in section 4. Private sector credit conditions and their dependence on the term structure of interest rates are dealt with in section 5. The autonomous behavior of the monetary aggregates and the forward premium for the exchange rate are briefly touched upon in section 6. Section 7 concludes with some observations on the relevance of the paper's findings for Czech monetary policy.

2 Optimizing by a Financial Institution under Cash Flow Constraints

The objective of sections 2 and 3 is to develop a formal framework to study the channels of monetary policy transmission within the financial sector of a transition economy, based upon individual optimization under uncertainty. The model proposed to this end departs from the conventional view of the representative financial firm as a classic portfolio optimizer under uncertainty by adding an additional variable to its current utility function. Said variable expresses, for the given moment, the assessment of netting asset incomes and liability service payments at all subsequent moments in the future, assuming that the asset and liability holdings are frozen at the levels attained at the moment of measurement. The proposed approach to the preference structure captures the spirit of the capital adequacy requirements imposed on banks and other financial intermediaries by regulatory authorities. A rationale for this seemingly subrational asset-liability management (ALM) argument in financial firm utility can be found, along with other capital market imperfections, in the fact that a number of assets in its portfolio are illiquid. The market participants are aware that the firm is unable to become 100% liquid on short notice. Therefore, agents must take into account the costs and benefits of firm ownership regardless of whether the company can be marketed immediately or not. Specifically, in transitional economies, a cash flow valuation of the mentioned form may express the high cost uncertainty about the future access to the markets, or it may be a substitute for the termination value component of standard portfolio optimization. Indeed, in the mainstream finance literature, the termination possibility is dealt with by means of the liquidation price of the firm based on the resale value of its assets at a random stopping time (see Karatzas et al., 1987). The asset-liability manager in a transition economy, however is forced to keep in mind a continuously updated measure of readiness to decisively redefine the company's activities or, at least, some form of profound restructuring of the company. Under such circumstances, the resale possibility may be either nonexistent or prohibitively expensive, and the direct cash flow measure seems to be the sole tangible alternative. Nevertheless, the definition to be given will be consistent with the resale alternative in an efficient capital market where, after all, the asset price should be fully reflected by future cash flows.

Another specific feature of period utility used in the model is its dependence on the current liquidity variable. This dependence serves as a substitute for a hard no-bankruptcy constraint (the same approach for continuous time portfolio optimization is discussed in Derviz, 1999a). The additional arguments in the period utility function, i.e. the current cash level variable and the future cash flow assessment variable, constitute two state variable constraints in the "soft" form. Among other things, this specification helps to restrict the optimal behavior of the financial intermediary to the paths with nonexplosive short positions.

Naturally, the additional elements in period utility described above entail several other important consequences for optimal investment and the overall outcome of the model. The fact will become clear as soon as the basic result of the optimal demand for new corporate securities is compared with the monetary policy impact on discount bond prices.

In the chosen definition of utility, I avoid any explicit measure of inflation. The decision-maker can take it into account by discounting all arguments of the utility function – dividends, current cash and future cash flows – by a common inflationary multiplier. Then, just as in standard portfolio optimizing models with money, the inflation factor will appear in all asset pricing results that involve marginal utilities and pricing kernels. In the present setting, however, the object of interest is the totality of "ALM effects," which will be shown to involve only *marginal utility ratios.* The common inflationary multiplier cancels out such ratios, making the most important ALM effects inflation-neutral. Thus, the inflationary effects, although implicitly present and easily recovered if necessary, are played down in the notations, so that the focus of the paper on nominal asset-liability constraints is not unduly blurred.

2.1 The Model

Consider a financial company which can trade in N securities of domestic or foreign origin comprising the set N. The bulk of these securities are corporate stocks and commercial paper. Time is discrete and changes take place between 0 and infinity. The number of shares/bonds of security k held by the company at time t is denoted by x_t^k, its price by P_t^k $(k = 1, ..., N)$. A negative value of x^k means that the company has a liability in the k^{th} security.

Security k is characterized by the flow of dividends (coupons and the principal if it is a bond), equal to ${\Gamma^k}_t$ of units of account at time t. Of course, in reality, time intervals between dividend payouts are long, so that this variable is different from zero only for a small number of dates. However, dividends will receive an additional characterization in the present model, which will be nontrivial in every period. At every time t, the market participants form an assessment of security k payments in the time moments following t: $(\gamma^k)_t^s, s \geq t+1$. For common stock, these are beliefs about future dividends; for fixed income securities, the known coupon and principal payments over the security's lifetime, adjusted by the default risk, if relevant. The definition should be appropriately modified to cover other security forms, e.g. indices or derivatives.

The amount by which the company increases (decreases if negative) the position in the k^{th} security between times $t-1$ and t is denoted by $\varphi_t^k : x_t^k = x_{t-1}^k + \varphi_t^k$.

To shorten the notations, I will use symbols $x, \Gamma, \gamma^s, \varphi$ and P to denote the vectors of security holdings, current payoffs, expected payoffs at time s, sales/purchases and prices, respectively, with the current date in the subscript. The security index k will also be omitted wherever possible. In fact, many results below can be formulated in an economy with only one composite security. Only some examples will require partitioning the set N.

Further, I introduce the notion of cash, or liquidity holdings of the company. Namely, it is assumed that all transactions (a sale of one security and a purchase of another) happen with the help of a liquid medium of exchange, whose current level in the company's till at time t is denoted by ${x^0}_t$. At every time period, income is added to the start-of-the-period level of x^0 while expenditures are subtracted from it. That is, no direct swaps of different securities are allowed, and the current level of cash indicates how far into debt the company is allowed to go in exercising the transactions (too negative cash levels are punished by disutility, see below).

In addition to the general security set N described above, there are pure discount bonds, distinguished by maturity alone. A discount bond maturing at time s pays one unit of account at that time. The time t position in the s discount bond will be denoted by $\Phi_t^s, s \geq t+1$, the current market price of one such bond by B_t^s. Thus, according to our definition, Φ_t^s is the totality of all discount bonds existing at time t and maturing at time s, *regardless of the issue date.* Accordingly, at every time t preceding the maturity they are priced equally (by B_t^s), so that all price deviations of individual bonds are ignored. Specifically, there is no default risk, and the defined discount bonds can be identified with the tools of the monetary policy traded in the money market.

The amount by which the company increases (decreases if negative) the position in the s bond between times $t-1$ and t will be denoted by $\varphi_t^s : \Phi_t^s = \Phi_{t-1}^s + \varphi_t^s, s \geq t+1$.

It is important to clarify what happens for $s = t$, i.e. how does one decide at time t what amount shall become available in cash at time $t+1$ without investing infinitely in the bonds maturing in the next period, i.e. Φ_t^{t+1}. The question is resolved by using a one-period ("overnight") interest rate i_{t+1}, at which the free end-of-period-t liquidity is deposited. This cash becomes part of the liquidity available at the beginning of period $t+1$ for further transactions.

In some periods, the company is supposed to pay dividends out of its free cash before making the overnight deposit. Just like the dividends or coupons it earns on securities of other issuers, these dividends would be nonzero only at selected predefined dates. However, to avoid cumbersome specifications of the payment regime, I assume that a contribution ρ_t to the dividend fund is made in every period t.

Looking at the specific situation of the deposit-taking financial institutions, i.e. banks and mutual funds, one should recognize the demand deposits of the nonbanking public as a special liability category in the model. For simplicity, I assume that the interest paid on demand deposits is zero, and ignore the exact motives of the public to hold them. The net increase in the deposit level at time t will be denoted by K_t, and the time t-expected net deposit increase at date $s \geq t+1$ by K_t^s. If the financial company is not entitled to take deposits, variable K shall be understood as the company-specific income term, which is not a function of the marketed securities' characteristics. The presence of exogenous income shocks represented by K will play no particular role in the present paper. It is, however, a convenient shortcut to define and explain the nonzero volumes of security trade caused by agent heterogeneity and should, therefore, be kept in mind when interpreting the results.

All other loan and deposit types, i.e. those that cannot be withdrawn at sight, are treated as regular securities belonging to the set N, as defined earlier. For example, I include in N the taken (or granted) credit lines, in spite of their "duality" with demand deposits, as pointed out by Kashyap et al., 1999.

Finally, I will need an auxiliary state variable x^f to denote the current state of financial technology available to the company. Without upgrading investment, the financial technology becomes obsolete. The obsolescence rate f works as the exponential decline rate of x^f. To offset it, the company can invest in an upgrade rate ρ^f. Altogether, the technology level follows the rule $x_t^f = x_{t-1}^f(\rho_t^f - f_t)$. By investing in x^f, the financial company prevents the administrative expenditure from becoming too high. In other words, it reduces the future transaction costs, as explained below.

The sources of exogenous uncertainty in the model are discrete-time stochastic processes Γ, K and f. They generate information filter $\mathrm{F} = (F_t)_{t\geq 0}$, so that F_t denotes information publicly available at the beginning of day t. Symbol $E_t[...]$ denotes the expectation conditional on F_t. The values Γ_t, K_t and f_t are known to the decision-maker at the start of period t, when he chooses the values of $\rho_t, \rho_t^f, \varphi_t$ and $\varphi_t^s, s \geq t+1$. Also, the optimizing financial firm is a price taker, so that the asset price vector P_t applicable during period t is known at the start of that period (accordingly, process P is adapted to filter F). Besides, the short-term interest rate i_{t+1} is known at time t as well. In order to make this assumption consistent with the requirement that all prices and return rates in the model are endogenous, I adopt the natural convention $B_t^{t+1} = \frac{1}{1+i_{t+1}}$. For reasonable preferences and transaction functions, this assumption implies $\varphi_t^{t+1} = 0$. Indeed, while dealing in the overnight debt market is costly, the same interest rate i can be earned costlessly if all the end-of-period-t-cash is deposited. Note that the time t assessments of date s cash flows γ_t^s, K_t^s and Φ_t^s are all known at time t. Naturally, in the case of the former two, it does not mean that the assessments are always correct, but rather that they are known to those who form them. In the case of discount bonds, the certainty follows from the assumption of zero default risk.

2.2 Transaction Costs

To generate nontrivial supply and demand schedules in the secondary markets for securities, it is necessary to introduce nonlinearity in the transaction variables φ and φ^s. Logically, this nonlinearity can enter either the utility function or the state-transition rules. I choose the latter variant, i.e. I introduce transaction costs which are nonlinear functions of the corresponding position change variables.

Take security k from the set N first. Nonzero transaction costs mean that, in order to add $\varphi^k > 0$ to the stock x^k, one must spend the amount $P^k j^k \varphi(^k) > P^k \varphi^k$. Analogously, by selling $\varphi^k < 0$ units of security k, one obtains the amount of cash $P^k j^k(-\varphi^k) < P^k(-\varphi^k)$. Consequently, the transaction function j^k can be defined as strictly increasing and strictly convex, with the value of zero corresponding to the zero transaction level. To accommodate the reduction effect of advanced technology on transaction costs, I assume that j^k is a strictly decreasing function of its second argument,

x^f. So, the technology level attained in the previous period drives down the cost level for the current transaction volume: $j^k_t = j^k(x^f_{t-1}, \varphi^f_t)$. Note the t subscript replacing the full list of arguments – the shorthand notation to be frequently utilized below. Another natural assumption is the zero marginal transaction cost at the origin: $j^k_\varphi(x,0) \equiv 1$ for all values of x (φ subscript indicates partial derivative).

As the principal example to keep in mind I invoke the linear-quadratic transaction function resembling the capital installation cost in Tobin's q models:

$$j^k(x,\varphi) = \varphi + \frac{b^k\varphi^2}{2x},$$

where b^k is a positive constant. For the values of φ^k/x^f not exceeding a certain negative level (i.e. for all but all too speedy sellouts), this functional form satisfies the partial derivative sign and the convexity requirements stated above.

It is assumed that the same beginning-of-period financial technology level has a reducing effect on transaction costs for trades in all securities in the current period. Then, variable x^f enters the transaction functions j^k for all k and the analogously defined transaction functions j^b in the discount bond market, for all s: ${j^b}_t = j^b(x^f_{t-1}, \varphi^s_t), s \geq t+1$. As regards the cost of technology upgrade itself, I assume the existence of a strictly increasing and concave function υ such that the upgrade at rate ${\rho^f}_t$ at time t costs $x^f_{t-1}\upsilon(\rho^f_t)$.

2.3 State-Transition Equations, Preferences and the Optimization Problem

Summing up the above definitions of incomes and expenditures, we are able to formulate the state transition equation for the liquidity variable, x^0, together with the remaining variables, i.e. x, Φ^s and x^f:

$$\begin{aligned} x^0_t = {} & (1+i_t)x^0_{t-1} + \Phi^t_{t-1} + x_{t-1}\cdot\Gamma_t - x^f_{t-1}\upsilon(\rho^f_t) + K_t - \\ & \rho_t - P_t \cdot j(x^f_{t-1}, \varphi_t) - \sum_{s \geq t+1} B^s_t j^b(x^f_{t-1}, \varphi^s_t), \end{aligned} \tag{1a}$$

$$x^k_t = x^k_{t-1} + \varphi^k_t, k = 1, \ldots, N, \tag{1b}$$

$$\Phi^s_t = \Phi^s_{t-1} + \varphi^s_t, s \geq t+1, \tag{1c}$$

$$x^f_t = x^f_{t-1}(\rho^f_t - f_t). \tag{1d}$$

Here and in the sequel, scalar products indicated by dots are used to simplify the notations concerning the security set N, wherever they do not cause ambiguity.

As stated in the introduction, I want the company to make a continuous assessment of future cash flows (positive and negative), as far as they are generated by the current asset holdings. For this purpose, let us introduce variable $R^s_t(s \geq t+1)$ as follows:

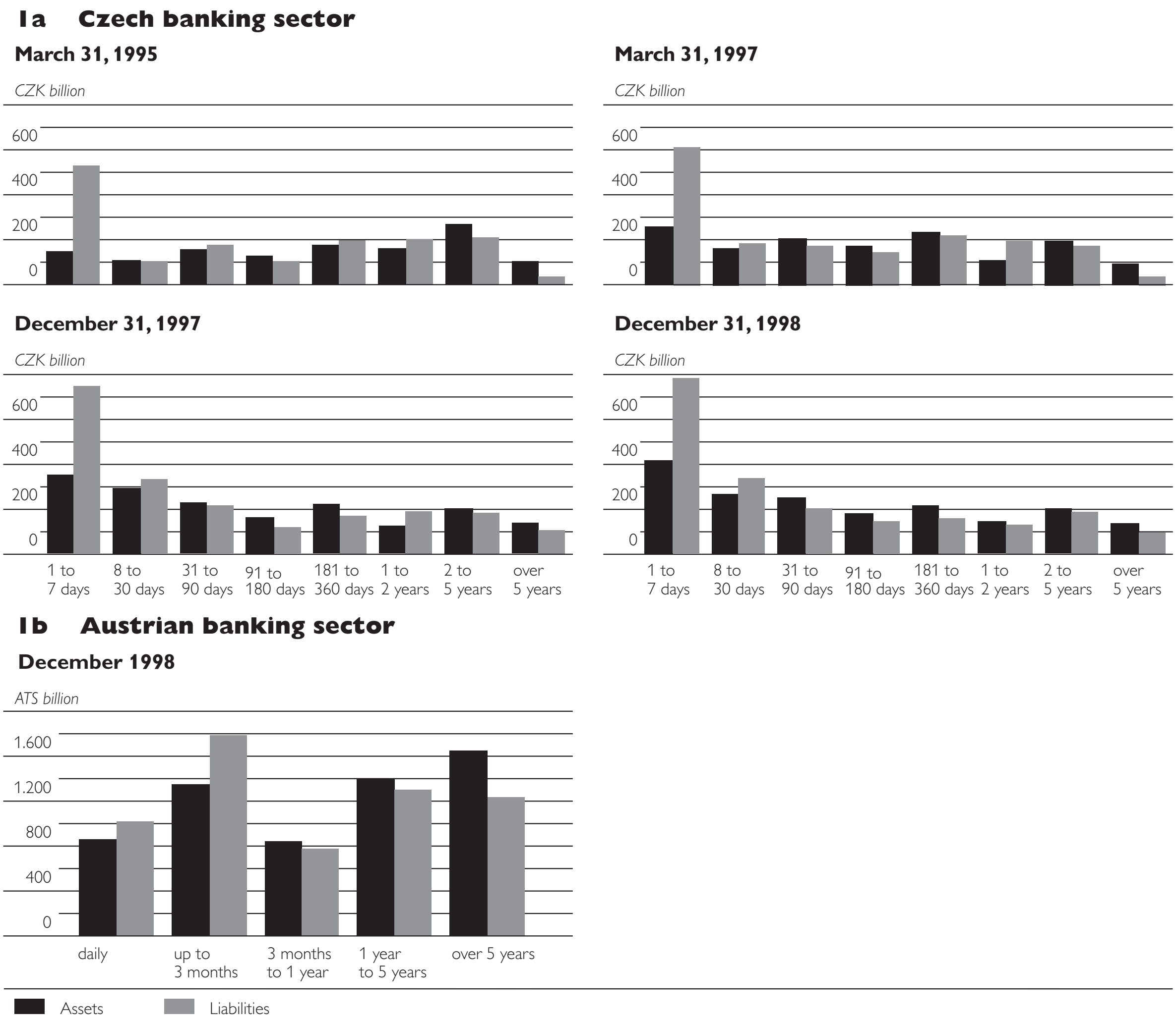
$$R^s_t = \Phi^s_t + x_t \cdot \gamma^s_t + K^s_t. \tag{2}$$

The first two terms on the right hand side of (2) are subjective assessments at date t of the net income from maturing discount bonds and the income from other securities at date s, respectively (the corresponding term is negative in the case of a negative position). Term K stands for the date t assessment of the exogenous income shock at date s. In the case of the deposit-accepting institutions, the main source of K is the net change of the deposit level.

A few words should be said about the dynamics of the net future income variable R_t^s. First of all, banking statistics confirm that its sign usually changes from period to period. Besides, the distribution of its highs and lows for a given measurement date t is variable in t and can differ from country to country. Chart 1 illustrates this with the example of future in- and out-payment schedules for the Czech and the Austrian banking sectors. The Czech case is covered by a sequence of four diagrams (chart 1 a). This was enabled by the daily data availability from the beginning of 1995. The last snapshot in chart 1 a (December 1998) is matched by the comparative diagram for Austrian banks in chart 1 b.

Chart 1

The Time Structure of Future Cash Flows

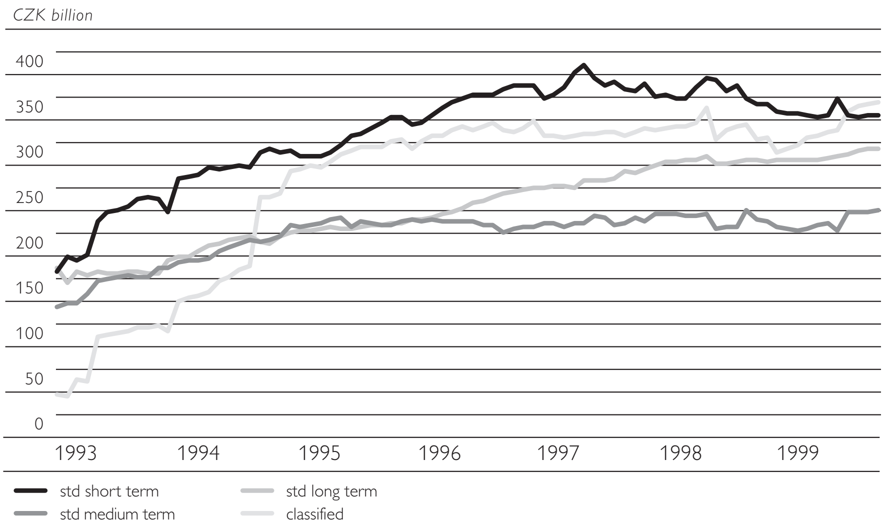


Source: OeNB, CNB.

Finally, R_t^s, regarded as a function of parameter t, can be very volatile if the financial sector portfolio contains many assets with a default risk. This is exactly the case in the Czech Republic, as chart 2 shows. Obviously, the dominant weight of classified loans in the commercial banks' portfolios (ranging from minimal to total losses) suggests a potential of frequent expectation revisions. All this supports the view adopted in this paper of the important role that the estimates of future net cash flows play in the decision-making of financial intermediaries, at least in the Czech economy today.

Chart 2

Credit Categories in the Balance Sheet of Czech Commercial Banks



Source: CNB.

Next I introduce the date t valuation of the net after-t income.

Let v be a strictly increasing and strictly concave function of the real variable, with $v(0) = 0$. Also, let $(a_t)_{t>0}$ be a sequence of positive and sufficiently big numbers, to be used in the after-t internal discount factors A_t^s as follows:

$$1 + A_t^s = \prod_{n=t+1}^{s} (1 + a_n), s \geq t + 1, A_s^s = 0.$$

Define the future cash flow valuation variable w as

$$w_t = \sum_{s \geq t+1} (1 + A_t^s)^{-1} v(R_t^s). \quad (3)$$

Weights $(1 + A_t^s)^{-1}$ express two things. First, their average level corresponds to the inflationary discount applied to the valuation of future cash flows. Second, their variability in parameter s expresses the relative importance of net cash flows on different dates in the future, be it in view of changes in the expected inflation rate across dates s, or for some other reason, e.g. special monitoring at preselected future moments. (Accordingly, if one is not interested in variable inflation or privileged monitoring dates, all parameters a_t can be assumed equal.) The weights must decrease with sufficient speed to make the sum in (3) converge. The period value function v goes to minus infinity more quickly than any straight line when its argument becomes increasingly negative, generating an increasingly adverse marginal valuation of growing dominance of income outflows over inflows. This happens thanks to the strict concavity of v. The same concavity property assures a decreasing marginal valuation of very high positive cash flows. If v were chosen very close to a linear function, then w_t would be almost proportional to the standard "fundamental" price of the modeled financial company, i.e. the discounted sum of future net cash flows. By choosing a strictly concave v instead, I stress the role of perceived market imperfections in the company asset valuation.

Observe that w_t in (3) contains no t-conditional uncertainties since, as was noted at the end of subsection 2.1, all components of R_t^s are known at time t.

Next, define the period utility function of the financial company as a smooth function $(x^0, w, \rho) \mapsto u(x^0, w, \rho)$, where the dependence on the dividend rate, ρ, is strictly increasing, concave and satisfies the Inada conditions. Observe that the *current liquidity level* x^0 and *the current valuation of future net investment incomes* (net cash flows) w are entering the period utility separately. That is, the measures of ALM performance and solvency are treated as imperfect substitutes. Otherwise, one would have to include variable x_s^0 in the future income variable R_t^s in (2). However, the usual ALM performance valuation practices speak in favor of the imperfect substitute definition chosen here.

Function u admits negative values of both x^0 and w, but is strictly concave in each of them with

$$\lim_{x^0 \to -\infty} u(x^0, w, \rho) = \lim_{w \to -\infty} u(x^0, w, \rho) = -\infty,$$

so that a fall into debt or an adverse expected future cash flow results in a big disutility. Strict concavity also generates diminishing marginal utility of increased cash holdings and future cash flows.

Let Θ be the time preference parameter of the financial company. Its optimization problem is that of maximizing

$$E\left[\sum_{t \geq 1} (1+\Theta)^{-t} u\left(x_t^0, w_t, \rho_t\right)\right] \tag{4}$$

with respect to time paths of the variables of choice φ, φ^s, ρ and ρ^f, subject to constraints (1), (2), (3) and given the initial asset and technology values x_0^0, x_0, Φ_0^s and x_0^f. Alternatively, (4) is to be maximized with respect to the trajectories of x^0, x, x^f, and Φ^s after φ, φ^s, ρ and ρ^f have been substituted away using the state-transition equation system (1). Note that transversality conditions on the state variables are not needed in this setting, since the restrictions on growth of optimal x^0 and w follow from their presence in the utility function.

The solution to (4) is described in the next subsection.

2.4 Solution of the Optimization Problem and Shadow Prices

As mentioned above the solution to the problem (1) to (4) can be obtained by substituting the decision variables with their expressions following from (1) (see Obstfeld and Rogoff, 1996, for the comprehensive exposition of the technique). Two other ways of finding a solution are by formulating and analyzing the Jacobi-Bellman equation, and by the Lagrange multiplier method. Under either approach, the imposed strict concavity conditions on the period utilities u and v and the strict convexity conditions on transaction functions j guarantee that the solution is an internal one, characterized by the first order conditions. Multiple explosive paths can be excluded by an appropriate transversality condition on the marginal utility of dividends, $u_\rho = \frac{\partial u}{\partial \rho}$, at infinity. The latter can either be

$$\lim_{T\to\infty}(1+i_{t+1})^{-1}\cdots(1+i_{t+T+1})^{-1}(1+\Theta)^{T+1}u_\rho(t+T+1)=0$$

or even weaker, just imposing the finiteness of the said limit. (The time argument by the marginal utility is a shorthand which will be used throughout the text to avoid long argument lists with individual time subscripts, for example, $u_\rho(t)=u_\rho(x_t^0,w_t,\rho_t)$. It shall be noted that marginal utility u_ρ is a complete analogue of the marginal utility of consumption found in traditional optimal portfolio models.

The list of first order conditions of optimality is given below. Symbols u_0 and u_w stand for the corresponding partial derivatives of the utility function; symbols $j_\varphi^s, j_x^s, j_\varphi$ and j_x have analogous meaning.

Optimal transactions are represented as follows:

$$P_t^k j_\varphi^k(x_{t-1}^f,\varphi_t^k)=Q_t^k,\ B_t^s j_\varphi^s(x_{t-1}^f,\varphi_t^s)=X_t^s,\ s\geq t+1,\ X_s^s\equiv 1, \tag{5a}$$

$$u_\rho(t)Q_t^k = u_w(t)\sum_{m\geq t+1}\frac{v'(R_t^m)}{1+A_t^m}(\gamma^k)_t^m + E_t\left[\frac{u_\rho(t+1)}{1+\Theta}\Gamma_{t+1}^k\right]+$$
$$E_t\left[\frac{u_\rho(t+1)}{1+\Theta}Q_{t+1}^k\right],\ k=1,\ldots,K, \tag{5b}$$

$$u_\rho(t)X_t^s = u_w(t)\frac{v'(R_t^s)}{1+A_t^s}+E_t\left[\frac{u_\rho(t+1)}{1+\Theta}X_{t+1}^s\right],\ s\geq t+1. \tag{5c}$$

The optimal dividend payment (marginal rate of substitution between dividend payments at two subsequent dates, in relation to the liquidity constraint) is represented as follows:

$$u_\rho(t)=u_0(t)+E_t\left[\frac{u_\rho(t+1)}{1+\Theta}\right](1+i_{t+1}). \tag{5d}$$

Optimal investment in financial technology (with the abbreviation $v(t)=v(\rho_t^f)$) is represented as follows:

$$u_\rho(t)v'(t)=E_t\left[\frac{u_\rho(t+1)}{1+\Theta}\left\{v'(t+1)\frac{x_{t+1}^f}{x_t^f}-v(t+1)-\right.\right.$$
$$\left.\left.\sum_{s\geq t+2}B_{t+1}^s j_x^b\left(x_t^f,\varphi_{t+1}^s\right)-P_{t+1}\cdot j_x\left(x_t^f,\varphi_{t+1}\right)\right\}\right]. \tag{5e}$$

Equation (5 a) describes the form of instantaneous demand for securities, or their supply, depending on the sign of the marginal transaction cost $j_\varphi - 1$: Remember that at the origin, the marginal transaction cost is zero and the partial derivatives j_φ are strictly decreasing in φ. These pricing schedules are formulated in terms of shadow values Q and X. The choice of names is motivated by the following. It can be verified that Q^k is equal to the ratio of the Lagrange multiplier of constraint (1b) to the Lagrange multiplier of cash constraint (1a), provided problem (4) is solved by the corresponding method. The same, with constraint (1b) replaced by (1c), is valid for X^s. Naturally, the rigorously correct terminology would assign the name "shadow price" to the multiplier/co-state variable itself. However, as I will not need the co-state corresponding to (1a), separately, reserving the denomination "shadow price" for the indicated ratios will cause no confusion.

Condition (5 a) can be interpreted in the following way. If the current price P^k of security k is greater than its shadow value, Q^k, the optimal value of φ^k must be negative, i.e. the firm sells the security shares; if P^k is lower than Q^k, the security shares are purchased (φ^k is positive). For the price at its shadow level ($P^k = Q^k$), there would be no transactions. The same is true for the current, B_t^s, and the shadow, X_t^s, price of the discount bond maturing at time s.

Equation (5 b) can be solved forward (given the transversality condition) with respect to Q_t. This (fundamental) solution would show that the shadow security price is the sum, for all future times, of the discounted dividend payments plus *the discounted weighted sums of the future dividend assessments in subsequent periods.* The weights are, naturally, determined by the subsequent marginal cash flow valuations $(1 + A_t^s)^{-1} v'(R_t^s)$. If the agents did not care about the value of future cash flows w, the shadow price would contain only the expected future dividends. Besides, under zero transaction costs $(j^k(x^f, \varphi^k) \equiv \varphi^k)$ or on condition of continuously clearing markets for security k and the existence of a representative investor ($\varphi^k \equiv 0$), the shadow and the actual price would be equal. In that case, (5 b) would reduce to the standard Consumption-based Capital Asset Pricing Model (CCAPM) formula in discrete time (see Ross, 1976; Breeden, 1979).

Equation (5c) describes the term structure of interest rates. It links the time t shadow price of the bond maturing at date s to the expected discounted shadow price of the same bond a period later plus the marginal rate of substitution between an expected income at time s and the current dividend payment. The latter term comes about only if the agents have separate preferences for the cash flow value variable w, which is the principal outstanding feature of the present model. Again, if there are no transaction costs or if the market for this bond clears in all periods under the representative agent condition, the shadow and the actual bond price coincide and (5 c) reduces to the usual term structure formula implied by the expectations hypothesis. The role of the stochastic discount factor/pricing kernel is played by ratio $\frac{u_\rho(t+1)}{(1+\Theta)u_\rho(t)}$, just like in standard discrete time term structure models (see Campbell et al., 1997). This ratio is characterized by (5 d). The latter condition, involving the marginal utility of cash, does not appear in standard

CCAPMs, but has its analogues in optimizing models with money in the utility function, such as Brock, 1974; Kouri, 1977; or Branson and Henderson, 1985. This equation can be characterized as the law of motion for the shadow price of liquidity, whose driving variable is the marginal utility of cash balances, u_0. The liquidity shadow price itself happens to equal u_ρ, as follows from the current value Hamiltonian optimization (for the shadow price technique in deterministic models with money, see Sidrauski, 1967; for stochastic models in continuous time, see Derviz, 1999a).

Finally, equation (5 e) links the current marginal cost of financial technology improvement to its discounted future value and the discounted future impact on liquidity. Discounting happens by means of the same stochastic factor as in (5 b) to (5 d).

2.5 Equilibrium

As was mentioned in subsection 2.1, the presence of future cash valuation w and the company-specific income factor K drives the present model away from the representative agent setup. Particularly, even if the market for a given security clears in aggregate, the mentioned trader heterogeneity generates a nontrivial subset of sellers and the complementary subset of buyers in every period. For the applications of the model to monetary policy transmission issues, it is important that there are always security markets of relevance that have a nontrivial exogenous supply side. More precisely, companies in nonbanking sector issue new debt and other securities belonging to set N. The consequences of the money market rate change for financial companies' demand for these new issues constitute the core of the financial-to-real link in the monetary transmission chain.

In the money market itself, supply and demand generated outside the modeled set of financial institutions play a crucial role. The most important example here is an intervention by the central bank effectuated through the key rate change of the discount bond with a selected maturity. This intervention roughly corresponds to the announcement of a supply/demand schedule of the central bank in the corresponding market segment (see Derviz, 1996, where a continuous time optimizing model addresses the analogous problem of the central bank supply/demand schedule in the foreign exchange market). It is especially important to remember that every such intervention in the discount bond market has an impact both on the immediate change in the liquidity level of the financial sector and on the reverse cash flow at a future date. Both effects have consequences for optimal demand *in all securities markets* and in the resulting new equilibrium of the yield curve, as stated by equations (5 a) to (5 d). These effects are, fortunately, rather transparent, since both the monetary authority and the financial companies take them into account in the course of decision-making. Another effect having to do with the change in the level of deposits, however, has the nature of an externality. Indeed, deposit-taking financial institutions regard the deposit flow as exogenous. Conversely the central bank ought to be aware of the consequences of infusing liquidity of amount $B_t^s j^b(x_{t-1}^f, \varphi_t^s)$ in period t, because at first such an infusion generates increased purchases of corporate paper. Subsequently, one is faced with a deposit-increasing potential in all periods following t.

Finally, there emerges a deposit-decreasing potential induced by $-\varphi_t^s$ (volume of the redeemed discount bills), in all periods following the redemption date s. Inversely, a liquidity contraction (negative φ_t^s) would generate a pair of effects that are mirror images of the ones described above.

In this paper, I adopt the view of a financial institution that takes investment decisions without endogenizing the deposit flow factor. Nevertheless, firms do take into account the consequences of aggregate monetary expansion or contraction for prospective deposit flows, e.g. when building up or depleting the buffer stock of liquid reserves to accommodate the anticipated deposit changes. The relevant information is, indeed, used in the formation of expectations on the K_t^s variable. The exogeneity of K means that, in spite of the information acquired about future Ks in aggregate, they do not belong to the private state variables controlled by an individual financial company. Unlike the latter, the monetary authority can make a direct use of the link between its money market actions and the deposit flow factor in the investment decisions of the financial sector. Or, at least, it can derive useful lessons for itself by analyzing the workings of this link observed from the outside.

In the next section, I derive a set of corollaries from the first order conditions of the financial company optimization problem, which are then used in a number of policy examples.

3 Consequences and Examples

The first order conditions (5 b to 5 d) of the financial company optimization problem (1) to (4) have a common structure of backward stochastic difference equations. To facilitate their analysis, I formulate an auxiliary result to be used throughout the text of this section.

Lemma: Let stochastic processes x, c and f in discrete time be adapted to an information filter F= $(F_t)_{t\geq 0}$, and let x satisfy the backward difference equation

$$x_t = f_t + E_t[(1 + c_{t+1})x_{t+1}], t \geq 0.$$

Put $1 + C_t^s = \prod_{\tau=t+1}^{s} (1 + c_\tau), C_s^s \equiv 0$. Then, given the value of x at $t = 0$,

$$x_t = (1 + C_0^t)^{-1} x_0 - \sum_{n=0}^{t-1} (1 + C_n^t)^{-1} g_{n+1}$$

for an F-adapted process g such that $E^t[g_{t+1}] = f_t$ for all $t \geq 0$. In addition, x satisfies the forward difference equation

$$x_{t+1} = (1 + c_{t+1})^{-1}(x_t - g_{t+1})$$

and the iterated backward equation

$$x_t = E_t\left[(1 + C_t^s)x_s \sum_{n=t}^{s-1} (1 + C_t^n) f_n\right], s \geq t.$$

Proof: The last equation can be obtained by straightforward iteration. A direct check shows that every solution of the backward equation must be of the form $x_t = (1 + C_0^t)^{-1}(x_0 + \zeta_t) - \sum_{n=0}^{t-1}(1 + C_n^t)^{-1} f_n$, where ζ

is a martingale with $\zeta_0 = 0$. One can then easily prove that $g_{t+1} = f_t - (1 + C_0^t)^{-1}(\zeta_{t+1} - \zeta_t)$ • .

It will be convenient to have a compact symbol for the stochastic discount factor mentioned in subsection 2.4. Therefore, put

$$\Lambda_{t_1}^{t_2} = \frac{u_\rho(t_2)}{(1+\Theta)^{t_2 - t_1} u\rho(t_1)}.$$

Evidently, the definition is admissible for any combination of t_1 and t_2; the formula $\Lambda_{t_1}^{t_2}\Lambda_{t_2}^{t_3} = \Lambda_{t_1}^{t_3}$ holds for any three time dates t_1, t_2 and t_3; and, finally, $\Lambda_{t_2}^{t_1} = (\Lambda_{t_1}^{t_2})^{-1}$.

3.1 The Term Structure of Interest Rates

The above lemma will now be applied to equation (5 c), which is first divided by $u_\rho(t)$ and then regarded as a difference equation for X^s. The role of $(1 + c_{t+1})$ is played by Λ_t^{t+1}, and the role of f_t by $\dfrac{u_w(t)}{u_\rho(t)} \dfrac{v'(R_t^s)}{1 + A_t^s}$ (note that character f is used here in a different meaning than in section 2). As a result, (5 c) is restated as

$$\begin{aligned} X_t^s &= \Lambda_t^0 X_0^s - \sum_{n=0}^{t-1} \Lambda_t^n \left(\frac{u_w(n)}{u_\rho(n)} \frac{v'(R_n^s)}{1 + A_n^s} - \Lambda_n^0 \varepsilon_{n+1} \right) \\ &= \Lambda_t^0 X_0^s - \sum_{n=0}^{t-1} \Lambda_t^n \frac{u_w(n)}{u_\rho(n)} \frac{v'(R_n^s)}{1 + A_n^s} + \Lambda_t^0 \sum_{n=0}^{t-1} \varepsilon_{n+1}, s \geq t, \end{aligned} \quad (6)$$

where ε is a purely random noise.

A special case of this formula is obtained when $s = t$. First, observe that in view of (5 d),

$$\begin{aligned} E_{s-1}[\Lambda_0^s] &= E_{s-1}[\Lambda_0^{s-1}\Lambda_{s-1}^s] = \Lambda_0^{s-1} E_{s-1}[\Lambda_{s-1}^s] \\ &= \Lambda_0^{s-1} \frac{1}{1 + i_s} \left(1 - \frac{u_0(s-1)}{u_\rho(s-1)} \right). \end{aligned}$$

Since $X_s^s = 1$ and the original date 0 is arbitrary, a special case of (6) can be obtained by putting $t = s$, taking $(s-1)$-conditional expectations of both sides and replacing 0 by t:

$$\begin{aligned} X_t^s = \Lambda_t^{s-1} \frac{1}{1 + i_s} \left(1 - \frac{u_0(s-1)}{u_\rho(s-1)} \right) + \\ \sum_{n=t}^{s-1} \Lambda_t^n \frac{u_w(n)}{u_\rho(n)} \frac{v'(R_n^s)}{1 + A_n^s}, s \geq t + 1. \end{aligned} \quad (7)$$

On condition of perfect markets, the sum on the right hand side of (7) would vanish, whereas X would be the prices of discount bonds undistorted by transaction costs. Equation (7) then reduces to the standard term structure formula.

In its general form, (7) implies the recursive formula

$$X_{t+1}^s = \Lambda_{t+1}^t \left(X_t^s - \frac{u_w(t)}{u_\rho(t)} \frac{v'(R_t^s)}{1 + A_t^s} \right), \quad (8)$$

from which it follows that the noise terms ε in (6) are zero.

Note that (7) is an equality between F_{s-1}-measurable random variables (although X_t^s is F_t-measurable), which means that it describes the term structure of interest rates, observed at date t, only indirectly. That is why it includes, among other things, the marginal valuation of s date cash flow assessments R_n^s made *at times after the present moment*. These are not particularly natural measures in practice. Therefore, the analysis based on (6), where only past values of R_n^s are involved, is often more convenient. To obtain a direct date t term structure formula, one must take t-conditional expectations of both sides of (7) and make a reasonable assumption about the mechanism of private R_n^s assessment updating. For example, under the assumption of a Markovian nature of all involved random processes and a Bayesian rationality of the optimizing firm, the outcome would be of the form

$$X_t^s = E_t\left[\Lambda_t^{s-1}\frac{1}{1+i_s}\left(1-\frac{u_0(s-1)}{u_\rho(s-1)}\right)\right] + G(t,s)\frac{v'(R_t^s)}{1+A_t^s} \qquad (9)$$

for some nonnegative F_t-measurable random variables $G(\cdot, s)$. The exact expression for function G proves to be very involved unless simplifying assumptions about u, v and the belief-updating procedure are made, and its derivation would be outside the scope of the paper.

According to (9), the *current shadow price* of the bond maturing at time s is influenced by the current valuation of the expected cash flow on the redemption date s. As follows from (6), high values of expected s income push the shadow price of the bond up, since the first derivative of v is a decreasing function. Therefore, it is more probable that the shadow price will exceed the actual one, and the bond will be purchased rather than sold (see the discussion in subsection 2.4).

Another consequence of (9) is that discount bonds have higher prices in the presence of cash flow constraints than in the standard models that do not include them. The difference is equal to the second term on the right hand side. Given that function G does not decrease in s and that the future cash flow values are bounded from above, (9) can serve as an explanation of frequent episodes of downward sloping yield curves observed in many transitional economies.

The shadow bond price corresponds to the equilibrium price level at which the market clears without external interventions. If such an intervention takes place, the equilibrium price and transaction volume depends on the whole supply/demand curve. The latter follows from the transaction costs, i.e. the marginal transaction function, as described by (5 a). The reservation price schedule for the redemption date s segment can be written as

$$B_t^s = \frac{X_t^s}{j_\varphi^b(x_{t-1}^f, \varphi_t^s)}.$$

For example of linear-quadratic transaction function mentioned earlier,

$$j_\varphi^b = 1 + \frac{b^s \varphi_t^s}{x_{t-1}^f}.$$

Therefore, the slope of this curve is an increasing function of parameter b^s and a decreasing function of the level of financial technology. That is, the less significant the role of the transaction cost factors expressed by parameter b is, the flatter the supply/demand schedules of the money market participants are.

We are particularly interested in the outcomes of a key money market interest rate change in this context. Suppose that the maturity at which the central bank intervenes is $s - t$ (e.g. 2W in the Czech case). Setting the key rate r_t^s for this maturity at time t is equivalent to announcing the price

$$M_t^s = \frac{1}{1 + r_t^s}$$

for the s bills at which the central bank is prepared to trade in period t and afterwards. Let us assume that in the previous period the s segment of the money market was in equilibrium with no trade, i.e. $B_{t-1}^s = X_{t-1}^s$. At date t, the outcome will depend on the relation of M_t^s and this date's shadow price X_t^s. If $M_t^s < X_t^s$, the financial sector will buy the bills from the central bank and reduce its liquidity; if $M_t^s > X_t^s$, it is the central bank that buys, while the liquidity flows into the financial sector. Since it is logical to expect a weak influence of transaction costs on the money market (equivalently, high price elasticity of demand or supply), even small changes in the key rate induce large movements of liquidity, both in the model and in reality. Observe that the conventional association of a monetary relaxation (tightening) with a reduction (hike) of the central bank rate tacitly relies on the stickiness of the shadow price X^s around the previous period's equilibrium value. However, if the shadow price moves substantially between the periods, the key rate change may fail to induce the expected change in the money supply.

Chart 3

Central Bank Interventions in the Money Market and the Liquidity Flow

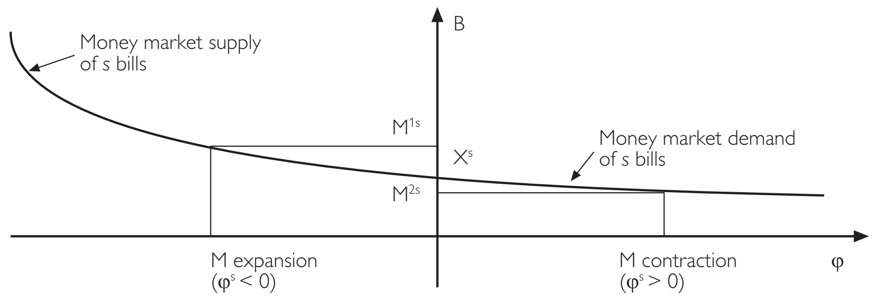


Chart 3 shows the two possible relations of the s bill shadow price and the central bank-imposed actual price during the intervention. In the first variant, the actual price M^{1s} is set higher than the shadow price, and monetary (abbreviated to M) expansion occurs. On the negative φ axis, the financial institution reservation price schedule of chart 3 is to be understood as inverse supply. In the second variant, the actual price M^{2s} lies under the shadow one, and the result is an M contraction. The same reservation price schedule now works in the area of positive φ values and has the meaning of s bill demand.

3.2 The Financial Sector's Demand for New Corporate Securities

Optimizing behavior of the financial institution described in subsection 2.4 implies the form of its demand for a generic security k from the set N.

Analogously to the previous subsection, the lemma will now be applied to equation (5 c), which is first divided by $u_\rho(t)$ and then regarded as a difference equation for Q^k. The role of $(1+c_{t+1})$ is again played by Λ_t^{t+1}, and the role of f_t by

$$\frac{u_w(t)}{u_\rho(t)}\sum_{m\geq t+1}\frac{v'(R_t^m)}{1+A_t^m}(\gamma^k)_t^m+\Lambda_t^{t+1}\Gamma_{t+1}^k.$$

As a result, one derives the following formula for the shadow value Q^k of security k:

$$Q_t^k=\Lambda_t^0Q_0^k-\sum_{n=0}^{t-1}\Lambda_t^n\left(\frac{u_w(n)}{u_\rho(n)}\sum_{m\geq n+1}\frac{v'(R_n^m)}{1+A_n^m}(\gamma^k)_n^m+\Lambda_n^{n+1}\Gamma_{n+1}+\Lambda_n^0\delta_{n+1}\right),$$

where δ is a purely random noise. According to (8), the latter can be transformed by making the substitution

$$\frac{u_w(n)}{u_\rho(n)}\frac{v'(R_n^m)}{1+A_n^m}=X_n^m-\Lambda_n^{n+1}X_{n+1}^m,$$

to get the following expression for Q_t^k:

$$\begin{aligned}Q_t^k=\Lambda_t^0Q_0^k+\sum_{n=0}^{t-1}\Lambda_t^n\sum_{m\geq n+1}(\Lambda_n^{n+1}X_{n+1}^m-X_n^m)(\gamma^k)_n^m-\\ \sum_{n=0}^{t-1}\Lambda_t^{n+1}\Gamma_{n+1}-\Lambda_t^0\sum_{n=0}^{t-1}\delta_{n+1}.\end{aligned}\tag{10}$$

This equation has the advantage of containing observable variables, and can, therefore, be subjected to a rigorous statistical test.

An important category of securities is the one with dividend or coupon payments extended over a finite horizon. Suppose that such a security, with the face value of unity, was issued at time 0, to be repaid at time T. Beside the principal at maturity, the security holder receives an uncertain flow of cash income of h_t for all time periods t between 1 and T.

As usual, valuation of this security type has a lot in common with that of the discount bonds. In particular, its finite life span allows one to derive pricing formulae that, in contrast with (10), contain no noise terms δ. The result obtained will also demonstrate the natural role of shadow price measures Q and X in the arbitrage-free properties of the asset market.

Proposition: The shadow price Q^h of the security issued at $t=0$, with a unit time T redemption value and a finite flow of random incomes $h_t(1\leq t\leq T)$, is given by

$$Q_t^h=\sum_{m=t+1}^{T}X_t^mE_t[h_m]+X_t^T, 0\leq t\leq T-1.\tag{11}$$

Proof: It is easiest to proceed by backward induction. Due to the final time span of security h, one can use the equality $Q_T^h = P_T^h = 1$ for the last day ex-dividend (shadow) price to derive the induction base from the first order condition (5 b):

$$\begin{aligned} Q_{T-1}^h &= \frac{u_w(T-1)}{u_\rho(T-1)} \frac{v'(R_{T-1}^T)}{1+a_T} E_{T-1}[1+h_T] + E_{T-1}[\Lambda_{T-1}^T(1+h_T)] \\ &= X_{T-1}^T E_{T-1}[1+h_T]. \end{aligned}$$

The second equality here follows from (8) for $t = T-1$. The induction step is now a trivial combination of (5b) and (8) for the current time value •.

The above proposition shows that the undisturbed arbitrage-free relationships between assets with finite life horizons exist and are valid for their *shadow prices,* i.e. those hypothetical prices for which the corresponding asset markets clear without outside participation. In the example of equation (11), these are shadow prices Q^h and X. Most important is that for *actual prices* (P^h and B in this example), no-arbitrage relationships are distorted by transaction costs and bid-ask spreads, i.e. the model behaves exactly like reality. However, no matter how much the actual market clearing price is distorted by transaction costs, the no-arbitrage property can be traced down to the shadow price values.

I now offer two applications of pricing equation (11).

Pricing of interest rate swaps: Consider a swap contract involving a fixed coupon rate c and the flexible interest rate r^Δ charged on discount bonds maturing Δ periods from the issuance date. (The relation to the discount bond price defined earlier is given by $B_t^{t+\Delta} = \frac{1}{1+r_{t+\Delta}^\Delta}$.) There are M payment instances, so that the life horizon of the swap agreement is $T = M\Delta$. Let us consider a contract with no default risk. Then, according to (11), the shadow price Q_0^{sw} of the swap (for the receiver of fixed payments) at time $t = 0$ is

$$Q_0^{sw} = \sum_{m=1}^{M} X_0^{m\Delta} E\big[c - r_{m\Delta}^\Delta\big] = \sum_{m=1}^{M} X_0^{m\Delta} c - \sum_{m=1}^{M} X_0^{m\Delta} E\big[r_{m\Delta}^\Delta\big].$$

The accepted convention defines the swap *fair rate* as such value of c for which the initial price P_0^{sw} of this contract (and, therefore, its shadow price Q_0^{sw} as well) is zero:

$$c = \frac{\sum_{m=1}^{M} X_0^{m\Delta} E[r_{m\Delta}^\Delta]}{\sum_{m=1}^{M} X_0^{m\Delta}}. \tag{12}$$

This equation is the reason why long swap rates are usually considered indicators of the expected future short interest rates. According to (12), the fair swap rate is a weighted sum of all expected short rates for the dates of coupon payments prior to and at maturity. The slope of the zero coupon yield curve determines which of these expected rates receive higher weights.

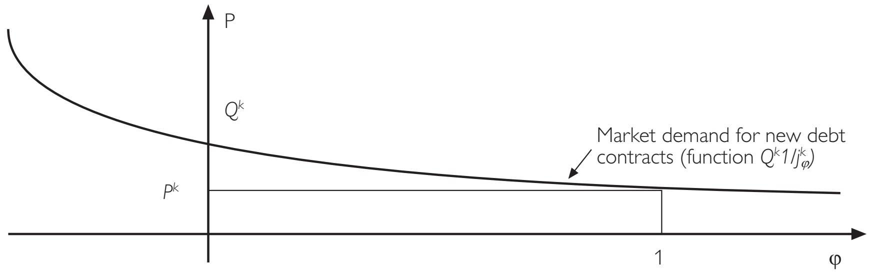
Interest rates on new corporate loans: This application deals with the demand for corporate outstanding debt, in other words, the supply of credit to the real sector. Price P^k in this case corresponds to the terms of credit, whereas Γ^k and γ^k denote the actual and the credit risk-adjusted anticipated interest and principal payments on the loan. The objective of the following exercise is to determine the fair coupon rate a on a one unit principal of a corporate loan issued at $t = 0$ and maturing at $T = \Delta M$. As in the previous example, coupons are to be paid at intervals Δ between $t = \Delta$ and $t = \Delta M$.

For simplicity, I consider a loan with no risk of default. Also, I assume that the loan can be traded in a secondary market. That is, the lender can sell the debt to another holder or exercise an equivalent operation, such as pledging it as collateral on another loan taken from a third party. Then, it is admissible to talk of the actual price P^k and the shadow price Q^k of this loan in the secondary market for any date up to the maturity of the contract. The shadow price is given by (11) with h_t replaced by a.

The relation of the actual and the shadow price in this example is illustrated by chart 4.

Chart 4

The Shadow and Actual Price of New Corporate Debt with Unit Principal



Just like in the case of the swap considered above, the loan has a specific price at the time of issue. By definition, one must have $\varphi_0^k = x_0^k = 1, P_0^k = 1$, since the borrowed principal equals unity. If the transaction function j^k of the lender is known, then the shadow price $Q_0^k = P_0^k j_\varphi^k(x_{-1}^f, 1) = j_\varphi^k(x_{-1}^f, 1)$ is known as well (see [5 a]). Substituting for the known values in (11), obtain

$$a = \frac{j_\varphi^k(x_{-1}^f, 1) - X_0^T}{\sum_{m=1}^{M} X_0^{m\Delta}} = j_\varphi^k(x_{-1}^f, 1)\frac{1 - B_0^T}{\sum_{m=1}^{M} X_0^{m\Delta}}. \tag{13}$$

The second equality in (13) follows from the natural assumption that $X_0^T = B_0^T j_\varphi^k(x_{-1}^f, 1)$ for the same transaction function j^k. Indeed, there is good reason to assume that the transaction function which services the operations with the discount bond maturing at $t = T$, is the same as the one applicable to the loan contract maturing at that date.

A slightly more involved case of corporate lending is represented by a loan with the same formal parameters, but taken out abroad. I will denote foreign variables corresponding to the domestic counterparts defined earlier with asterisks. The coupon paid out in domestic currency is denoted by a^*. The actual price of the loan contract (in foreign currency terms) is $P^* = 1/S$, where S is the nominal exchange rate. The (foreign currency) shadow price of the same contract at time 0 is $Q_0^* = P_0^* j_\varphi^*(x_{-1}^f, 1) = \frac{j_\varphi^*(x_{-1}^f,1)}{S_0}$. At the coupon payment times $t = \Delta m, m = 1, \dots M$, the expected payments to the foreign lender (in his currency) are $h_t^* = a^* E[\frac{1}{S_t}]$. Then, according to the proposition, the shadow price of the loan satisfies the equation

$$j_\varphi^*(x_{-1}^f, 1) = Q_0^* = \sum_{m=1}^{M} X_0^{*\Delta m} E\left[\frac{1}{S_{\Delta m}}\right] a^* + X_0^{*\Delta M} E\left[\frac{1}{S_{\Delta m}}\right],$$

which implies the following international analogue of (13):

$$a^* = \frac{j_\varphi^*(x_{-1}^f, 1) - X_0^{*T} E\left[\frac{S_0}{S_T}\right]}{\sum_{m=1}^{M} X_0^{*m\Delta} E\left[\frac{S_0}{S_{\Delta m}}\right]} = j_\varphi^*(x_{-1}^f, 1) \frac{1 - B_0^{*\Delta M} E\left[\frac{S_0}{S_{\Delta M}}\right]}{\sum_{m=1}^{M} X_0^{*m\Delta} E\left[\frac{S_0}{S_{\Delta m}}\right]}. \quad (14)$$

Analogously to (13), I assume in the second equation above that the transaction function j^* is the same for the T discount bond and the fixed coupon loan with maturity T.

3.3 Cash Flow Determinants of the Forward Exchange Rate Premium

In this subsection, I analyze the difference between the yield differential and the exchange rate change, i.e. the disparity term in the generalized uncovered interest rate parity formula (Derviz, 1999 b), which is the same as the forward exchange rate premium.

Assume that there is a subset $F \subset N$ of securities that earn their returns abroad (in short, *foreign securities).* For a given foreign security $f \in F$, the price and dividend in foreign units of account are P^{*f} and Γ^{*f}. The domestic investor, who uses home cash units, faces the price equal to $P^f = SP^{*f}$ and the dividend equal to $\Gamma^f = S\Gamma^{*f}$. The foreign shadow price $Q_t^{*f} = P_t^{*f} j_\varphi^f(x_{t-1}^f, \varphi_t^f)$ of security f is defined in the same way as the domestic shadow price Q^f in (5 a), so that $Q_t^f = S_t Q_t^{*f}$ for all times t. (Observe that this time, character f is used in a sense different from that in subsections 2.4, 3.1 and 3.2.) Next, I define the *effective total returns* y^k and y^{*f} on any domestic security $k \in N \setminus F$ and foreign security $f \in F$, by

$$1 + y_{t+1}^k = \frac{Q_{t+1}^k + \Gamma_{t+1}^k}{Q_t^k}, 1 + y_{t+1}^{*f} = \frac{Q_{t+1}^{*f} + \Gamma_{t+1}^{*f}}{Q_t^{*f}}.$$

Values y^k and y^{*f} stand for the yields earned by an investor facing the transaction costs.

Now, dividing (5 b) by the shadow security price and using the above definition of effective total yields, I rewrite the first order condition (5 b) for $k \in N \setminus F$ and $f \in F$ as

$$u_\rho(t) = u_w(t) \sum_{m \geq t+1} \frac{v'(R_t^m)}{1 + A_t^m} \frac{(\gamma^k)_t^m}{Q_t^k} + E_t\left[\frac{1 + y_{t+1}^k}{1 + \Theta} u_\rho(t+1)\right], \quad (15a)$$

$$u_\rho(t) = u_w(t) \sum_{m \geq t+1} \frac{v'(R_t^m)}{1 + A_t^m} \frac{(\gamma^f)_t^m}{Q_t^f} + E_t\left[\frac{1 + y_{t+1}^{*f}}{1 + \Theta} \frac{S_{t+1}}{S_t} u_\rho(t+1)\right], \quad (15b)$$

and apply the lemma to process $u_\rho(t+1)$ in both these difference equations.

The result is

$$u_\rho(t+1) = \frac{1+\Theta}{1+y^k_{t+1}} \left(u_\rho(t) - u_w(t) \sum_{m \geq t+1} \frac{v'(R^m_t)}{1+A^m_t} \frac{(\gamma^k)^m_t}{Q^k_t} + \eta^k_{t+1} \right),$$

$$u_\rho(t+1) = \frac{S_t}{S_{t+1}} \frac{1+\Theta}{1+y^{*f}_{t+1}} \left(u_\rho(t) - u_w(t) \sum_{m \geq t+1} \frac{v'(R^m_t)}{1+A^m_t} \frac{(\gamma^f)^m_t}{Q^k_t} + \eta^k_{t+1} \right)$$

for some processes η^k and η^f for which $E_t \lfloor \eta^{k,f}_{t+1} \rfloor = 0$. These two equations can be combined to render

$$1 + y^k_{t+1} = (1 + y^{*f}_{t+1}) \frac{S_{t+1}}{S_t} \frac{1 - \frac{u_w(t)}{u_\rho(t)} \sum_{m \geq t+1} \frac{v'(R^m_t)}{1+A^m_t} \frac{(\gamma^k)^m_t}{Q^k_t} + \frac{\eta^k_{t+1}}{u_\rho(t)}}{1 - \frac{u_w(t)}{u_\rho(t)} \sum_{m \geq t+1} \frac{v'(R^m_t)}{1+A^m_t} \frac{(\gamma^f)^m_t}{Q^f_t} + \frac{\eta^f_{t+1}}{u_\rho(t)}}. \quad (16)$$

Taking conditional time t expectations and approximating the right hand side of (16), one gets

$$E_t \left[y^k_{t+1} - y^{*f}_{t+1} \right] \approx E_t \left[\frac{\Delta S_{t+1}}{S_t} \right] + h^0$$

$$+ \frac{u_w(t)}{u_\rho(t)} \sum_{m \geq t+1} \frac{v'(R^m_t)}{1+A^m_t} \left(\frac{(\gamma^f)^m_t}{Q^f_t} - \frac{(\gamma^k)^m_t}{Q^k_t} \right). \quad (17)$$

Here, h^0 is the term originating from conditional covariances. It is usually small and close to a constant. Equations (16) and (17) render two formulations of the generalized uncovered asset return parity. The approximate form (17) is closer to the standard statements of uncovered parity in that it separates the forward exchange premium from the yield differential in an additive way.

The interpretation of the forward premium follows from that of the terms in brackets summed up on the right hand side of (17). The ratios $\frac{(\gamma^k)^m_t}{Q^k_t}$ and $\frac{(\gamma^f)^m_t}{Q^f_t}$ are time t-expected effective period m returns on investment in k and f, for the domestic investor. In the home country of security f, however, the same return on investment is evaluated as $\frac{(\gamma^{*f})^m_t}{Q^{*f}_t}$. The difference in returns appearing in (17) is equal to

$$\frac{(\gamma^f)^m_t}{Q^f_t} - \frac{(\gamma^k)^m_t}{Q^k_t} = \frac{S^t_m}{S_t} \frac{(\gamma^{*f})^m_t}{Q^{*f}_t} - \frac{(\gamma^k)^m_t}{Q^k_t},$$

where S^t_m is the date m spot exchange rate expected at date t. Let us take the simplest case when the *national* returns on investment are equal for both securities, i.e. $\frac{(\gamma^{*f})^m_t}{Q^{*f}_t} = \frac{(\gamma^k)^m_t}{Q^k_t}$, $m \geq t+1$. Then a high expected depreciation in late periods (high values of S^t_m) generates a high forward premium, so that the depreciation in the next period is not expected to be too high, and vice versa. Contributions of future expected depreciations to the current forward premium are weighted by $\frac{v'(R^m_t)}{1+A^m_t}$. Since v' is a decreasing function, periods with high expected cash flow contribute less to the forward premium than periods with low cash flow. According to (17), the forward premium is a nonconstant autoregressive process.

4 Transmission of CNB Rates along the Yield Curve

The results of the model discussed in the previous two sections indicate that the properties of monetary transmission in the modeled economy are significantly dependent on the properties of the term structure of interest rates. The latter, in turn, is determined by a number of factors beyond of the short rate statistics, unlike in the standard theories. In particular, the shape of the yield curve in the present model accommodates three distinct aspects of optimal behavior. The most traditional one is intertemporal risk sharing, expressed by the marginal utility of current dividends entering the stochastic discount factor. Another conventional aspect is the solvency requirement, which, somehow less traditionally, is in this model associated with the marginal utility of current cash holdings. The third aspect is more novel and has to do with the asset-liability management decisions of financial intermediaries. Technically, the presence of these decisions in the model is reflected by the marginal utility of the net expected future cash flow as well as other future income valuation terms in the term structure formulae.

In the sections to follow, I investigate the ability of the model to take account of monetary policy actions and their transmission, most importantly via the credit channel.

Since the central banks of many countries control overnight rates, the question of short-long rate transmission is most often posed in the same way in theory, i.e. the instantaneous rate statistics are linked to the prices of all other discount bonds. This approach would be difficult to apply to the Czech situation, where monetary policy is exercised by means of 2W repo limit rates and some other auxiliary rates of longer maturities, particularly 3M. The overnight rate is not controlled and, indeed, shows a lot of excessive technical (nonfundamental) volatility out of line with other money market instruments. The model of sections 2 and 3 accommodates this property by making the one-period interest rate endogenously dependent on the current liquidity and preference structure of the agents, in contrast with all other rates, where explicit market segments are subsumed. The short-long rate transmission is traced to the mutual dependence of the prices of discount bonds that mature later than in the period immediately following the current one.

The transmission of the 2W repo rate into the immediately corresponding maturities in the money market can be regarded as an indirect consequence of the model asset price formulae. Indeed, let the 2W repo be expected to remain constant during the next $T = \Delta M$ periods from now, where Δ=14 days and M is the number of two-week periods before the maturity of the discount bond whose return we want to know.

Let $t = 0$ be the date when the 2W bond market segment is in equilibrium with no net flow of liquidity to or from the central bank. In terms of the notations of subsection 2.4, this means that the shadow price X_0^Δ of the 2W discount bond is now equal both to the repo bill price and the actual price B_0^{Δ} of the 2W bond. Now, consider a swap agreement of M fixed payments of a rate c against M payments of the actual (variable) 2W money market rates (equal to $\frac{1}{B_t^{t+\Delta}} - 1, t = \Delta, 2\Delta, \dots, M\Delta$, at time intervals of Δ. As follows from the formula for the swap rate derived in subsection 3.2, c is a weighted sum of the expected future values of the 2W rate, with the weights determined by the current shape of the yield curve. If the 2W segment of the money market is expected to remain in equilibrium with the unchanged level of the repo rate for the next T days, then the swap rate c must be equal to this central bank key rate as well. In other words, the market is indifferent between the fixed and the flexible rate of return for maturity $T = \Delta M$, since it offers the same rate on both sides of the swap agreement. Therefore, the annualized return on the T discount bond must also be fairly close to the same value. The same conclusion is valid for the other maturities, even if they are not exactly divisible by Δ. That is, a swift transmission of the key rate into other money market rates is achieved on condition of a reliable time horizon of the constant key rate policy, whereas the discount bond returns stay close to the key rate *for the maturities lying inside this constant policy interval.*

The model offers one more inference concerning the shape of the zero-coupon yield curve. Namely, it predicts a much higher occurrence of downward sloping yield curve episodes than the term structure models coming from standard portfolio optimization settings. A formal argument in support of this statement was presented in subsection 3.1. Informally, the presence of constraints on payment maturity mismatches (in the form of an additional state variable in the utility function measuring individually assessed future net income) is a factor that pushes up the prices of discount bonds with long maturities. Equivalently, long interest rates are pushed down. This effect becomes more pronounced for periods in which the current state of the balance sheet predicts large negative cash flows in the future, e.g. an accumulation of debt service payments within a substantial time interval. The intuition behind this phenomenon has to do with the positive correlation between the predictable debt service accumulation in a future period and the current period prices of hedging instruments (in the present context, of the discount bonds) related to the same future dates. In other words, when a financial institution knows it will have to disburse money later, its cash flow constraint dictates it to compensate this negative future cash flow now by trading some of the current liquidity against future liquidity.

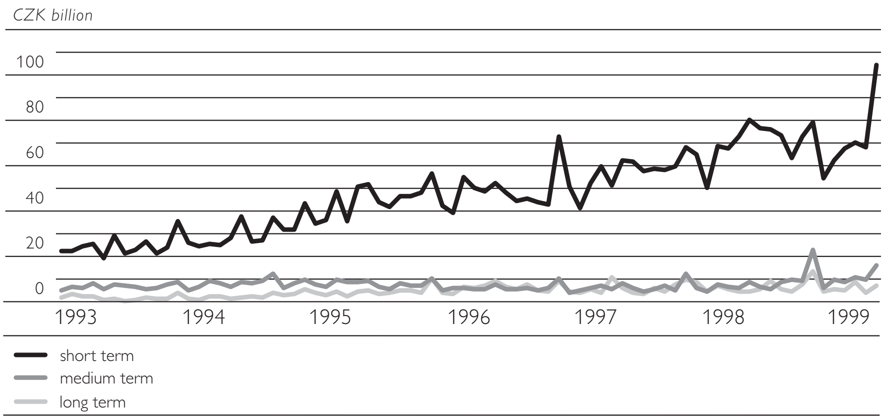
The traditional theory would typically result in a flat or upward sloping yield curve, depending on the short rate statistics. Therefore, the depressing effect of the cash flow constraint in the present model is insufficiently offset by the short rate factor at times at which the market expects its decrease. What one gets is a fairly realistic picture of the term structure in a transitional economy, where positive and negative slopes can be observed equally often.

5 The Shadow Interest Rate on New Loans and the Actual Credit Conditions

In the Czech economy, there is a distinct difference between the short-term and the medium-to-long-term credits granted by banks to nonbanks. While the volume of short-term loans has been growing on average during the reform years, credit volumes in the other two categories have been stagnating (chart 5). As regards the interest rates, all three categories more or less reflect the developments in the money market, even if the connection of the short-term credit conditions with the corresponding PRIBOR (Prague Interbank Offered Rate) rates is much more pronounced than in the other two groups (chart 6). For the above reason and given its weight in the total credit volume, short-term credit seems to be the best object with which to analyze the transmission mechanism.

Chart 5

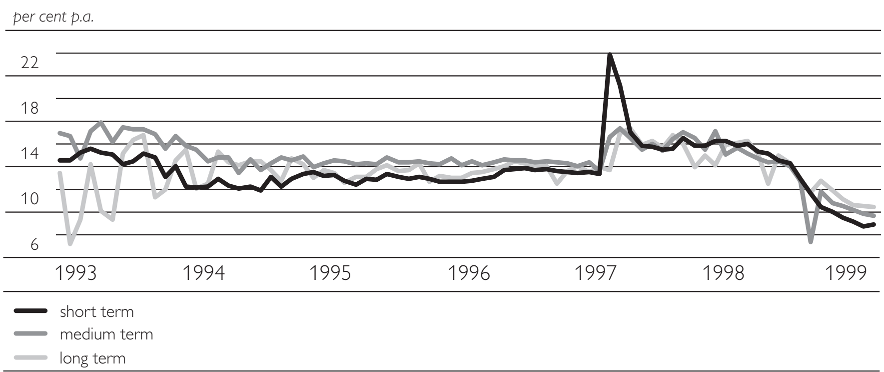
New Credit Volumes in the Czech Economy



Source: CNB.

Chart 6

Interest Rates on Newly Granted Loans

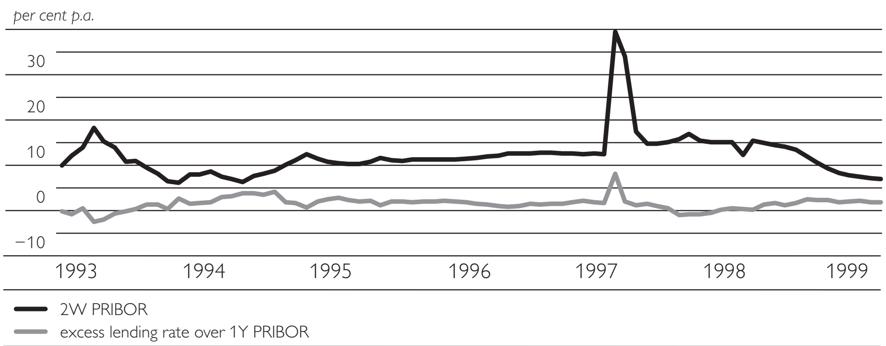


Source: CNB.

The dependence of new corporate loan rates on the money market rates will be checked in three ways. First of all, I investigate the general wisdom about the new loan rate being a simple markup over the chosen interbank rate. Most typical for the Czech reported conventions is the 1Y PRIBOR. Chart 7 shows the difference between the average interest rate charged on a new short-term loan and the average 1Y PRIBOR value for the same month, together with a simple indicator of the monetary policy stance, expressed by the 2W PRIBOR. (Deviations of the latter from the 2W repo rate set by the Czech National Bank prove to be negligible.)

Chart 7

New Lending Rate Markup over Czech Money Market Rates and the CNB's Monetary Policy



Source: CNB.

There are two messages to be read off chart 7. First, the data do not confirm the claimed simple mark up rule for setting new loan interest rates. The credit conditions seem to be much softer than such a rule would imply. Near-zero or negative deviations from 1Y PRIBOR, falling into periods of strict monetary policy, are particularly conspicuous. Second, the overall correlation of the mark up with the monetary conditions is doubtful. In short, despite the widespread view of the Czech private credit market as an area of bank dominance, the data suggest a fairly limited market power of lenders over borrowers. At some points in time, borrowers were able to negotiate credit conditions that hardly remunerated the lenders at all.

Next, I regress the same short-term lending rate as above, which I denote r^{sh}, on the four relevant money market rates, i.e. the PRIBOR values for three, six and nine months and one year, i^{3M}, i^{6M}, i^{9M} and i^{1Y}. The result of this regression exercise is the following (standard errors in parentheses):

$$\begin{array}{llllll} r^{sh} = & 5.41 & + 1.32 i^{3M} & - 1.65 i^{6M} & - 0.58 i^{9M} & + 1.59 i^{1Y}, \\ & (0.46) & (0.24) & (0.90) & (1.41) & (0.77) \end{array}$$

R^2=0.86, the standard error of regression is 0.86, and the Durbin-Watson statistic is 2.01.

Let me say a few words about the risk of multicollinearity in the above equation. Several findings on Czech money market rates suggest that the only rate suffering from probable full spanning by other maturities is i^{9M}. However, an alternative regression omitting this maturity renders qualitatively similar results. Therefore, I will refer to the formulation including all four rates in the regression, so that I can compare it with the nonlinear formula for r^{sh} resulting from the model of sections 2 and 3, to be spelled out shortly. Another comment has to do with alternative regressions of lending rates on money market rates, known from the literature, particularly those utilizing first differences of the variables instead of their levels (Bernanke and

Blinder, 1992; or Cook and Hahn, 1989). For the purpose of the present study, first differences would not be the right tool, since we are looking for explicit credit-granting mechanisms and not just the general impact of the interbank market on credit conditions. Therefore, the regression of first differences, whatever its outcome might be, is unable to provide enough information about the lending policy of banks.

Arguably, several properties of the obtained regression outcome are counterintuitive. For example the constant value is too high and does not correspond to any reasonable markup following from the direct comparison of time series means for r^{sh} and PRIBOR rates. However, an attempt to estimate a restricted equation (without the intercept) leads to even more nonsensical results. Furthermore, while the i^{1Y} coefficient is too big, the sizes and signs of the remaining three coefficients are hard to justify. On the other hand, satisfactory values of R^2, DW, and a number of other diagnostic tests suggest that, although the variables for the regression have been chosen well, the simple linear functional form of the equation has been unfortunate.

The disappointing result of the above linear regression does not mean that a better model would not shed more light on the link between private credit and money market interest rates. The formulae of subsection 3.2 provide a result that can serve as an indicator of the right functional form to replace the unsatisfactory linear one. Namely, the lending rate r set by an optimizing financial institution is given by (see equation (13) of subsection 3.2)

$$r = g \frac{r_0^T}{(1 + r_0^T \sum_{m=1}^{M} X_0^{m\Delta}}$$

In this equation, $T = M\Delta$ is the loan maturity date, r_0^T is the return rate of a discount bond with maturity $T, X_0^{m\Delta}$ is the shadow price of the discount bond maturing at $t = m\Delta, m = 1, \ldots, M$,while the latter are the debt service dates at which the coupon r is to be paid. Term g, which expresses the transaction costs and other microstructural features of the market for private debt, is model-specific. Analogous coefficients $g^{m\Delta}$ are present in the markets for the corresponding discount bonds and are connected to their shadow prices by the rule $X_0^{m\Delta} = B_0^{m\Delta} g^{m\Delta} = \dfrac{g^{m\Delta}}{1 + r_0^{m\Delta}}$, where $B_0^{m\Delta}$ is the actual date zero price of the bond. Shadow prices coincide with actual prices ones when the markets clear without trades with outside buyers or sellers (i.e. those that are not explicitly modeled as the representative financial firm).

I will call the rate r defined above the *shadow interest rate* on the loan of unit principal with maturity date T. Its meaning is the value of the interest rate which the borrowers must pay if they wish to get a loan of the size which they fix themselves. In reality, the bargaining between banks and nonbanking borrowers should result in a joint determination of both the size and the interest on the granted loans, roughly corresponding to the intersection of the supply and the demand schedule, like in any other market. Nevertheless, the shadow rate is a useful benchmark if one wants to measure the "price" of keeping a constant volume of granted credit in a changing environment.

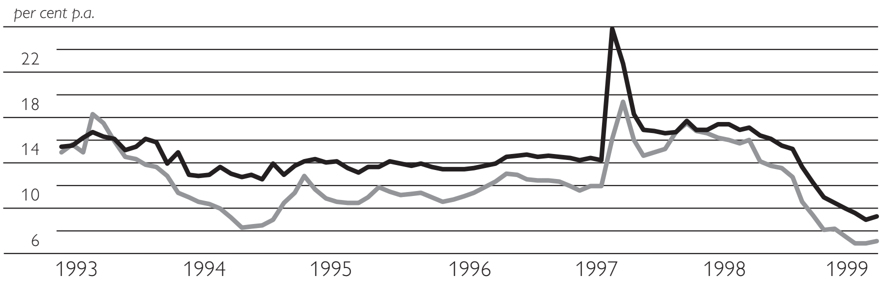
Under the assumption that all microstructural coefficients $g^{m\Delta}$ are close together and nearly constant (the latter is denoted by g^T), the shadow interest rate is equal to

$$r = \frac{g}{g^T} \frac{r_0^T}{\sum_{m=1}^{M} \frac{(1+r_0^T)}{(1+r_0^{m\Delta})}}.$$

This is a value that can be calculated using the available data on the money market rates. The microstructural coefficients can be eliminated by assuming that g is close to g^T and that their ratio is almost a constant. The shadow and the actual rates on a one-year loan with quarterly coupon payments are given in chart 8 a. The chart shows a much better correspondence between the two rates than in the case of the 1Y PRIBOR and the new loan rate. In other words, the shadow rate hints at the true nonlinear relation that exists between the PRIBORs and the private lending rate. The model prescribes

Chart 8a

Actual and Shadow Short-Term Credit Conditions in the Czech Republic

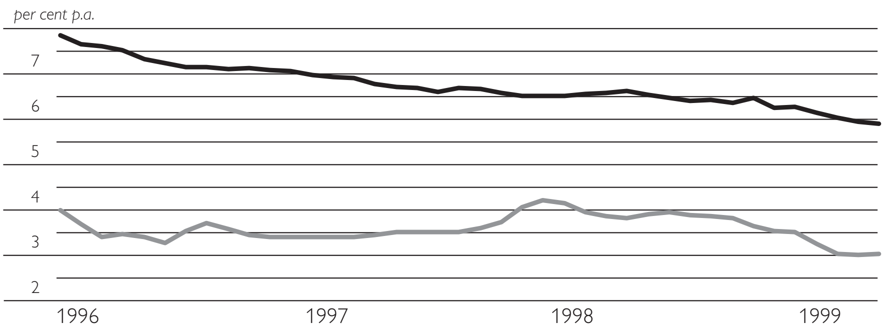


new short-term corporate lending rate
shadow 1Y corporate lending rate

Source: CNB.

Chart 8b

Actual and Shadow Short-Term Credit Conditions in Austria



actual 1Y new corporate lending rate
shadow 1Y lending rate

Source: OeNB.

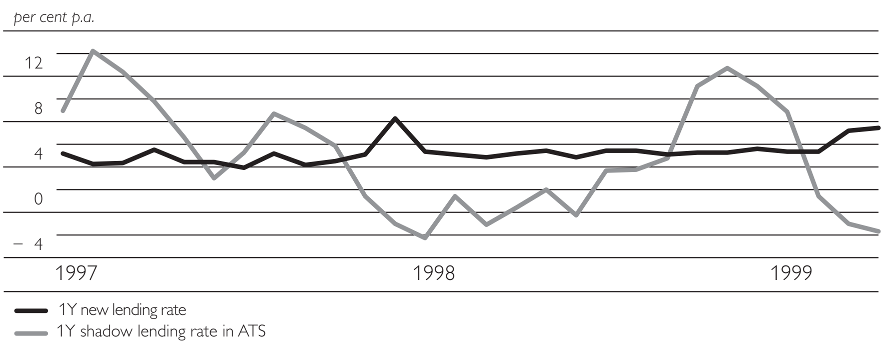
an almost constant distance between the shadow and the true rate at all times, given that the volume of credit remains at a constant level. When this distance falls short of the usual level, it indicates an overall tightening of credit conditions and predicts a reduction in the volume of granted loans. This can be identified with the *credit channel* of monetary transmission. The notion of the shadow rate helps to separate it from the money, or interest rate channel, which presupposes a direct transmission of money market rates.

Chart 8 b features the actual and the shadow short-term lending rate for the Austrian economy. Here, the correspondence is very weak. The reason may be a difference in microstructure compared to the Czech market. Indeed, the data on Austrian lending rates indicate a very broad range of values, plus a considerable margin over the money market rate. The market power of the banks over clients in Austria is a factor that makes the interbank-corporate rate spreads a variable with a nontrivial law of motion of its own.

The calculations carried out in subsection 3.2 also cover the case of a loan taken out abroad, and render the corresponding shadow rate. There, exchange rate expectations play a predominant role. Therefore the calculated shadow rate values based on *ex post* exchange rate movements (chart 9) are very much influenced by forex market volatility. That is why the *ex post* shadow rate may sometimes seem prohibitively high and at other times permissively low. However, the presence of hedging instruments in the forex market suggests that the shadow rate of interest for foreign loans is to be calculated on the basis of forward rather than spot exchange rate values. In that case, it can become a much better indicator of the credit conditions for domestic firms in foreign markets.

Chart 9

Actual and Shadow Lending Rate for Czech Clients in Austria



Source: CNB.

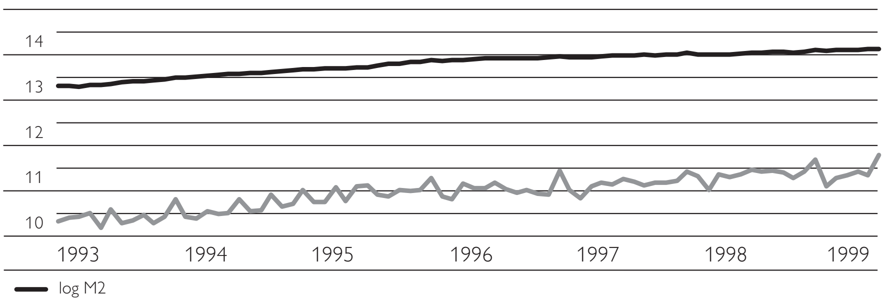
6 Portfolio Choice and Monetary Policy Effects

The discussion in the previous sections was aimed at showing that monetary policy transmission, viewed through the lens of an optimizing agent in the financial sector, is a process that materializes in the behavior of the money market instrument prices. Within the chosen modeling approach, the latter are the discount bond rates that define the term structure of the economy's interest rates. The central bank's key rate decisions are transmitted as far as the horizon of its expected constancy can be extended in the beliefs of the market participants. Open market operations, on the other hand, in addition to guiding the current amount of liquidity, also change the predicted cash flow schedules of financial institutions (a formal treatment of open market operations was given at the end of subsection 3.1). Consequently, asset prices, which depend on these cash flow assessments, often react in such a way that the immediate consequence of a decision by the central bank to ease or tighten monetary policy has a side effect. One example is the yield curve slope predicted by the model, as discussed in section 4. The other is the credit channel effect discussed in section 5, namely the input of the totality of the yield curve values up to the loan horizon into the shadow lending rate characterizing the credit conditions for the nonbank sector. Naturally, in their impact on the yield curve, policy measures and exogenous factors mix. In longer maturities, the exogenous factors clearly dominate, so that the corresponding parts of the term structure and of asset prices in general display a highly autonomous behavior almost unaffected by the policy measures. Below, I give two more examples of the limits that monetary policy has in the pursuit of its traditional goals, namely control of the quantity of money and control of the exchange rate through central bank operations in the money market.

Control of M2: Between February 1996 and December 1997, the Czech National Bank conducted a policy of monetary base targeting. After the policy was changed to direct inflation targeting in January 1998, the monetary aggregates remained the key indicators. Particular attention was paid to the M2 aggregate (cash plus most types of domestic and foreign currency deposits with maturities of up to two years) and its extension, called L (M2 plus the official bills in the hands of nonbank public). These two broad indicators

Chart 10

CZK Money Quantity and New Credit



Source: CNB.

were assigned the decisive role for the central bank's inflation target. A low sensitivity of the aggregates to the policy measures was a disturbing circumstance. Instead, M2 (and L, which is not significantly different) was following its own pace of autonomous growth. It is interesting to note, recalling our discussion of credit to nonbanks in section 5, that the broad money growth rate almost exactly coincides with the average growth rate of the new credit volume in the Czech economy (chart 10). The credit volume, however, shows an additional volatility, which is independent of the money quantity variable.

The mentioned policy insensitivity of M2, however disappointing it may be for a central banker, is well explained by the model with an explicit choice-theoretic foundation for the portfolio decisions of the banking sector. To show this in the setup of the present model, it is necessary to associate the broadest possible money aggregate in the economy with a wealth measure of a representative financial firm. I base the argument on the fact that, while the analyzed broad measure of money is the liability side of this agent's balance sheet, it can be captured equally well by the asset side in the form of aggregate financial wealth. As in most other applications of the model, it is convenient to use the notion of shadow prices.

In the notations of section 2, let x be the vector of private security amounts in the financial sector portfolio, and Φ be the vector of the discount bonds (indexed by maturity dates) in the same portfolio. The respective shadow price vectors are denoted by Q and X. Respecting the nature of the model, I must also take into consideration the shadow price V of an auxiliary variable x^f expressing the level of financial technology in the economy (section 2 contains a detailed description). Preferences of the representative financial firm, expressed by the period utility function u, give rise to a stochastic process denoted $\Lambda_t^{t+1} = \dfrac{u_\rho(t+1)}{(1+\Theta)u_\rho(t)}$ (it is the analogue of the stochastic discount factor of the standard portfolio optimization models under uncertainty; see Duffie, 1992). Here, ρ stands for the dividends disbursed by the financial sector to the public per period. Parameter Θ is the time preference rate. Symbol u_ρ is used to denote the corresponding partial derivative. The time argument in the utility is a shorthand indicating at what date its true arguments were measured and substituted into the formula. It is assumed that the utility dependence on ρ is of the HARA (hyperbolic absolute risk aversion) type.

Define the shadow ex-dividend wealth as

$$W = x^0 + Q \cdot x + X \cdot \Phi + V x^f$$

Under certain conditions on homogeneity and the multiplicative structure of the utility and transaction functions, it can be shown that

$$E_t\big[\Lambda_t^{t+1}(W_{t+1} + \rho_{t+1})\big] = W_t + \rho_t - \beta \frac{u(t)}{u_\rho(t)}$$

for some constant β dependent on risk aversion.

As follows from the above formula, the cum-dividend wealth grows in line with the long-term average rate of dividends which the financial firms disburse to their owners according to an internal optimality rule. All the other shocks are dampened because of the portfolio reshuffling decisions that to a large extent offset the changes in asset supplies (the exact reason is the induced change in relative prices). In aggregate, all that remains is a very indirect reaction of a decision variable inside the utility function. Although the chosen wealth measure is based upon the shadow rather than the actual prices, the difference caused by the microstructure effects should be very small in aggregate. Also, although the defined wealth stands for the broadest possible money aggregate, the difference from the conventionally used measure M2 or L is relatively small in the Czech case. The reason is the relatively small weight of long-horizon instruments in the financial wealth. Accordingly, any high monetary aggregate that one would choose to follow would exhibit a very low sensitivity to monetary policy measures.

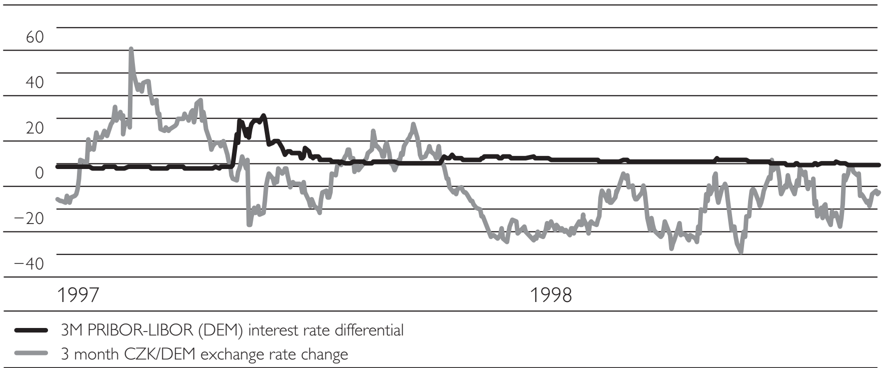
Key rate transmission into the spot exchange rate: No matter how much discredited, the dogma persists in the minds of development economists and policy advisers that freely floating currency depreciation can be achieved by cutting short-term interest rates. Historically, this atavistic belief originates in the textbook uncovered interest rate parity (UIP) statement. Even though both the empirical and the theoretical deconstruction of UIP is by now almost complete (see, e.g., McCallum, 1994; Meredith and Chinn, 1998; a survey and a model for the Czech currency can be found in Derviz, 1999), policy recommendations of the mentioned sort do not cease to crop up. The reality behind UIP is such that it is not – and cannot ever be – valid for the short end of the money market. Instead, there exists strong evidence in favor of the uncovered parity between *yields on long maturity instruments.* However, the latter are endogenous variables whose direct control by monetary policy tools is impossible. A credible permanent reduction in the short rates is, indeed, a precondition for the corresponding fall at the long end of the yield curve. Nevertheless, this is seldom a result of monetary policy alone. Rather, the financial markets must be convinced that the fundamental condition of the economy has irrevocably moved it deep into the low rate area. Therefore, the relevant question for the policymaker should not be the magnitude of the rate cut needed to achieve a specific exchange rate increase but, instead, how much of the key rate reduction is consistent with the internal balance and price stability progress. As a reward, the overall descent of the yield curve can be a good protection against uncontrolled capital inflows and a sharp appreciation of the currency.

The evidence on what was happening with the yield curve and the Czech koruna exchange rate with respect to the CNB's key rate policy is no different from other examples known from empirical literature. The poor performance of the classical UIP is shown in chart 11. In contrast, chart 12 features a strikingly close correspondence of short-term moves of the yield differential between Czech and German five-year government bonds and the logarithm of the nominal CZK/DEM exchange rate. Just a few short episodes of the country premium revision disturb the uncovered total return parity (a theoretical explanation of the country premium moves is outlined in subsection 3.3). For comparison, the uncovered parity of the Austrian schilling against the U.S. dollar, measured with the ten-year government

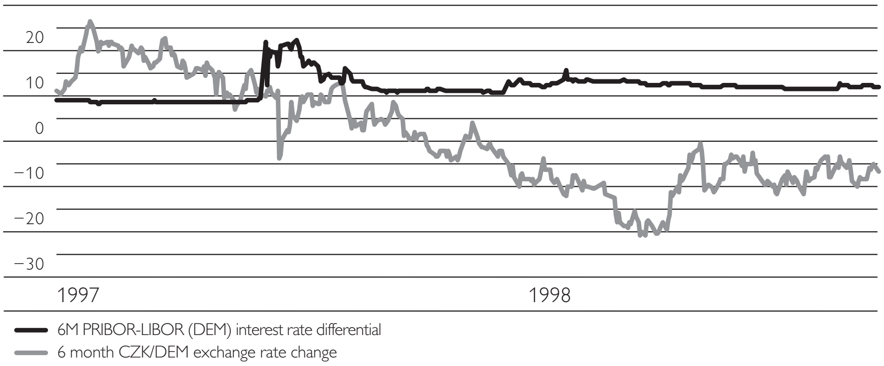
Chart 11

Czech-German Interest Rate Differential and the Exchange Rate Changes

Three Months



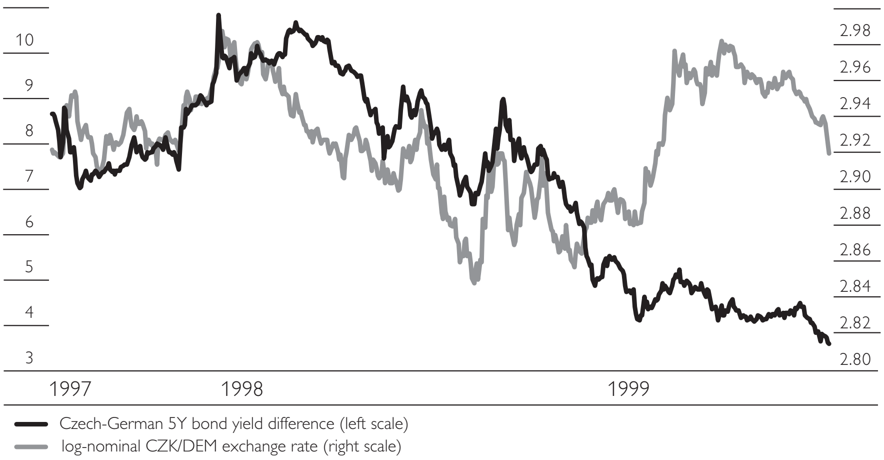
Six Months



Source: CNB.

Chart 12

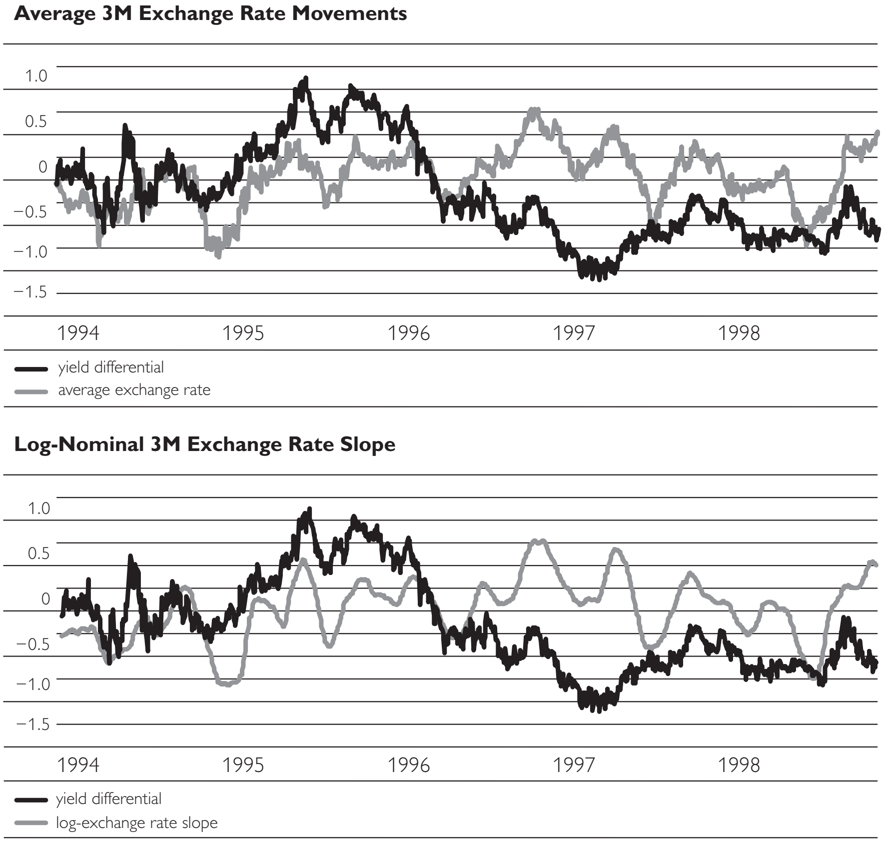
Czech-German Five-Year Government Bond Yield Difference and the Nominal CZK/DEM Exchange Rate



Source: CNB.

Chart 13

Austrian vs. U.S. Ten-Year Government Bond Yields and the ATS/USD Exchange Rate



Source: OeNB.

bond yield differential, is illustrated in chart 13. One observes that the time horizon for which the relative exchange rate movements confirm the uncovered parity rule is different for the two currencies. It lies between two weeks and two months for CZK, compared to three to four months for the ATS and other EMU currencies. An explanation may be hidden in the different typical holding times of Czech and EMU country government bonds, since they probably attract different types of international investors. A detailed treatment of the uncovered return parity of the exchange rate is given in Derviz, 1999 b.

7 Conclusion

The paper has identified a number of specific features of monetary policy transmission inside the Czech financial sector during transition. It has visualized the (uneven) performance of financial intermediaries as the key factor behind the credit conditions in the Czech economy. The model proposed to explain the functioning of the credit channel of monetary transmission is based on an optimizing decision-making of a financial institution restricted by liquidity and cash flow constraints. The environment of portfolio decisions is stochastic in discrete time. The key property of the solution is the existence of the so-called shadow prices of assets in the financial institution portfolio, which alone obey the standard no-arbitrage rules, while the actual prices deviate from their arbitrage-free "shadows" for transaction cost and other microstructural reasons.

The following features can be regarded as the main contributions of the proposed model to the analysis of the transmission mechanism in a transitional economy.

– The model explains the observed long periods of a negatively sloped yield curve, in that it points at additional factors pushing up the discount bond prices;
– It reveals a nonlinear implicit dependence of new loan rates on money market rates;
– It draws a distinction between the "naive" and the microstructurally adjusted arbitrage-free relationships between asset prices;
– It gives an explanation of the empirically plausible autoregressive property of the forward exchange rate premium and offers a structural decomposition of the latter.

The following lessons for Czech monetary policy can be drawn by confronting the formal analysis with the empirical evidence on the properties of transmission mechanism:

1. The credit channel is present in the Czech economy and cannot be ignored. Its current state can be assessed by comparing the actual and the shadow interest rates on new debt. The consequences of the credit channel blockage can be particularly severe for corporate debt with short maturities.
2. The short-long interest rate transmission can be explained by the cash flow effect in the term structure. Since the cash flow variable is likely to be volatile in transitional economies, the asset-liability management considerations of firms in the financial sector can either suppress the original monetary policy signal of the key rates or multiply its effect to an undesirable magnitude. Therefore, key rate change decisions must be avoided at times of upward movements of either a part or the entirity of the yield curve.

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