OESTERREICHISCHE NATIONALBANK
EUROSYSTEM

WORKING PAPER 146

The Optimal Mix Between Funded and Unfunded Pensions Systems When People Care About Relative Consumption

Markus Knell

Statement of Purpose

The Working Paper series of the Oesterreichische Nationalbank is designed to disseminate and to provide a platform for discussion of either work of the staff of the OeNB economists or outside contributors on topics which are of special interest to the OeNB. To ensure the high quality of their content, the contributions are subjected to an international refereeing process. The opinions are strictly those of the authors and do in no way commit the OeNB.

Imprint: Responsibility according to Austrian media law: Günther Thonabauer, Secretariat of the Board of Executive Directors, Oesterreichische Nationalbank
Published and printed by Oesterreichische Nationalbank, Wien.
The Working Papers are also available on our website (http://www.oenb.at) and they are indexed in RePEc (http://repec.org/).

Editorial

In this paper the author derives the optimal portfolio mix between a funded and an unfunded pension system when people care about their consumption relative to a reference group. Pay-as-you-go systems with fixed contribution rates have the property that pension benefits are tied to labor income. This lowers the uncertainty of individuals' future relative position and thus increases the attractiveness of unfunded systems. The paper shows analytically that in an OLG model the optimal share of funding decreases with the strength of individuals' concern for relative standing. A calibrated version of the model that uses data for various countries and time periods suggests that the sensitivity of the optimal share of funding to the concern of relative standing is also quantitatively important. For reasonable assumptions about reference standards it is typically around 20%.

September 1, 2008

The Optimal Mix Between Funded and Unfunded Pensions Systems When People Care About Relative Consumption

Markus Knell*
Oesterreichische Nationalbank
Economic Studies Division

September 2008

Abstract

In this paper I derive the optimal portfolio mix between a funded and an unfunded pension system when people care about their consumption relative to a reference group. Pay-as-you-go systems with fixed contribution rates have the property that pension benefits are tied to labor income. This lowers the uncertainty of individuals' future relative position and thus increases the attractiveness of unfunded systems. The paper shows analytically that in an OLG model the optimal share of funding decreases with the strength of individuals' concern for relative standing. A calibrated version of the model that uses data for various countries and time periods suggests that the sensitivity of the optimal share of funding to the concern of relative standing is also quantitatively important. For reasonable assumptions about reference standards it is typically around 20%.

Keywords: Pension Systems; Social Security; Risk sharing; Portfolio Choice; Relative Consumption
JEL-Classification: H55; G11; E60

*Otto-Wagner-Platz 3, POB-61, A-1011 Vienna; Phone: (++43-1) 40420 7218, Fax: (++43-1) 40420 7299, Email: Markus.Knell@oenb.at. The views expressed in this paper do not necessarily reflect those of the Oesterreichische Nationalbank. I thank Helmut Elsinger, Martin Summer, an anonymous referee and seminar participants at the Universities of Göteborg and Zürich for valuable comments and suggestions.

1 Introduction

The comparison between funded and unfunded pension systems normally starts with the comparison of expected returns. It is well-known that the implicit rate of return of an unfunded pay-as-you-go (PAYG) system is equal to the growth rate of the wage sum which is lower than the real rate of interest as long as the economy is in a dynamically efficient state (cf. Abel et al., 1989; Feldstein, 1996). Empirically, this is borne out by the fact that for industrialized countries the average stock market returns typically exceed the growth rate of wages and of GDP by a considerable margin. Dimson et al. (2002), e.g., report for the period of 1900-2000 and for 16 countries an average real return on equity of 5.1% while the growth rate of real per capita GDP over the same period and for the same group of countries was only 2 % (Maddison, 2003). Looking at these figures it thus seems suggestive to conclude that a funded system clearly dominates an unfunded one.

There are, however, three limitations to this apparent advantage of a fully funded pension system, as summarized by N. Barr: "A straightforward comparison between rates of return, however, does not compare like with like. A full analysis needs to include (a) the costs of the transition from PAYG to funding, (b) the comparative risks of the two systems, and (c) their comparative administrative costs" (Barr, 2000, 26). In this paper, I want to focus on the second (and arguably most important) issue mentioned by N. Barr—the comparative risks of funded and unfunded pension systems. In the same data sources quoted above one finds, e.g., that the average annual standard deviation of equity returns in the sample of 16 countries is 22.7% while for the growth rate of national income it amounts to only 5.1% (Dimson et al., 2002; Maddison, 2003).

A number of recent papers discuss how the presence of risk might change the relative attractiveness of funded and unfunded systems (e.g., Dutta et al., 2000; Matsen and Thøgersen, 2004; de Menil et al., 2006). There exists, however, a property of PAYG systems that is not fully taken into account in the existing literature and that might further diminish the apparent advantage of fully funded systems. In an appropriately designed defined contribution PAYG system the average pension is closely (and in principle perfectly) tied to the level of average current wages. If wages grow fast then pension benefits will increase at the same speed and if they show a disappointing performance then pensions will also be adjusted in tandem. In other words, the *relative* pension level (the ratio of the average pension to the average wage level) is held constant in such a defined contribution PAYG system. Pensions might be (and in reality will be) lower than the after-contribution wage of the active population but this relative position itself is deterministic over time, irrespective of the degree of wage uncertainty. If people are not only concerned about absolute consumption but also about their level of consumption relative to some reference group (that includes active workers) then this property will strengthen the advantages of an unfunded system where both the future absolute pension benefit *and* the future relative position are less uncertain than in a funded system. In fact, this is an advantage of the PAYG system that is often overlooked in public discussions.

In this paper, I analyze and discuss this argument based on relative consumption preferences in greater detail. First, I develop a simple model in order to present the main logic of the argument in an intuitive manner. In particular, I work with a two-period OLG model where individuals' utility is assumed to depend on a weighted average of absolute own consumption and the difference between absolute consumption and the consumption level of a reference group. The pension system is assumed to collect contributions that are used for a funded and an unfunded (PAYG) pillar. I compute the optimal total contribution rate to the pension system and the optimal share of funding (i.e. the percentage of total contributions that should be invested into the funded pillar) that maximize individuals' (ex-ante) expected utility. The model allows for closed form solutions for the optimal parameters of the pension system. It comes out that for risk-averse individuals the optimal degree of funding is lower if asset returns are more volatile and if people are more concerned about their relative standing. The neglect of risk and of social comparison motives will thus overstate the case for a funded system.

The importance of relative consumption motives and of the related phenomenon of habit formation has recently been studied in a variety of economic fields, ranging from the analysis of savings and consumption behavior (Frank 1985; Carroll, 1998), tax policy (Boskin and Sheshinski, 1978; Abel, 2005), the relation between inequality and growth (Knell, 1999) to the equity premium puzzle (Abel, 1990; Campbell and Cochrane, 1999). This paper is in particular related to the latter field where it was argued that the existence of comparisons (either to a reference group or to own past consumption) might diminish the return advantage of stocks and thus increase the equilibrium equity premium. The results of the present paper are distinct from this literature, however, in so far as the "PAYG assets" differ from bonds in the important property that they really eliminate the uncertainty about the future relative position. Completely risk-free assets (e.g., government bonds) are characterized by a deterministic absolute future payoff but they cannot promise a future return that is directly (and proportionally) related to future wages. If bond markets and labor markets diverge then a pension income that is based on a pure bond portfolio could turn out to be associated with an unfavorable position vis-à-vis active workers. The perfect correlation of pension benefits with the income of active workers is a unique feature of PAYG assets.

There exists ample psychological, sociological and economic evidence that the concern for relative standing is an important human trait (e.g., Frank, 1985; Weiss and Fershtman, 1998; Clark et al., 2008). One area of research that is particularly relevant in this context concerns the issue of poverty. In public discussions and political resolutions it is often stressed that the prevention of old age poverty is among the central functions of well-designed pension systems.[1] Poverty, however, is an inherently relative concept and most research in this area works with some notion of "relative poverty" (e.g. Sen, 1983). Accordingly, in industrialized countries the poverty line or the minimum subsistence level is defined as some percentage of median income (mostly between 50% and 60%). The conceptualization of relative consumption used in this paper allows to clearly capture the notion of such a poverty threshold in the framework of a formal model.

In the second part of the paper I construct a multi-period version of the model that is calibrated to real-world data on asset returns and GDP growth rates (as a proxy for the returns of the PAYG pillar). The estimations use return data that are based on various time periods (1900-1999, 1950-1999) and countries or country groups and they employ commonly used values for the curvature of the utility function and the discount factor. In order to get reasonable values for the strength of the concern for relative standing I use the above-mentioned literature on poverty lines and results from choice experiments. These lines of research suggest a plausible range of values for the importance of relative standing that is used for the estimations. The different risk-return-profiles between equity returns and GDP growth rates imply that even in the absence of social comparison motives it is optimal that the pension system contains a sizable unfunded pillar. Depending on the country and the sample period chosen for the simulations the optimal size of the funded part ranges between 7% (for France) and 91% (for the Anglo-Saxon country group). As soon as the existence of a concern for relative standing is taken into account these shares fall, however, to considerably lower levels, as predicted by the theoretical model. For a moderate concern for relative standing (one that implies a poverty line of 30% of average income) the optimal share of funding does not exceed 20% for a single country and is typically even lower than that. The robustness of these results is discussed with respect to different assumptions concerning the degree of relative risk aversion, the home bias, the

[1] "The first [function of pension systems] is redistribution of income towards low-income pensioners and prevention of destitution in old age. The second is helping workers maintain living standards during retirement by replacing income from work at an adequate level" (OECD, 2005, 16). Similarly, N. Barr describes the three main objectives of pension programmes as: "poverty relief, consumption smoothing, and insurance (the last in respect, for example, of the longevity risk)" (Barr, 2000, 39). Also the World Bank stresses the importance of adequate pensions that prevent old age poverty in their most recent report (Holzmann and Hinz, 2005).

correlation between the rates of return, the influence of management fees and population decline, the composition of the portfolio and the definition of the reference group. In general terms, the results are robust to changes in these assumptions. The different assumptions about the reference groups are particularly important since there does not exist a generally accepted notion of their most appropriate form. Besides the benchmark specification (where the reference level is given by the average, society-wide) consumption level I also use two alternative assumptions: "old-age comparisons" (where workers do not have reference groups and only pensioners compare their level of consumption with the one of the active population) and "age-specific reference groups" (where it is assumed that the "span of comparisons" only covers generations of a similar age). The results indicate that the assumptions about reference groups have a non-negligible impact on the results and both alternative specifications are associated with higher degrees of optimal funding. For all assumptions one can, however, again observe that the concern for relative standing considerably lowers the attractiveness of funding.

The paper is structured as follows. In the next section I present a simple model and derive solutions for the optimal design of the pension system in this framework. Section 3 contains the calibrated version of the model and it estimates the optimal contribution rate and the optimal share of funding using real-world data. I also briefly compare the results of this paper to the related literature. Section 4 concludes.

2 A Simple Model

2.1 The set-up of the model

In this section I present a simple model that suffices to provide the main intuition for the later empirical exercise. Individuals are assumed to live for two periods — young and old. They supply a fixed amount of labor (normalized to 1) in the first period and earn a wage w_t. The contribution rate to the PAYG pension system is given by τ^U. Consumption in the first period can be written as $c_t^y = (1-\tau^U)w_t - s_t$, where s_t denotes the private savings of the young. In the second period of their lives individuals do not work anymore and their consumption c_{t+1}^o is given by the benefits of the PAYG pension system and by the revenues from their investments. The rate of return on these investments in the financial market is denoted by r_{t+1}. The pension benefit p_{t+1}^U is determined by the balanced budget condition of the PAYG pension system: $\tau^U w_{t+1} N_{t+1} = p_{t+1}^U N_t$, where N_t is the size of the generation born in period t. Under the assumption of a constant population the pension benefit comes out as: $p_{t+1}^U = \tau^U w_{t+1}$. The contributions $\tau^U w_t$ paid by generation t thus have a return ("notional interest rate") of $g_{t+1} = \frac{w_{t+1}}{w_t} - 1$.[2] The consumption in old age c^o_{t+1} is then given by:

$$c^o_{t+1} = \tau^U w_t(1 + g_{t+1}) + s_t(1 + r_{t+1}) \tag{1}$$

Instead of assuming that the pension system only runs a PAYG scheme and that only individuals invest in the financial market one could as well assume that the pension system takes the investment decisions on behalf of the individuals. In particular, I will assume that the pension system collects an additional percentage τ^F_t of gross wage w_t that exactly corresponds to the individuals' optimal amount of saving, i.e. $s_t = \tau^F_t w_t$. These contributions are invested into the financial market where they earn the same rate of return r_{t+1} that can be achieved by private investments. Furthermore, it is assumed that the contribution rate to the funded pillar is constant over time (i.e. $\tau^F_t = \tau^F$, $\forall t$) which is in parallel to the constancy of the contribution rate to the unfunded pillar. Under these assumption one can use (1) to write the total pension benefit p_{t+1} as:

$$p_{t+1} = c^o_{t+1} = w_t \left[\tau^U(1 + g_{t+1}) + \tau^F(1 + r_{t+1})\right] = \tau w_t \left[(1 - \lambda)(1 + g_{t+1}) + \lambda(1 + r_{t+1})\right], \tag{2}$$

where $\tau \equiv \tau^U + \tau^F$ denotes the total contribution rate to the pension system and $\lambda \equiv \frac{\tau^F}{\tau}$ stands for the share of the total contribution rate that is devoted to the funded pillar (i.e. to investments into the financial market). This is the specification of the problem that will be used in the following. In particular, I will take the viewpoint of a social planner who has to decide on the optimal values of τ and λ, while the individuals just consume what is determined by the pension system. This formulation is, however, completely equivalent to the situation where the social planner only decides about the optimal contribution rate τ^U to the PAYG system while taking individuals' optimal savings decisions into consideration.

In contrast to other papers in the literature, I allow for the possibility that individuals care not only about absolute consumption but also about the level of their own consumption relative to the one of a reference group. In particular, it is assumed that the utility

[2] In related papers dealing with questions of intergenerational risk-sharing, the authors often distinguish between various kinds of defined benefit and defined contribution specifications (cf. Thogersen, 1998; Bohn, 2001; Wagener, 2003). This distinction, however, only matters if there are demographic fluctuations (changes in cohort size and/or life expectancy) or if pension benefits are defined in absolute terms and not in terms of replacement rates. These issues are not the focus of the current paper. Furthermore, the assumed determination of pension benefits according to a defined contribution scheme is fairly similar to the actual practices in countries like Sweden, Germany and Austria.

function for the representative individual of generation t is given by[3]:

$$U_t = \frac{1}{1-\alpha}\left(c_t^y - \theta d_t^y\right)^{1-\alpha} + \frac{\delta}{1-\alpha}\left(c_{t+1}^o - \theta d_{t+1}^o\right)^{1-\alpha} \tag{3}$$

Here d_t^y (d_{t+1}^o) denotes the consumption of generation t's reference group when they are young (old). The parameter $\theta \geq 0$ measures the importance of the concern for relative standing, δ stands for the time discount factor and α measures the curvature of the utility function. For $\theta = 0$ equation (3) reduces to the usual CRRA function.[4] It is assumed throughout the paper that the parameter values of the crucial variables are always determined in a way such that $c_t^y \geq \theta d_t^y$ and $c_{t+1}^o \geq \theta d_{t+1}^o$. If these conditions were not fulfilled then the utility function would not be well defined.[5]

There exists a long literature on the question what constitutes the most appropriate specification of people's reference groups. A standard assumption in this context is that it consists of the average consumption level in a society and I will employ this assumption for the later empirical estimations. For the example in this section I will, however, use the simplifying assumption that comparisons are only relevant in old age and that therefore young individuals do not have a reference group while pensioners compare themselves solely to the active population. Expressed in the language of the model this assumption thus implies that $d_t^y = 0$ and $d_{t+1}^o = c_{t+1}^y = (1-\tau)w_{t+1}$. This assumption is not very realistic but it allows for closed form solutions and is therefore useful to develop some intuition.[6]

[3] For $\alpha = 1$ the utility function becomes $U_t = \ln\left(c_t^y - \theta d_t^y\right) + \delta \ln\left(c_{t+1}^o - \theta d_{t+1}^o\right)$.

[4] The coefficient of relative risk aversion $RRA \equiv \frac{U''(c)}{U'(c)}c$ for a utility function of the form $U = \frac{(c-\theta d)^{1-\alpha}}{1-\alpha}$ is given by $RRA = \frac{\alpha c}{c-\theta d}$, Thus only for $\theta = 0$ it is equal to α, while for $\theta > 0$ it is decreasing in c and increasing in θ (cf. Meyer and Meyer, 2005).

[5] There exist some controversies about the appropriate functional form to model the relative standing motive. In equation (3) I use an "additive approach" where individuals are assumed to look at the difference between their own level of consumption c and the reference standard θd. This specification is, e.g., employed by Ljungvist and Uhlig (2000) and Knell (1999). Other authors (e.g. Gali, 1994; Abel, 2005) have chosen a "ratio approach" where the comparison motive is modelled as $\frac{c}{d^\theta}$. The choice between the two approaches is not innocuous and the use of a specification in terms of ratios might overturn some of the results that are derived in the framework of the additive model. It is ultimately an empirical question which approach better captures people's concern for relative standing. There exists a handful of studies (Clark and Oswald, 1998; Johansson-Stenman et al., 2002) that compare the two approaches although so far with rather inconclusive results. I stick to the additive approach since I regard it as a more accurate reflection of individual attitudes towards social comparisons.

[6] One could motivate this specification by assuming that people make upward comparisons, i.e. that people have a stronger tendency to compare themselves to individuals that are better off (cf. Duesenberry, 1949, 101) and that total pension benefits are lower than post-contribution wages (i.e. $p_t \leq (1-\tau)w_t$). Alternatively, one could also interpret this assumptions as an indication of habit formation. This follows from the fact that $(1-\tau)w_{t+1}$ is not only the consumption of the active population in period $t+1$ but

Finally, it is assumed at the moment that wages are constant and deterministic ($g_t = 0, \forall t$) and that the rate of asset return can only take on two values: high and low. In particular, r_t has an equal probability to take on the values $r_H = \mu_r + \varepsilon_r$ and $r_L = \mu_r - \varepsilon_r$, respectively. It holds that $E(r_t) = \mu_r$ and $\sigma_r^2 = \varepsilon_r^2$. It is assumed that $r_L < 0$, i.e. that the bad asset market outcome is worse than the constant wage growth. Without this condition the asset market would always dominate the PAYG system under the assumption of a two-point distribution. In this simple model the volatile interest rate is thus the only source of uncertainty. In the empirical section the assumption of constant wages will of course be lifted.

2.2 The optimal design of the pension system

In order to analyze the relative attractiveness of funded and unfunded pensions systems I compute the optimal size of the pension system τ^* and the optimal share of the funded pillar λ^* that maximize expected utility.

Inserting the assumptions about the reference groups and asset returns into the utility function (3) one can derive an explicit expression for expected utility:

$$E[U_t] = -\frac{w_t^{1-\alpha}}{\alpha - 1}\left\{(1-\tau)^{1-\alpha} + \frac{\delta}{2}\left[\begin{array}{c}[\tau(1+\lambda r_H) - \theta(1-\tau)]^{1-\alpha} + \\ [\tau(1+\lambda r_L) - \theta(1-\tau)]^{1-\alpha}\end{array}\right]\right\} \qquad (4)$$

It is assumed that $0 \leq \lambda^* \leq 1$, i.e. that short sales of the financial asset are excluded. The optimal contribution rate and the optimal share of funding can now be calculated from the first-order conditions. The results are summarized in the following proposition for the case of log utility ($\alpha = 1$). The results for the general case with $\alpha \neq 1$ and the proofs an be found in the appendix.

Proposition 1 *Under the assumptions that only old people make comparisons, wages are constant, asset returns are described by a two-point distribution and the utility function is logarithmic ($\alpha = 1$) the optimal contribution rate τ^* and the optimal share of funding λ^* are given by:*
(i) $\tau^* = \frac{\delta+\theta(1+\delta)}{(1+\theta)(1+\delta)}$ *and*
(ii) $\lambda^* = \min[1, \hat{\lambda}]$*, where* $\hat{\lambda} = \frac{\delta(1+\theta)}{\delta+\theta(1+\delta)}\frac{\mu_r}{\varepsilon_r^2 - \mu_r^2} \geq 0$.
(iii) It holds that: $\frac{\partial \tau^*}{\partial \delta} \geq 0$, $\frac{\partial \tau^*}{\partial \theta} \geq 0$, $\frac{\partial \hat{\lambda}}{\partial \delta} \geq 0$, $\frac{\partial \hat{\lambda}}{\partial \theta} \leq 0$, $\frac{\partial \hat{\lambda}}{\partial \mu_r} \geq 0$ *and* $\frac{\partial \hat{\lambda}}{\partial \varepsilon_r} \leq 0$.

---

also equals own past consumption (for $g = 0$). The reference standard $d_{t+1}^o = (1-\tau)w_{t+1} = (1-\tau)w_t$ could thus also be viewed as a habit stock. For positive wage growth, however, this is no longer true and I thus prefer the interpretation as a social reference standard.

Proposition 1 contains a number of interesting results. Starting with the optimal contribution rate, the proposition indicates that τ^* increases in both δ and θ. If people care more about the future period (high δ) and/or if they are more concerned about their relative standing during retirement (high θ) then the social planner will secure a higher and more stable income in old age and implement a higher contribution rate τ. In the absence of social comparisons this optimal contribution rate is simply given by $\tau^* = \frac{\delta}{1+\delta}$.

For the discussion of the optimal share of funding I will concentrate on the case where $\lambda^* < 1$ (i.e., where the short sales constraint is not binding). First, one can observe that a higher return on equity ceteris paribus increases the advantage of the funded pillar, i.e. $\frac{\partial \hat{\lambda}}{\partial \mu_r} \geq 0$. In particular, if one assumes that the worst asset return r_L is the same as the return of the PAYG system (i.e. $r_L = g = 0$ or $\mu_r = \varepsilon_r$) then $\lim\limits_{\varepsilon_r \to \mu_r} \hat{\lambda} = \infty$ and the pension system only uses the funded pillar. On the other hand, however, the attractiveness of the funded pillar decreases with the riskiness of asset returns ($\frac{\partial \hat{\lambda}}{\partial \varepsilon_r} \leq 0$). The return advantage of the funded system ($\mu_r > 0$) is thus increasingly counteracted by higher return uncertainty. For sufficiently high levels of asset return uncertainty the pension system will in fact use exclusively the unfunded pillar, i.e. $\lim\limits_{\varepsilon_r \to \infty} \hat{\lambda} = 0$. Furthermore, the proposition indicates that a higher concern for relative standing is also associated with a lower optimal share of funding ($\frac{\partial \hat{\lambda}}{\partial \theta} \leq 0$). The same asset return uncertainty is more cumbersome if individuals care a lot about the level of consumption of their reference group.

The intuition behind this result is straightforward. Risk-averse individuals dislike situations with uncertain payoffs since the disutility of a bad state outweighs the additional utility of an equally sized favorable outcome. If people are concerned about their relative standing then they fear that a bad shock might drive them near to the reference standard θd_{t+1}^o, which can be interpreted as the socially defined subsistence level (or the poverty line). The more they approach this subsistence level the larger the disutility will get. A higher concern for relative standing (measured by θ) therefore means that the dislike for fluctuating outcomes will become stronger and that the optimal share of funding will decrease. Individuals will increasingly prefer the unfunded pillar that promises lower but at the same time less uncertain returns.[7] Put differently, an increase in θ has the same qualitative effect as an increase in α since both changes will increase the degree of relative risk aversion which for the functional form used in (3) is given by $RRA = \frac{\alpha c}{c - \theta d}$ (see FN 4). Due to this non-linear relation, however, the quantitative impact of increases in α and

[7] Note that the social planner will always choose a λ^* that guarantees that even in the worst case scenario with $r_{t+1} = r_L$ the consumption in old age is strictly above the minimum consumption level, i.e. that it holds that $c_{t+1}^o \geq \theta d_{t+1}^o = \theta c_{t+1}^y$ (see appendix).

in θ on the degree of relative risk aversion differs. For calibrating the model it is thus important to distinguish between the two concepts.

The crucial feature that underlies the main results of proposition 1 is that a PAYG pension system fixes the relative position in old age. This is in fact not a consequence of the specific assumptions on which this proposition is based but it is rather more general. In particular, it also holds for uncertain wage growth and for different reference groups. This can be seen most easily for the case where the PAYG system provides the only source of old-age income (i.e. $\tau^F = \lambda = 0$). In this case a defined contribution pension system implies that for a stationary demographic development the pension benefit is proportional to the income of active workers. To see this, note that under these circumstances the consumption of young individuals (workers) in period t is given by $c_t^y = (1-\tau)w_t$ while consumption of the old is given by $c_t^o = \tau w_{t-1}(1+g_t) = \tau w_t$. Thus for any combination of the two consumption levels the reference consumption levels d_t^y and d_t^o will be directly proportional to w_t. Therefore the relative position of young and old individuals vis-à-vis their respective reference groups will be constant over time even if the growth rate g_t fluctuates.

3 Empirical Estimations of the Optimal Pension System Design

3.1 Model specification for the estimations

In order to study the empirical implications of the model it has to be extended to a multi-period framework. In particular, the utility function of the representative member of generation t is now assumed to be given by:

$$U_t = \sum_{y=1}^{Y} \delta^{y-1} \frac{1}{1-\alpha} \left(c_{t,t+y-1} - \theta d_{t,t+y-1} \right)^{1-\alpha}, \tag{5}$$

where Y is the (constant) life expectancy, $c_{t,s}$ is the consumption of generation t in period s and $d_{t,s}$ stands for the consumption of the reference group of generation t in period s. Furthermore, it is assumed that the cohort size is constant and that all generations work for Z periods and receive a pension payment for the remaining $Y-Z$ periods. In each period individuals earn a wage w_t that is identical for all working cohorts (i.e., no seniority profile) and they pay a fixed contribution rate τ. A share λ of total contributions is used for the funded pillar and a share $(1-\lambda)$ for the unfunded pillar. In order to match the model with the available data, I assume that $Y = 3$, $Z = 2$ and that one period lasts for 20 years. This corresponds to a situation where individuals start to work at the age of 20, retire at the age of 60 and die at the age of 80, a "life-cycle" that is broadly in line with the actual constellation in industrialized countries.

The pension payment for generation t in the retirement period is denoted by $p_{t+2} = c_{t,t+2} = p^U_{t+2} + p^F_{t+2}$, where p^U_{t+2} (p^F_{t+2}) is the pension benefit from the unfunded (funded) pillar. The unfunded pension is calculated in a "quasi-actuarial" manner (cf. Lindbeck and Persson, 2003) where the notional interest rate is equal to the growth rate of wages (see appendix). It can be shown that under this assumption the PAYG pension is always strictly proportional to the current wage level as has already been the case in the 2-period model. The funded pension is equal to the accumulated capital from the contributions to the funded pillar (see appendix).

For the benchmark case I assume that the reference standard is now given by society-wide average consumption:

$$d_{t,t+y-1} = \bar{c}_{t+y-1} \text{ for } y = 1, 2, 3 \tag{6}$$

where $\bar{c}_t = \frac{1}{Y}\sum_{i=1}^{3} c_{t-i+1,t}$ stands for average consumption in period t. Given the structure of the model lifetime utility of generation t depends on asset returns up to period $t+2$ (the last period of his or her life). In the other direction, however, the members of generation t are also influenced by economic variables that materialized before they were even born in as far as these variables will affect consumption of older generations and thus also the average consumption level. In particular, the reference standard $\bar{c}_t$ in generation t's initial period of life includes the consumption of generation $t-2$ and thus generation t is affected by macroeconomic variables that affect this generation's asset returns.

For the simulations one has therefore to deal with the important question of which utility concept should be used to derive the optimal design of the pension system. In particular, under the assumption of society-wide comparisons (cf. (6)) expected lifetime utility of generation t depends not only on the initial wage w_t and the future returns r_{t+1}, r_{t+2}, g_{t+1} and g_{t+2}, but also on the present and past returns r_t, r_{t-1}, g_t and g_{t-1} (which are known when generation t starts to work). There exist two possibilities for calculating expected utility. Either one treats w_t and the past and present returns as given which means that the future rates of return are the only sources of uncertainty in the model. Or one can fix w_t and calculate expected utility under the assumption that *all* (past, present and future) rates of returns are uncertain. In the related literature these different concepts of expected utility are treated under different names, e.g.: traditional vs.

Rawlsian (Matsen and Thøgersen, 2004), true vs. ex-ante (Hassler and Lindbeck, 1997) or ex-post vs. ex-ante (Wagener, 2003) risk-sharing. In this paper, I use the concept of ex-ante (or Rawlsian) risk-sharing since I want to look at the choice between funded and unfunded pension systems from the perspective of an intertemporal social planner who evaluates utility for a generation behind the veil of ignorance. The problem of such a planner can be described as a situation where he has to consider *all* possible histories of returns that might have lead to some given wage level w_t and where he has to choose the optimal levels of τ and λ given these possible histories.

3.2 The data

For the data on asset returns and wage growth I make use of two sources. Dimson et al. (2002) provide data on real equity returns for a number of countries and 10 non-overlapping decades from 1900-1999. I take these decades data to derive the geometric mean and the standard deviation (SD) of 20-year returns based on 9 overlapping periods (1900-1919, 1920-1939, ..., 1980-1999). These summary statistics are reported in the upper panel of Table 1 for a number of countries and pooled groups of countries. The three countries covered (US, UK and France) have in common that they are rather large economies for which it is not unreasonable to assume that the performance of their domestic financial markets is a good proxy for the investment opportunities faced by domestic investors. In addition to the three countries I also report the data for the country group "Anglo-Saxon" that comprises the return data for the US, the UK, Canada and Australia. In particular, it is assumed that the equity returns for this group of countries are realizations from the same underlying stochastic process. The summary statistics for this pooled country are thus based on all 36 observations of 20-year returns for the four individual countries. I do not report the available data for other large countries (Germany, Italy, Japan, Spain) since their returns seem to be overly influenced by the war period.[8] I use, however, the summary statistics for a second pooled country ("Group of Nine (G9)") that is based on the individual return data for the nine large economies included in the dataset (US, Japan, Germany, France, UK, Italy, Spain, Canada and Australia).

`Insert Table 1 about here`

Using data from Maddison (2003) one can also calculate the growth rates of real per capita GDP for the same sequence of overlapping 20 year intervals. Since data on real

[8] In fact, most simulations based on the data for these countries imply an optimal share of funding close to zero.

wage growth are not available for most countries over the time span from 1900 to 1999 it is assumed that per capita GDP is a reasonable proxy for wage developments. Geometric averages and standard deviations of real growth are replicated in the third and forth column of Table 1. The last column of the table reports the coefficient of correlation between equity returns and GDP growth rates.

It has been argued that the data from 1900 to 1999 cover some exceptional periods (years of depression, two world wars etc.) and that they are therefore not a reasonable guide for future investment returns and growth rates. In order to account for this (not completely uncontroversial) argument I also construct summary statistics for the post-war period starting in 1950. For this short time-period the 20-year return data for single countries are, however, problematic since they are based on a very small number of observations. For the sake of comparison, I nevertheless report the numbers in the lower panel of Table 1 for the same three individual countries as in the upper panel. The summary statistics for the pooled countries "Anglo-Saxon" and "Group of Nine (G9)" are based on more underlying observations and their data can thus be regarded with a higher level of confidence. The results below indicate, however, that the main result concerning the impact of the concern for relative standing on the optimal share of funding is not very sensitive to the choice of the data sample.

Table 1 shows that equity returns are higher but at the same time much more volatile than GDP growth. For the US, e.g., the mean equity return over the average 20-year period was 243.6% (corresponding to an average annual return of roughly 6.4%) with a standard deviation of 263.6%. There exist also considerable differences in the risk-return profile between countries. France, e.g., had both a lower average equity return and a higher standard deviation than the Anglo-Saxon countries (US, UK and the pooled country Anglo-Saxon). The exceptional cross-country differences during the first part of the last century are also reflected in the data for the country-group G9 that shows a particularly large standard deviation (386.3%).

The differences between countries (and pooled countries) are less pronounced in the data sample 1950-1999. The summary statistics for France and also for the G9 are now more in line with the risk-return profile of the Anglo-Saxon countries (US, UK and Anglo-Saxon). On the other hand, however, it is remarkable that this shorter period is in general characterized by higher 20-year returns (255.9% for the nine large countries) and a higher standard deviation (493.6%) than the longer period.

The data for real per capita GDP, on the other hand, show a different picture. Real GDP in the US, e.g., has increased by only 46.8% over the average 20-year period between 1900 and 1999 (corresponding to an average annual growth rate of roughly 1.9%). The fluctuations in GDP growth are, however, also considerably less pronounced than the volatility of equity returns. The standard deviation is typically lower than the mean (and often considerably lower) while the contrary is true for equity returns. The last column of Table 1 contains the correlation between the growth rates of GDP and of equity returns. The results here are not very consistent. For the longer period the correlation is typically positive and low (with the exception of the UK and the pooled country G9) while for the shorter period it is typically negative and rather large. Since the data for the short period are based on a small number of observations these values should not be regarded as very compelling and the correlation coefficients based on the longer sample are certainly more reliable.[9] Below I will also report the results of simulations that use different values of the correlation coefficient in order to see how sensitive the results are to changes in this parameter.

3.3 Calibration and Simulation

In order to simulate the model one has to calibrate a number of parameters that are related to individual preferences: α, δ and θ. For the curvature of the utility function I set $\alpha = 3$. This is close to the values chosen in the related literature (Matsen and Thøgersen, 2004; de Menil et al., 2006) and it is within the range of values that are regarded as plausible in this context (cf. Feldstein and Ranguelova, 2001). For the discount factor I choose a value of $\delta = 0.55$ which corresponds to an annual discount rate of 3%.

There exists less literature on the appropriate values for the strength of the concern for relative standing θ. It is, however, possible to gauge plausible values by referring to two classes of evidence. The first stems from the experimental choices between hypothetical societies conducted by Johansson-Stenman et al. (2002). In particular, in the experiments individuals are asked to make repeated choices between two societies that differ with respect to the average income and the income of the respondent's hypothetical grandchildren. For an assumed utility function one can then get an indication of an individual's concern for relative standing by looking at the societies between which the respondent is indifferent. The overall results in Johansson-Stenman et al. (2002) show some differences depending on the specific characteristics of the choice experiments and the assumed form of the utility function. For most cases, however, the value of the concern

[9] This conclusion is also supported by the numbers based on 10-year-return data. Under the assumption that equity returns and growth rates are jointly lognormally distributed the correlation coefficient should be the same for periods of any length n. The correlations coefficients based on the 10-year-return data are in fact more similar to the data in the upper panel (e.g. for the US: 0.072, for the UK: 0.6 and for G9 0.48).

for relative standing is in the range between 0.2 and 0.5.

As a second source of evidence for plausible parameter values of θ one can use the literature on the poverty line. The poverty line is defined as the minimum level of income that is deemed necessary in order to achieve an adequate standard of living and to participate in social life. Most developed countries use a poverty line that is specified in terms of relative income. Institutions like the OECD and the EU employ a threshold of 50% or 60% of national median equivalised household income. The poverty line in the model of this paper is given by θd_t, where the reference standard refers, however, to the mean and not to the median. Since the median income is typically lower than the mean income the values of 50% or 60% would overstate the true weight of the relative component. In order to correct for this bias one can make a crude adjustment. In particular, median income in the US (using data from the US Census Bureau from 2004) has been 71.6% of mean income. 50% (60%) of median income thus corresponds to 36% (43%) of mean income.

The results from the experimental choices and the definition of the poverty line thus suggest that parameter values for θ between 0.2 and 0.4 can be regarded as reasonable. For the sake of comparison the estimations will report results for the optimal pension design for values of θ between 0 and 0.5.

For the simulation of return data it is assumed that $R \equiv 1+r$ and $G \equiv 1+g$ are jointly lognormally distributed where the expected values, variances and covariances are derived from the data in Table 1.[10] For each simulation run 5 data points are drawn both for equity return and for GDP growth. For a given set of simulated data points lifetime utility for generation $t = 3$ (and for a fixed level of initial income w_3) is evaluated for various values of τ, λ and θ. This measure corresponds to the concept of "Rawlsian expected utility" (see above and Matsen and Thøgersen, 2004). The "optimal" (i.e. utility-maximizing) values of τ and λ for country i and a concern for relative standing θ are denoted by $\tau_i^*(\theta)$ and $\lambda_i^*(\theta)$. In the course of the simulations it might happen that for a specific constellation of the return data the level of consumption $c_{t,t+y-1}$ is below the minimum consumption $\theta d_{t,t+y-1}$ in some period y. A social planner will try to choose values for τ and λ that prevent such a situation as shown in the solution to the simple model of section 2. For the continuous distribution used for the simulations, however, this outcome can only be excluded for all cases if the weight of the funded pillar is zero. Otherwise there always exist extreme constellations where any choice of $\lambda > 0$ might lead to a period where consumption is below the subsistence level. This circumstance can induce

[10] For the US 1900-1999 sample this means, e.g., that $E(R) = 3.44$, $SD(R) = 2.64$, $E(G) = 1.47$, $SD(G) = 0.12$ and $Cor(R, G) = 0.07$. In the appendix the simulation from the bivariate lognormal distribution is described in detail.

an unrealistically cautious behavior of the social planner. In order to account for this I allow that for any combination of τ and λ a certain number of simulation runs (up to 10%) might be associated with a situation where the constraint $c_{t,t+y-1} \geq \theta d_{t,t+y-1}$ is binding. These simulations runs are then discarded when I calculate the expected utility for the specific combination of pension parameters.[11] The simulations have shown, however, that almost no constellation of optimal parameters τ and λ involves such extreme periods of time.

3.4 Benchmark results

The benchmark simulations start from the assumption that each country only invests into its home market and that the portfolio consists exclusively of equity. Both of these assumptions will be relaxed below where I will also calculate the optimal share for internationally diversified portfolios and for mixed portfolios.

Figure 1, panel (a) shows the pattern of $\lambda_i^*(\theta)$ and $\tau_i^*(\theta)$ for the country-data based on the period 1900-1999 while panel (b) does the same thing for the data based on the shorter period.

`Insert Figure 1 about here`

Disregarding the presence of social comparisons ($\theta = 0$) one observes that for some — particularly Anglo-Saxon — countries the optimal share of funding λ^* is rather large. For the US it is estimated as 76% (for the long period) and 71% (for the short period) and for the simulations that use data from the other Anglo-Saxon countries it comes out between 49% (UK, long period) and 91% (Anglo-Saxon, long period). Including, however, also data for other countries leads to a different picture. For France, e.g., the optimal share of funding is only 7% (using data from 1900-1999) or 45% (using data from the shorter period). For the country G9 that pools the return data for all large economies the optimal share of funding is calculated as 20% (using data from 1950-1999).[12] These results indicate that even without the consideration of preferences for relative consumption the

[11] The rationale behind this procedure is the following. At certain extreme periods of time the normal pension benefits will not be enough to secure a decent standard of living in old age. It will then be necessary to amend the pension benefits with supplementary payments that are financed from the general budget. When designing the pension system the social planner accepts these extreme events and the associated budgetary responsibilities in exchange for a more favorable average rate of return of the pension system in normal times.

[12] Including the return data for 5 additional smaller countries (the Netherlands, Belgium, Sweden, Switzerland and Denmark) and constructing a country "Group of Fourteen" gives similar results (λ^* for $\theta = 0$ is in this case 25%).

return advantage of equity can be considerably thwarted by the higher risk of stock market investments.

If the existence of reference standards and relative poverty is taken into account ($\theta > 0$) the optimal share of funding is reduced for all countries. In fact, as illustrated in Figure 1, the simulation results indicate that for all 10 data examples shown the optimal share of funding decreases monotonically (i.e. $\frac{\partial \lambda_i^*(\theta)}{\partial \theta} < 0$) and in a non-negligible way. If one defines, e.g., the social reference standard as 20% of average wages (a rather modest definition) the optimal share of funding for the US falls from 76% (71%) to only 29% (20%). Similar reductions can be observed for all Anglo-Saxon countries where λ_i^* falls to values between 20% to 30%. The decrease in the optimal share is less pronounced (although still considerable) for the countries that had a lower weight of funding even for $\theta = 0$. For the G9, e.g., the optimal share is reduced to a values between 3% and 9%. A reference standard weight of $\theta = 0.4$ is associated with even lower values for λ^*, typically around or below 10%.

For $\tau_i^*(\theta)$ (the optimal total contribution rate to the pension system) the differences between the various data samples seem to be less distinct than for the optimal share of funding. For the simulations based on the longer period τ^* is between 0.15 and 0.22 while it is slightly lower for the shorter period (both for the case where $\theta = 0$). In general it holds that the optimal level of the total contribution rate is lower in those countries where the optimal share of funding is higher. This is of course the expected result since a larger proportion of investments into the financial market allows to achieve a given level of expected old-age income with a smaller amount of savings. An increase in the concern for relative standing increases the optimal level of the contribution rate. For $\theta = 0.4$ the optimal contribution rate is now typically between 22% and 25%. As was the case for the optimal share of funding the differences in the optimal contribution rates between the various data samples are smaller for higher values of θ.

3.5 Estimation under alternative assumptions

The results of the last subsection should only be understood as approximate calculations that are based on a number of assumptions. In the following, I want to analyze the robustness of the results to changes in some of the crucial assumptions. The panels of Figures 2 to 3 report the effect of these changes for two "representative" countries—the US (based on the sample period 1900-1999) and the pooled country group G9 (based on the short period from 1950-1999).

`Insert Figures 2 and 3 about here`

The reported optimal shares of funding might be biased downwards because people are less risk-averse than implied by the assumption $\alpha = 3$. In order to investigate this argument I have reestimated the optimal share $\lambda_i^*(\theta)$ for various values of the **curvature parameter** α as shown in panel (a) of Figure 2 (for the US) and Figure 3 (for the G9). These figures also include the benchmark estimation with $\alpha = 3$. As expected, the estimations are quite sensitive to changes in this crucial behavioral parameter. For the US and assumption $\theta = 0$, e.g., the values range between $\lambda^* = 1$ (for $\alpha = 1$) and $\lambda^* = 0.44$ (for $\alpha = 5$). For medium-ranged concern for relative standing and for the G9, however, these differences are less pronounced. For $\theta = 0.3$ the values for λ^* are between 0.03 and 0.5 and for the less extreme value of $\alpha = 2$ the differences to the benchmark are even smaller.[13] The optimal contribution rate increases in α (which is the mirror image effect of the decrease in λ^*). The differences here are, however, smaller and only for $\alpha = 1$ the effect is larger than two percentage points.

The benchmark estimations of Figure 1 also (implicitly) assume that a country's funded pillar is exclusively invested in domestic stocks, i.e. that the pension system is characterized by a complete home bias. Although there exists much controversy concerning the sources behind and the future development of the empirically observed home bias it is nevertheless interesting to deal with this issue by also calculating the optimal share of funding for an assumption that is the exact opposite: a **perfectly diversified portfolio**. For this I use the "world index" assembled by Dimson et al. (2002). This presents the performance of a sixteen-country portfolio weighted by market capitalization and making up around 88% of today's world stock market. This world index reflects the gains from international diversification from the perspective of an US investor. The optimal share of funding for a pension system that invests into this world index (not shown) is close to the values of the US: $\lambda^* = 0.65$ (for $\theta = 0$), $\lambda^* = 0.27$ (for $\theta = 0.2$) and $\lambda^* = 0.16$ (for $\theta = 0.3$). International diversification thus does not necessarily seem to strengthen the case for the funded pillar when compared to the results of a portfolio with home bias.

Another factor that could influence the results is the **correlation between equity returns and GDP growth rates**. A discussed above, the estimations of coefficient of correlation are not very reliable as they are based on a rather small number of observations. Matsen and Thøgersen (2004) quote a number of papers that have studied the correlation

[13] In fact, also in traditional portfolio choice problem the sensitivity of the optimal portfolio share of a risky asset with respect to changes in the risk-return profile decreases with higher degrees of risk aversion. Since increases in α and in θ alike will increase relative risk aversion (see FN 4) both of these changes will reduce the sensitivity of the portfolio share with respect to changes in the underlying parameters. (I am grateful to an anonymous referee for pointing this out).

between domestic stock market returns and GDP growth and that have also come to rather mixed and inconclusive results. In order to analyze the effect of the correlation I have repeated the simulations with changed values of ρ (between -0.5 and +0.75). The results are shown in panel (b) of Figures 2 and 3. For the US, the optimal contribution rate and the optimal size of the funded pillar seem to be quite insensitive to the coefficient of correlation. The value of λ^* slightly increases in ρ which might be due to the fact that for high levels of correlation the PAYG pillar is less advantageous as a hedge against possible bad outcomes on the stock market. This results is in line with Matsen and Thøgersen (2004) although the magnitude of the effect is larger in their modelling framework. For the G9 country group (Figure 3 (b)) I get, however, the opposite result. Now an increase in the correlation *lowers* the optimal share of funding and also the size of the effect is larger than for the case of the US. These results thus confirm the rather mixed findings in the related literature concerning the role of the correlation. Nevertheless, for $\theta = 0.3$ the optimal share of funding is again almost identical for both countries across all different values of ρ. This highlights a special property of PAYG systems. It not only offers an asset that shows less fluctuations than stock returns. It also provides an asset that is perfectly correlated with wage growth and thus with the development of the income of the reference group. If the concern for relative standing is important then this suggest a high investment into the unfunded pillar, almost irrespective of the specific return structure that is available on the stock market.

Both, the historical data on equity returns and the GDP growth rates might be sub-optimal predictors for the future development, although for different reasons. As far as the return data are concerned one has to note that the unfunded and funded pillars have different **administrative costs**. In order to account for this I have looked at the possible effect of these costs by reducing the average annual equity return by 0.5% while assuming that the costs for running the unfunded system are negligible. These administrative costs are in line with empirical studies on this issue (cf. James, 2005). As far as the growth rate of GDP is concerned it has been argued that changes in the demographic structure (declining fertility and mortality rates) will lower the growth rate of the wage bill and thus the rate of return of the unfunded pillar. Although it is hard to tell how important these effects will be (in particular since increases in migration and in labor force participation might partly counteract them) I have also run simulations where annual GDP growth was lowered by 0.5%. The results are shown in panel (c) of Figures 2 and 3. As expected, the presence of management fees lowers the optimal share of funding while the assumption of a declining population increases λ^*. For G9 the effects are quite large for $\theta = 0$ while they are smaller for the US. For values of θ between 0.2 and 0.3, however, the effects are again almost negligible.

One could also argue that a portfolio that is composed solely of stocks is overly risky and does not reflect the typical investment behavior of pension funds. In order to deal with this argument I have used data on real bond returns (again from Dimson et. al, 2002) to construct **mixed portfolios** that consist of varying proportions of equity and bonds. The optimal shares of funding for these portfolio selections are shown in panel (d) of Figures 2 and 3. As is apparent from these pictures the mixed portfolios are always associated with a smaller degree of optimal funding. The only exception is for the US data where for some values of θ the portfolio with 75% equity and 25% bonds implies similar and sometimes slightly higher values of λ^* than the equity-only portfolio. For larger proportions of bonds the simulations suggest much lower shares of funding and a portfolio that invests primarily in bonds is completely dominated by the unfunded system. The reason for these results is to be found in the return structure of equity and bonds. For the mixed portfolios the standard deviations are lower than in the benchmark case with 100% equity while at the same time the reduction in total return is moderate for portfolios with equity shares up to 25%. This increases the attractiveness of the funded pillar based on such portfolio mixes. A further increase in the share of bonds, however, only seems to lower the expected portfolio return without entailing a noticeable reduction of its standard deviation.

The assumptions discussed so far are, however, not the only factor that might have an impact on the estimation of the optimal shares. One could, e.g., also argue that not all people have the same reference standard and that to the contrary **reference groups differ** with respect to age, geographical regions, socioeconomic characteristics etc. If, e.g., people have a tendency to compare themselves to individuals of a similar age then the impact of increases in θ might be smaller than in the benchmark case since movements in own consumption and the consumption of the reference group will be more synchronized. In order to study this argument I have repeated the analysis with different assumptions concerning reference groups. In the first case I assume "old-age comparisons", i.e. (as in the model of section 2) workers do not have reference groups and only pensioners compare their level of consumption with the one of the active population. Since this seems to be a rather extreme scenario I also study the case of "age-specific reference groups" where it is assumed that the "span of comparisons" only covers "neighboring generations". This means that young workers look at the consumption of old workers, pensioners compare themselves with the old workers and only old workers look at both young workers and pensioners. This scenario appears to be a reasonable middle ground between the assumption of society-wide comparisons and the assumption of completely isolated generation-specific comparisons. The results of these simulations are shown in panel (e) of Figures 2 and 3. Interestingly, both alternative assumptions concerning reference groups are associated with a larger optimal share of funding. Also the sensitivity to the strength of the concern θ is less pronounced than in the benchmark case. This is particularly noticeable for the case of "old-age comparisons". These findings suggest that in the benchmark case of society-wide comparisons the optimal share of funding decreases in θ not only because bad asset market returns diminish pensioners' relative standing but also because good asset market returns lead to high pension benefits which will depress the relative standing of workers. The position of the lines in panel (e) suggest (for both the US and G9) that the latter effect is in fact stronger than the first. The in-between case of age-specific comparisons leads to quite similar results as the benchmark scenario and in addition it is still the case that an increase in θ from 0 to 0.3 almost halves the optimal share of funding.

Overall, I believe that the results stated in Figures 1 to 3 give a good indication of the direction of the effects that arise if a concern for relative standing is taken into account. The robustness exercises show that changes in the underlying assumptions have typically rather moderate effects on the optimal total contribution rate but that they can sometimes have a considerable impact on the estimated optimal shares of funding. This is in particular true for the assumptions concerning the curvature of the utility function, the correlation between GDP growth and equity returns (for G9) and alternative reference groups. In every case, however, one could observe that an increase in the concern for relative standing is associated with large reductions in these optimal shares. In fact, for medium-ranged assumptions about the reference group dependence (θ is around 0.3) the sensitivity to changes in the basic assumptions seems to be less pronounced. For almost none of the cases and datasets considered the optimal share of funding exceeds 20% when the strength of the relative comparison motive is assumed to be $\theta = 0.3$.

3.6 Related Literature

There exists a small number of papers that contain results that are related to my estimations, in particular the three papers by Dutta et al. (2000), Matsen and Thøgersen (2004) and de Menil et al. (2006). Although these papers do not take a possible concern for relative standing into account they have undertaken (in one way or another) empirical estimations for the optimal share of funding that can be related to the results of the paper for the case with $\theta = 0$. A direct and detailed comparison between the four papers is, however, quite difficult since the studies show considerable differences concerning the analyzed countries and time periods and the used methods and data.

For this reason I only want to refer briefly to the paper by de Menil et al. (2006) that is most closely related to my approach. They use a two-period OLG model with exogenous interest rates and wages where individuals are assumed to take optimal savings decisions and where there exists a purely PAYG public pension system. The authors analyze the determinants of the optimal savings rate (into the funded pillar) and the optimal contribution rate (to the unfunded system) in the framework of the theoretical model. They then proceed to present estimations for the optimal values of these variables by assuming a CARA utility function and using data (from 1950 to 2002) for the US, the UK, France and Japan. Furthermore, they assume that the working period is twice as long as the pension period (the same assumption as in the estimations above). But instead of simulating a multi-period model they stick to the two-period model and "condense" the various years into two-period (or lifetime) measures by using appropriately aggregated values of the annual data. For the same coefficient of relative risk aversion as in the benchmark case of the present paper ($\alpha = 3$) the estimates for the optimal shares of funding λ^* are reported as: 0.44 (US), 0.31 (UK), 0.18 (France) and 0.12 (Japan).[14] It is interesting to note that despite the considerable differences in the estimation procedure the ranking of these countries with respect to the size of the funded pillar is identical to my calculations although the sizes of the estimated λ^* are quite different.[15]

4 Conclusion

In this paper, I have dealt with the issue of optimal portfolio choice between funded and unfunded pension systems when people care about their standing relative to a reference group. Relative consumption is an important factor in order to evaluate the well-being of pensioners, in particular when one is concerned about the threat of old-age poverty. In fact, the importance of pensions systems to deal with the prevention and relief of poverty among older people was recently emphasized by a number of international organization (OECD, 2005; Holzmann and Hinz, 2005). I have developed a formal model that includes a consumption reference standard for old individuals which is defined as a certain percentage

[14] These values can be found in Table 2 of de Menil et al. (2006) where my τ^* corresponds to the value (written in their notation): $\left(\theta^* + \frac{S^*}{E(\omega_1)}\right)$ and my λ^* corresponds to their $\frac{\frac{S^*}{E(\omega_1)}}{\theta^* + \frac{S^*}{E(\omega_1)}}$.

[15] The numbers reported in Figure 1 for these four countries are: 0.76 (US), 0.49 (UK), 0.07 (France) and 0 (Japan; not shown in Figure 1). The total contribution rates derived in de Menil et al. (2006) are on average 0.27 which is slightly higher than my estimates.

of labor income and that has the natural interpretation as a socially defined subsistence level (or as a poverty line).

It was shown that the optimal total contribution rate increases with the strength of the concern for relative standing while the optimal share of funding decreases with this parameter. This is an implication of the fact that in properly designed defined contribution PAYG systems pensions are perfectly tied to future wages. Accordingly, in PAYG systems there exists only uncertainty about the level of future wages (and thus about future absolute consumption) but not about the future relative position. For funded systems, on the other hand, both the absolute and the relative positions are uncertain and thus the attractiveness of funding is diminished by higher levels of equity return risk and/or a stronger concern for relative standing.

In the final part of the paper I have developed a multi-period version of the model in order to estimate the optimal shares of funding for various countries and time periods. Using data on equity returns and income growth rates from 1900 to 1999 the empirical estimations suggest that for the case where people are assumed to care solely about their absolute level of consumption the optimal share of funding very much depends on the data used for the simulations. For data from the US (1900-1999), e.g., the benchmark results indicate an optimal share of funding of 76% while using data for the G9 from 1950-1999 suggest a much lower share of only 20%. Once the concern for relative standing is taken into account these differences between data samples become less important and for all cases considered in this paper the optimal share of funding drops considerably if one assumes a reasonable strength of the concern for relative standing. For $\theta = 0.3$, e.g., the optimal share of funding is typically below 20%. This finding has been confirmed by an extensive number of robustness tests concerning the degree of relative risk aversion, the correlation between returns, the use of mixed portfolios, the presence of management fees and declining population and the existence of alternative reference groups. Overall one can thus conclude that the high risk of equity taken together with people's concern for relative consumption lowers the attractiveness of a funded pension system that is associated with its return advantage.

References

Abel, A. (1990), 'Asset Prices under Habit Formation and Catching Up with the Joneses', *American Economic Review* **80**(2), 38–42.

Abel, A. (2005), 'Optimal Taxation When Consumers Have Endogenous Benchmark Levels of Consumption', *The Review of Economic Studies* **72**(1), 21–42.

Abel, Andrew B, e. a. (1989), 'Assessing dynamic efficiency: Theory and evidence', *Review of Economic Studies* **56**(1), 1–19.

Barr, N. (2000), Reforming Pensions: Myths, Truths, and Policy Choices, Imf working paper 139.

Bohn, H. (2001), Social Security and Demographic Uncertainty: The Risk Sharing Properties of Alternative Policies, *in* J. Campbell & M. Feldstein, eds, 'Risk Aspects of Investment Based Social Security Reform', University of Chicago Press, pp. 203–241.

Boskin, M. J. & Sheshinski, E. (1978), 'Optimal redistributive taxation when individual welfare depends on relative income', *The Quarterly Journal of Economics* **92**(4), 589–601.

Breyer, F. (1989), 'On the Intergenerational Pareto-Efficiency of Pay-As-You-Go Financed Pension Systems', *Journal of Institutional and Theoretical Economics* **145**, 643–658.

Campbell, J. & Cochrane, J. (1999), 'Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior', *Journal of Political Economy* **107**(2), 205–251.

Carroll, C. D. (1998), Why do the rich save so much?, NBER Working Papers 6549, National Bureau of Economic Research, Inc.

Clark, A. E. & Oswald, A. J. (1998), 'Comparison-concave utility and following behaviour in social and economic settings', *Journal of Public Economics* **70**(1), 133–155.

Clark, Andrew, E., Frijters, Paul, Shields & Michael, A. (2008), 'Relative Income, Happiness, and Utility: An Explanation for the Easterlin Paradox and Other Puzzles', *Journal of Economic Literature* **46**(1), 95–144.

de Menil, G., Murtin, F. & Sheshinski, E. (2006), 'Planning for the Optimal Mix of Paygo Tax and Funded Savings', *Journal of Pension Economics and Finance* **5**(1), 1–25.

Dimson, E., Marsh, P. & Staunton, M. (2002), *Triumph of the Optimists: 101 Years of Global Investment Returns*, Princeton University Press, Princeton, NJ.

Duesenberry, J. S. (1949), *Income, Saving and the Theory of Consumer Behavior*, Harvard University Press, Cambridge, MA.

Dutta, J., Kapur, S. & Orszag, J. M. (2000), 'A Portfolio Approach to the Optimal Funding of Pensions', *Economics Letters* **69**(2), 201–206.

Feldstein, M. (1996), 'The Missing Piece in Policy Analysis: Social Security Reform', *American Economic Review* **86**(2), 1–14.

Feldstein, M. & Ranguelova, E. (2001), 'Individual risk in an investment-based social security system', *American Economic Review* **91**(4), 1116–1125.

Frank, R. (1985), *Choosing the Right Pond. Human Behavior and the Quest for Status*, Oxford University Press, New York and Oxford.

Gali, J. (1994), 'Keeping Up with the Joneses: Consumption Externalities, Portfolio Choice, and Asset Prices', *Journal of Money, Credit and Banking* **26**(1), 1–8.

Geanakoplos, J., Mitchell, O. S. & Zeldes, S. P. (1999), Social Security Money's Worth, *in* O. S. Mitchell, R. J. Myers & H. Young, eds, 'Prospects for Social Security Reform', University of Pennsylvania Press, pp. 79–151.

Holzmann, R. & Hinz, R. (2005), *Old-Age Income Support in the 21st Century. An International Perspective on Pension Systems and Reform*, The World Bank, Washington, D.C.

James, E. (2005), Reforming Social Security: Lessons from Thirty Countries, Ncpa policy report no. 277, june 2005.

Johansson-Stenman, O., Carlsson, F. & Daruvala, D. (2002), 'Measuring Future Grandparents' Preferences for Equality and Relative Standing', *Economic Journal* **112**(479), 362–383.

Knell, M. (1999), 'Social Comparisons, Inequality, and Growth', *Journal of Institutional and Theoretical Economics* **155**(4), 664–695.

Krüger, D. & Kubler, F. (2006), 'Pareto Improving Social Security Reform when Financial Markets are Incomplete!?', *American Economic Review* **96**(3), 737–755.

Law, A. M. & Kelton, D. M. (2000), *Simulation Modeling and Analysis*, McGraw-Hill Higher Education.

Lindbeck, A. & Persson, M. (2003), 'The Gains from Pension Reform', *Journal of Economic Literature* **41**, 72–112.

Ljungqvist, L. & Uhlig, H. (2000), 'Tax Policy and Aggregate Demand Management under Catching Up with the Joneses', *American Economic Review* **90**(3), 356–366.

Maddison, A. (2003), *The World Economy: Historical Statistics*, OECD Development Centre, Paris.

Matsen, E. & Thogersen, O. (2004), 'Designing Social Security - A Portfolio Choice Approach', *European Economic Review* **48**(4), 883–904.

Meyer, D. J. & Meyer, J. (2005), 'Risk Preferences in Multi-period Consumption Models, the Equity Premium Puzzle, and Habit Formation Utility', *Journal of Monetary Economics* **52**(8), 1497–1515.

Sen, A. (1983), 'Poor, Relatively Speaking', *Oxford Economic Papers* **35**(2), 153–169.

Thogersen, O. (1998), 'A Note on Intergenerational Risk Sharing and the Design of Pay-As-You-Go Pension Programs', *Journal of Population Economics* **11**, 373–378.

Wagener, A. (2003), 'Pensions as a Portfolio Problem: Fixed Contribution Rates vs. Fixed Replacement Rates Reconsidered', *Journal of Population Economics* **16**(1), 111–134.

Weiss, Y. & Fershtman, C. (1998), 'Social Status and Economic Performance: A Survey', *European Economic Review* **42**(3), 801–820.

5 Appendix

Section 2.2: The optimal design of the pension system

Proof of proposition 1:

Maximizing expected utility (4) with respect to τ and λ leads to FOCs that can be transformed into:

$$\frac{\delta}{2}\left\{\begin{array}{c}[\tau(1+\lambda r_H)-\theta(1-\tau)]^{-\alpha}(1+\lambda r_H+\theta)+\\ [\tau(1+\lambda r_L)-\theta(1-\tau)]^{-\alpha}(1+\lambda r_L+\theta)\end{array}\right\}=(1-\tau)^{-\alpha} \tag{7}$$

$$[\tau(1+\lambda r_H)-\theta(1-\tau)](r_H)^{-\frac{1}{\alpha}}=[\tau(1+\lambda r_L)-\theta(1-\tau)](-r_L)^{-\frac{1}{\alpha}} \tag{8}$$

Equations (7) and (8) lead to:

$$\frac{\tau}{1-\tau}(1+\lambda r_H)-\theta=\left(\frac{\delta(1+\theta)\varepsilon_r}{(-r_L)}\right)^{\frac{1}{\alpha}}\equiv\Psi \tag{9}$$

$$\lambda=\frac{\tau-\theta(1-\tau)}{\tau}\frac{(r_H)^{\frac{1}{\alpha}}-(-r_L)^{\frac{1}{\alpha}}}{r_H(-r_L)^{\frac{1}{\alpha}}+(-r_L)(r_H)^{\frac{1}{\alpha}}}\equiv\frac{\tau-\theta(1-\tau)}{\tau}\Gamma \tag{10}$$

Using (9) and (10) one can solve for the optimal levels of τ and λ. They come out as:

$$\tau^*=\frac{\Psi+\theta(1+\Gamma r_H)}{\Psi+(1+\theta)(1+\Gamma r_H)} \tag{11}$$

$$\hat{\lambda}=\frac{\Psi\Gamma}{\Psi+\theta(1+\Gamma r_H)} \tag{12}$$

Note that $\Psi\geq 0$ and $\Gamma\geq 0$. The latter follows from the facts that $(-r_L)\geq 0$ (due to the assumption $r_L\leq g=0$) and that $(r_H)\geq(-r_L)$. $\hat{\lambda}$, however, can be larger than 1 (thereby violating the no-short-sales constraint) and so the optimal share is defined as $\lambda^*=\min[1,\hat{\lambda}]$. For τ^* it holds that $0\leq\tau^*\leq 1$.

Setting $\alpha=1$ reduces (11) and (12) to the equations stated in proposition 1, i.e. $\tau^*=\frac{\delta+\theta(1+\delta)}{(1+\theta)(1+\delta)}$ and $\hat{\lambda}=\frac{\delta(1+\theta)}{\delta+\theta(1+\delta)}\frac{\mu_r}{\varepsilon_r^2-\mu_r^2}$. Note that $\varepsilon_r^2-\mu_r^2\geq 0$ due to the assumption that $r_L\leq 0$ (and thus $\mu_r\leq\varepsilon_r$). It follows that:
$\frac{\partial\tau^*}{\partial\delta}=\frac{1}{(1+\theta)(1+\delta)^2}\geq 0$, $\frac{\partial\tau^*}{\partial\theta}=\frac{1}{(1+\theta)^2(1+\delta)}\geq 0$
$\frac{\partial\hat{\lambda}}{\partial\delta}=\frac{\theta(1+\theta)}{[\delta+\theta(1+\delta)]^2}\frac{\mu_r}{\varepsilon_r^2-\mu_r^2}\geq 0$, $\frac{\partial\hat{\lambda}}{\partial\theta}=-\frac{\delta}{[\delta+\theta(1+\delta)]^2}\frac{\mu_r}{\varepsilon_r^2-\mu_r^2}\leq 0$
$\frac{\partial\hat{\lambda}}{\partial\mu_r}=\frac{\delta(1+\theta)}{\delta+\theta(1+\delta)}\frac{\varepsilon_r^2+\mu_r^2}{(\varepsilon_r^2-\mu_r^2)^2}\geq 0$, $\frac{\partial\hat{\lambda}}{\partial\varepsilon_r}=-\frac{2\delta(1+\theta)}{\delta+\theta(1+\delta)}\frac{\varepsilon_r\mu_r}{(\varepsilon_r^2-\mu_r^2)^2}\leq 0$

Note that for $\tau=\tau^*$ and $\lambda=\lambda^*$ it holds that $c_{t+1}^o\geq\theta d_{t+1}^o$, $\forall t$. In order to see this write $c_{t+1}^o-\theta d_{t+1}^o=w_t[\tau(1+\lambda r_{t+1})-\theta(1-\tau)]\equiv\Phi$. In the case where $\hat{\lambda}<1$ the expression Φ can be written as:

$$\Phi = \frac{\delta(1+\theta)}{(1+\theta)(1+\delta)}\left[1+\frac{\mu_r r_{t+1}}{r_H\left(-r_L\right)}\right]$$

For the bad outcome ($r_{t+1} = r_L$) this reduces to $\Phi = \frac{\delta(1+\theta)}{(1+\theta)(1+\delta)}\left[\frac{\varepsilon_r}{r_H}\right] \geq 0$. In the case where $\lambda^* = 1$ the expression Φ can be written as:

$$\Phi = \frac{1}{(1+\theta)(1+\delta)}\left[\delta(1+\theta)+\left[\delta+\theta(1+\delta)\right]r_{t+1}\right]$$

It holds that $\Phi \geq 0$ if $\left[\delta(1+\theta)+\left[\delta+\theta(1+\delta)\right]r_{t+1}\right] \geq 0$ or (inserting $r_{t+1} = r_L$) if $\varepsilon_r \leq \frac{\delta(1+\theta)}{\delta+\theta(1+\delta)} + \mu_r \equiv \tilde{\varepsilon}_r$. But $\lambda^* = 1$ implies that $\hat{\lambda} \geq 1$ which holds for $\frac{\delta(1+\theta)}{\delta+\theta(1+\delta)}\frac{\mu_r}{\varepsilon_r^2-\mu_r^2} \geq 1$ or $\varepsilon_r \leq \sqrt{\mu_r\left(\frac{\delta(1+\theta)}{\delta+\theta(1+\delta)}+\mu_r\right)} \equiv \hat{\varepsilon}_r$. Since $\hat{\varepsilon}_r \leq \tilde{\varepsilon}_r$ it can be concluded that also in the case where $\lambda^* = 1$ it holds that $\Phi \geq 0$ or $c^o_{t+1} \geq \theta d^o_{t+1}$.

Section 3.1: The model specification for the estimations

In every period t the social planner puts $\lambda\tau w_t$ into the funded part and $(1-\lambda)\tau w_t$ into the unfunded part of the pension system. The unfunded pillar is organized as a notional defined contribution (NDC) system where the notional interest rate ni_t states how contribution to the (fictitious) pension account are revalued. At the time of retirement generation t has accumulated a (fictitious) capital $\Omega^U_{t,t+2}$ on the account of the unfunded pillar that is given by:

$$\Omega^U_{t,t+2} = (1-\lambda)\tau w_t(1+ni_{t+1})(1+ni_{t+2}) + (1-\lambda)\tau w_{t+1}(1+ni_{t+2}) \tag{13}$$

Similarly, the (real) capital $\Omega^F_{t,t+2}$ in the funded pillar is given by:

$$\Omega^F_{t,t+2} = \lambda\tau w_t(1+r_{t+1})(1+r_{t+2}) + \lambda\tau w_{t+1}(1+r_{t+2}) \tag{14}$$

The notional interest rate is set equal to the growth rate of wages: $ni_t = g_t$. Inserting this into (13) leads to $\Omega^U_{t,t+2} = p^U_{t+2} = 2(1-\lambda)\tau w_{t+2}$. Note that the PAYG pillar of the pension system has a balanced budget in every period. This follows from the fact that total income of the PAYG system in period $t+2$ is given by $2(1-\lambda)\tau w_{t+2}$ which equals total expenditures given by p^U_{t+2}. This is a stylized PAYG system that nevertheless resembles the general practice of many real world unfunded pension systems (e.g. Sweden, Germany, Austria).[16] Due to the assumption that there is only one period of retirement

[16] The fact that the PAYG system is balanced in every period also holds in the case where there are

it is not necessary to specify how the pension capital of the funded pillar is annuitized, i.e. transformed into annual pension installments. It simply holds that $p^F_{t+2} = \Omega^F_{t,t+2}$. In the general case, however, with $Y - Z > 1$ one had to make additional assumption about this annuitization.

Under the assumption of society-wide comparisons (cf. (6)) lifetime utility for generation t can be written as:

$$U_t = \frac{1}{1-\alpha} \sum_{y=1}^{Y} \delta^{y-1} \left(c_{t,t+y-1} - \theta \bar{c}_{t+y-1} \right)^{1-\alpha} \tag{15}$$

Note that consumption for all periods of life can be written in the form $w_t\Theta_1$, where Θ_1 is a function of the returns r and g. By similar transformations also average consumption $\bar{c}_t$ can be expressed in the form $w_t\Theta_2$ where Θ_2 is now a rather complicated function of lags and leads of r and g. Calculating $V_t \equiv \frac{U_t}{w_t^{1-\alpha}}$ thus gives a measure for lifetime utility that is independent of the initial wage *level* of generation t and instead only depends on the stochastic properties of the growth rates of wages and asset prices. In order to assess the relative attractiveness of funded and unfunded pension systems for a given initial state I use the measure V_t in the simulations.

Section 3.3: The simulation

The sources for the data on real equity returns and GDP growth rates are described in section 3.2 and the means μ_j, standard deviations σ_j and correlations ρ are shown in Table 1 for $j = r, g$. The procedure for simulating data points that possess these stochastic properties is the following. It is assumed that $R \equiv 1 + r$ and $G \equiv 1 + g$ are jointly lognormally distributed with $E(R) = 1 + \mu_r \equiv \hat{\mu}_r$, $E(G) = 1 + \mu_g \equiv \hat{\mu}_g$, $Var(R) = \sigma_r^2$, $Var(G) = \sigma_g^2$, $Cor(R, G) = \rho$ and $Cov(R, G) = \rho\sigma_r\sigma_g$. Given this information one knows that the two variables $\tilde{R} \equiv \ln R$ and $\tilde{G} \equiv \ln G$ are jointly normally distributed with $\tilde{R} \sim N(m_r, s_r^2)$ and $\tilde{G} \sim N(m_g, s_g^2)$ and with $Cor(\tilde{R}, \tilde{G}) = \tilde{\rho}$. The following relations hold (e.g. Law and Kelton, 2000, chap. 8.5): $m_j = \ln\left(\frac{\hat{\mu}_j^2}{\sqrt{\hat{\mu}_j^2+\sigma_j^2}}\right)$ and $s_j^2 = \ln\left(1 + \frac{\sigma_j^2}{\hat{\mu}_j^2}\right)$ for $j = r, g$. Furthermore $Cov(\tilde{R}, \tilde{G}) = \ln\left(1 + \frac{Cov(R,G)}{\left|\hat{\mu}_r\hat{\mu}_g\right|}\right)$ and $\tilde{\rho} = \frac{Cov(\tilde{R},\tilde{G})}{s_r s_g}$. The Mathematica command *MultinormalDistribution* is employed to draw sequences of bivariate normally distributed variables $\tilde{R}_i$ and $\tilde{G}_i$. From these $r_i = e^{\tilde{R}_i} - 1$ and $g_i = e^{\tilde{G}_i} - 1$ can be calculated.

The simulation is based on $Y = 3$ and $Z = 2$. For each simulation run I draw time series for r_i and g_i from $i = 1, \ldots, 5$ and evaluate V_t at time $t = 3$. Due to the fact

multiple periods of retirement (i.e. where $Y - Z > 1$). In this case one has to assume in addition that pensions are annually adjusted with the growth rate of wages (i.e with ni_t).

that average consumption depends on past variables 5 data points are needed to calculate lifetime utility measures (from $t - Y + 1$ to $t + Y - 1$, where now $t = 3$ and $Y = 3$).

The results reported in the paper are based on 1.000 simulation runs. For λ and τ I consider values between 0 and 1 with a chosen step length of 1/100 while for θ I use values between 0 and 0.5 with a steplength of 1/10. In the simulation the problem may arise that (for $\theta > 0$) in some simulation runs $c_{3,3+y-1} - \theta \bar{c}_{3+y-1} < 0$ for some $y = 1, \ldots, 3$. I describe in section 3.3 how I deal with these cases.

Table 1

20 Year Equity Returns and Per Capita GDP Growth for Selected Countries

	Real Return on Equities Over Twenty Years		**Real Per Capita GDP Growth Over Twenty Years**		
	Mean	*Standard Deviation*	*Mean*	*Standard Deviation*	*Correlation*
	1900-1999				
US	243.6%	263.6%	46.8%	11.6%	0.073
UK	207.8%	296.9%	33.3%	17.2%	0.481
France	93.2%	337.1%	48.6%	30.0%	-0.016
Anglo-Saxon	246.9%	255.2%	40.0%	17.2%	0.045
Group of Nine (G9)	152.3%	386.3%	47.3%	50.5%	0.484
	1950-1999				
US	302.9%	337.3%	56.5%	8.6%	-0.751
UK	349.9%	374.5%	49.6%	4.2%	-0.373
France	284.9%	427.0%	64.9%	32.3%	-0.729
Anglo-Saxon	266.7%	279.1%	53.3%	11.3%	-0.222
Group of Nine (G9)	255.9%	493.6%	74.6%	59.6%	0.416

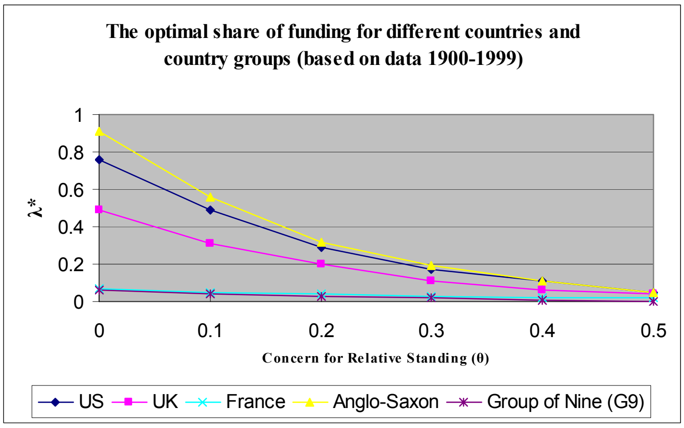
Note: Own calculations based on Dimson et al. (2002) and Maddison (2003). The (geometric) means are calculated from 20 year periods, for the upper panel periods between 1900 and 1999 and for the lower periods between 1950 and 1999. The (sample) standard deviations and correlations are based on the same data of 20-year returns. For the summary statistics of the country group "Anglo-Saxon" the data for four Anglo-Saxon countries (US, UK, Canada and Australia) are pooled. "Group of Nine (G9)" also refers to a country group that pools the return data of all large economies (US, Japan, Germany, France, UK, Italy, Spain, Canada and Australia).

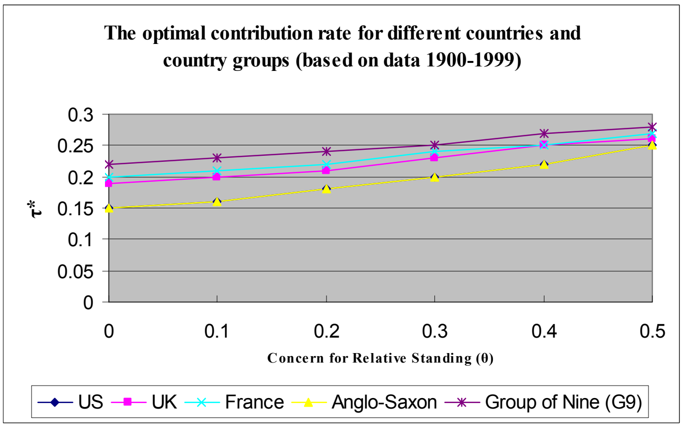
Figure 1

The optimal share of funding λ^* and the optimal contribution rate τ^*
as a function of the concern for relative standing θ (different countries and time periods)

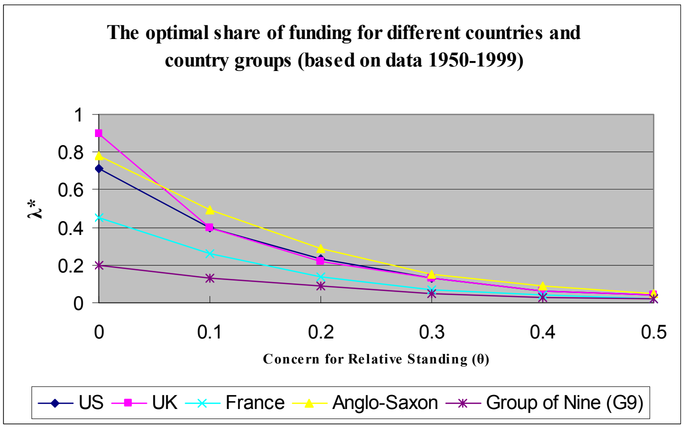
The optimal share of funding (λ)*	*The optimal contribution rate (τ*)*

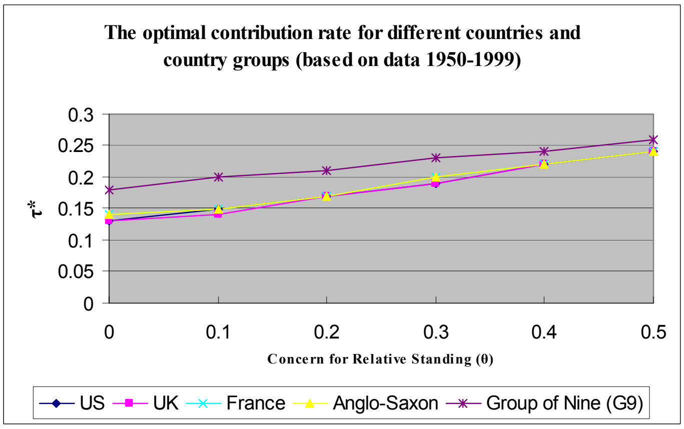
(a) Sample period: 1900-1999





(b) Sample period: 1950-1999





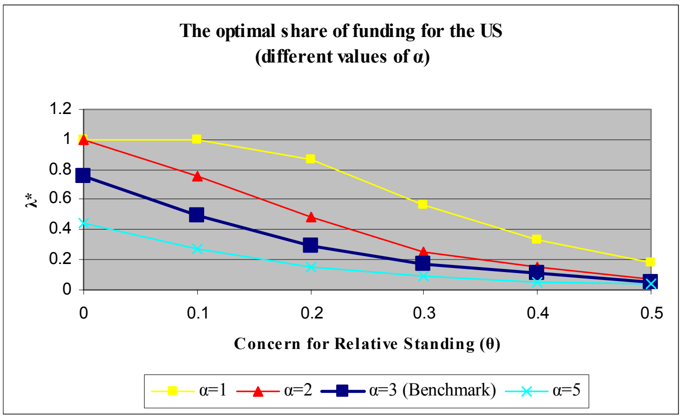
Note: The picture illustrates the optimal (i.e. utility-maximizing) contribution rate τ^* and share of funding λ^* in a model with life expectancy of Y=3 and a retirement age of Z=2. It is assumed that equity returns and GDP growth rate are jointly lognormally distributed with the values for the means, standard deviations and correlations given in Table 1. The calculations for the optimal τ^* and λ^* in dependence of θ are based on 1.000 simulation runs.

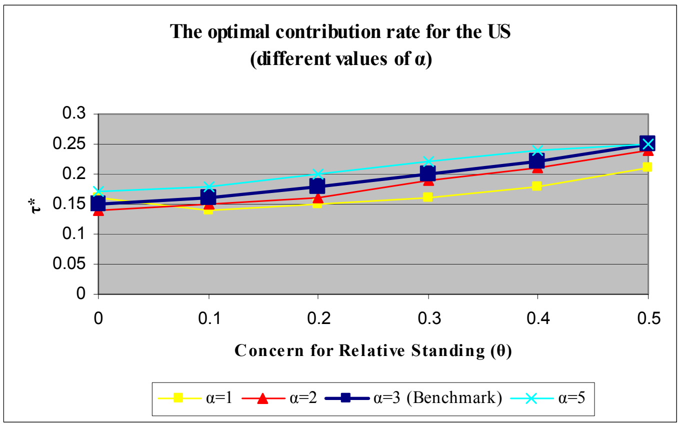
Figure 2

The optimal λ^* and τ^* for the US (1900-1999) under various alternative assumptions

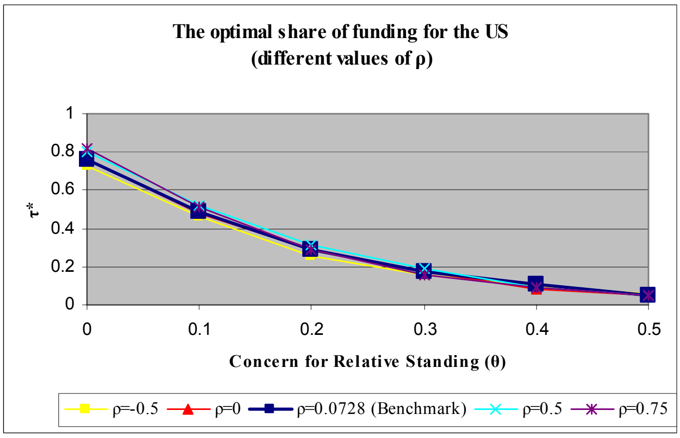
The optimal share of funding (λ)*	*The optimal contribution rate (τ*)*

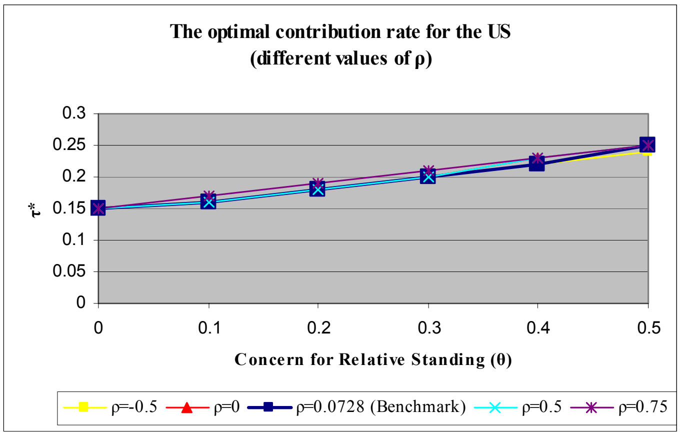
(a) Different curvatures of the utility function α



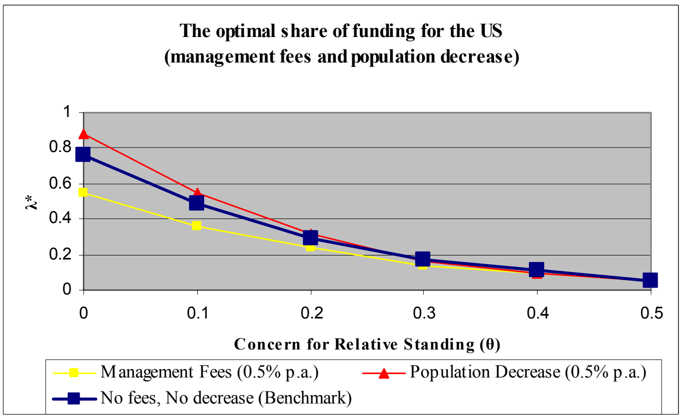


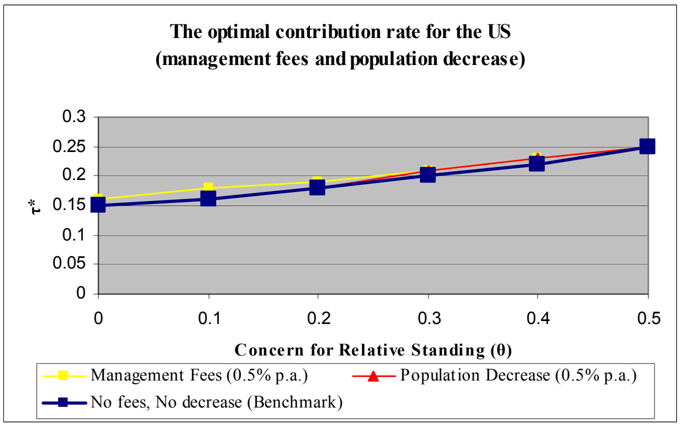
(b) Different correlations between equity returns and GDP growth





(c) Different management fees and populations growth rates



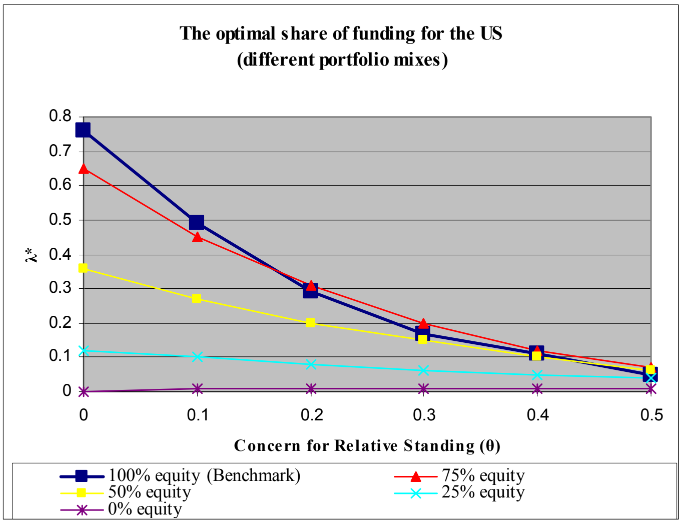


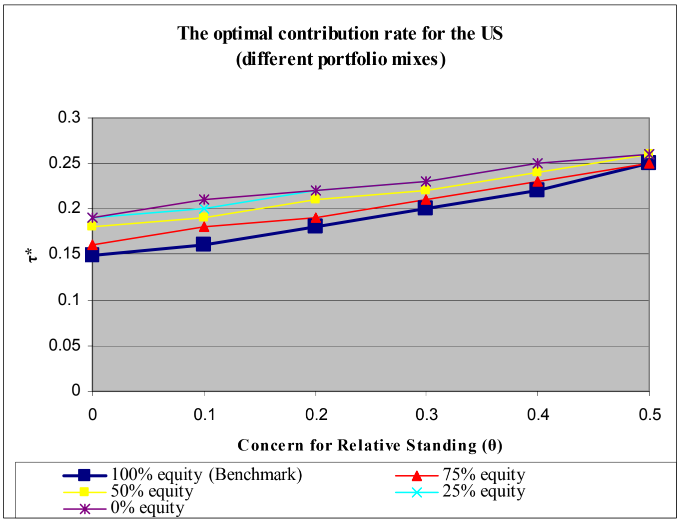
Note: Figure continued on the next page.

Figure 2 (continued)

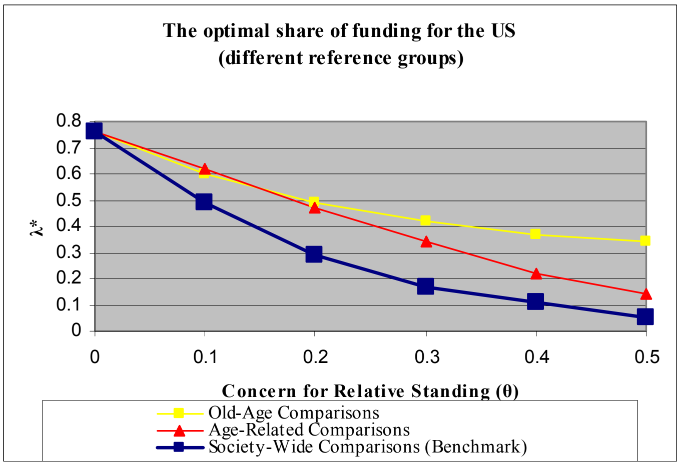
The optimal share of funding (λ)*	*The optimal contribution rate (τ*)*

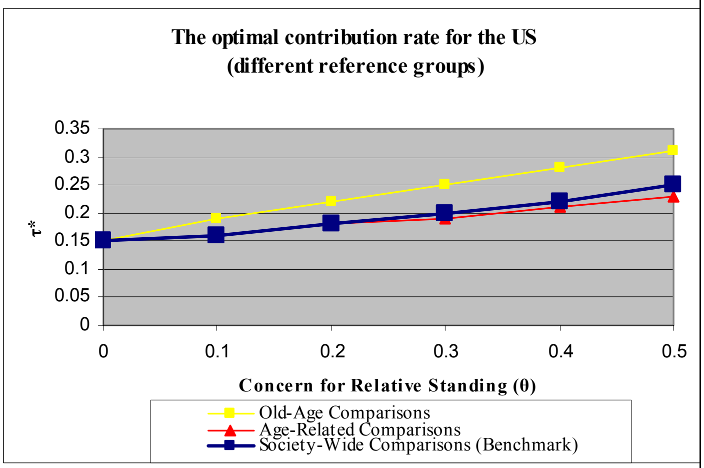
(d) Different portfolio mixes between equity and bonds





(e) Different reference groups





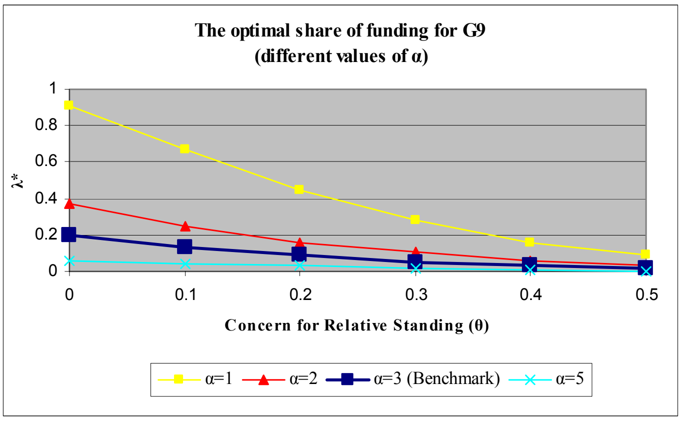
Note: The optimal values are derived as explained in the note to Figure 1. The alternative assumptions are explained in the text. The data are based on the period 1900-1999.

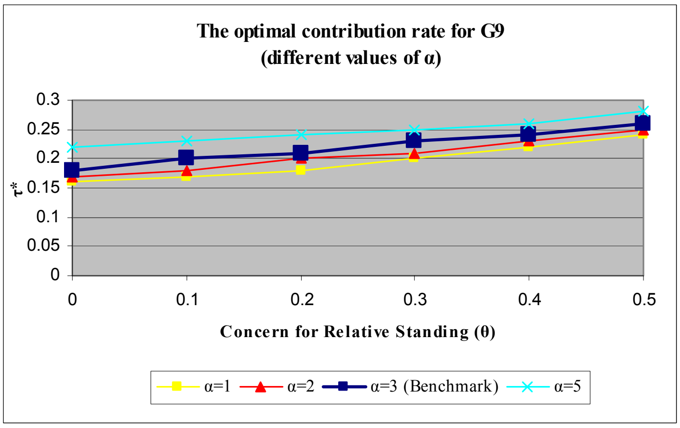
Figure 3

The optimal λ^* and τ^* for the country group G9 (1950-1999) under various alternative assumptions

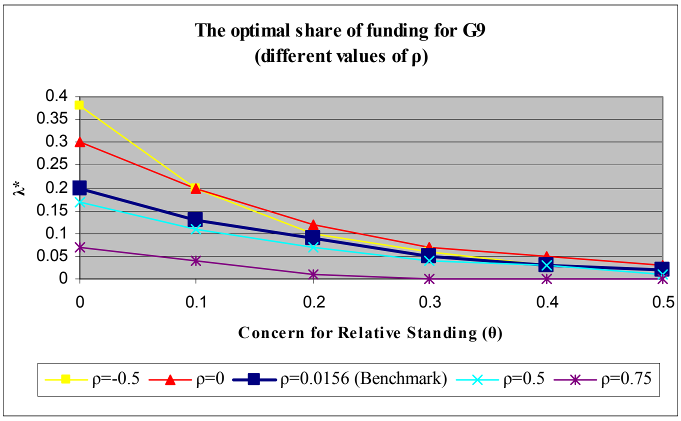
The optimal share of funding (λ)*	*The optimal contribution rate (τ*)*

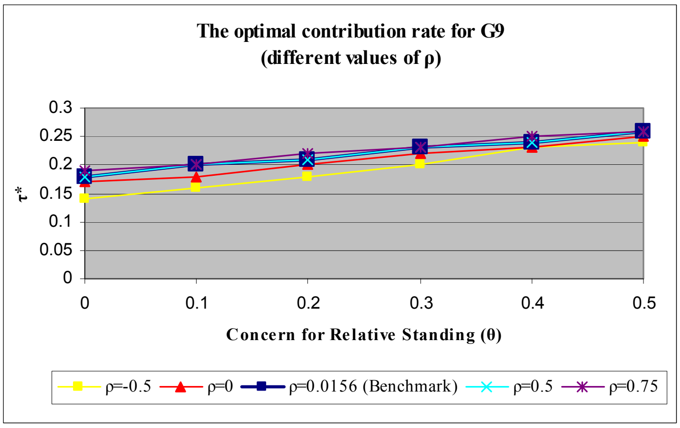
(a) Different coefficients of relative risk aversion α



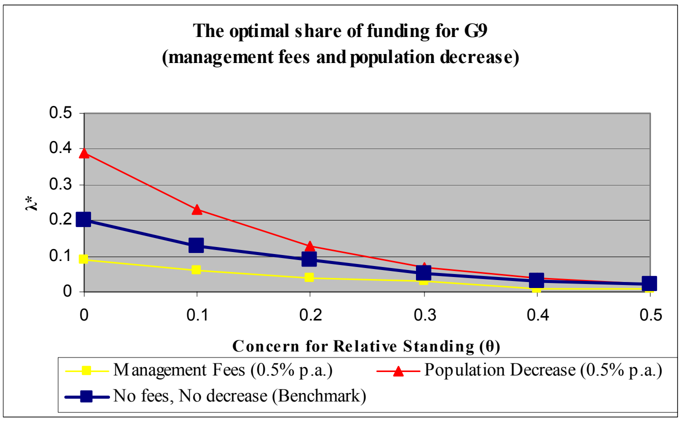


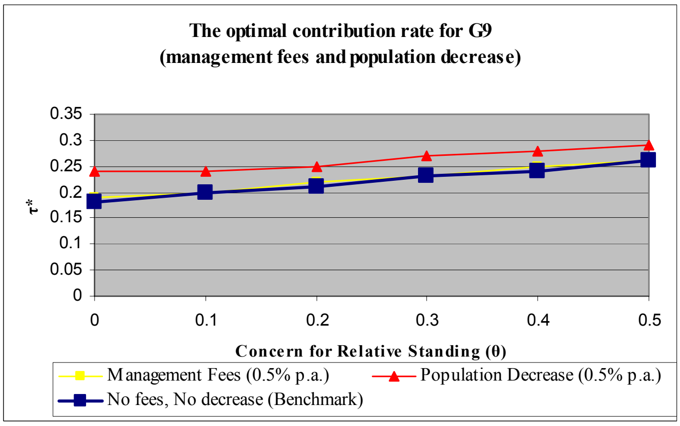
(b) Different correlations between equity returns and GDP growth





(c) Different management fees and populations growth rates



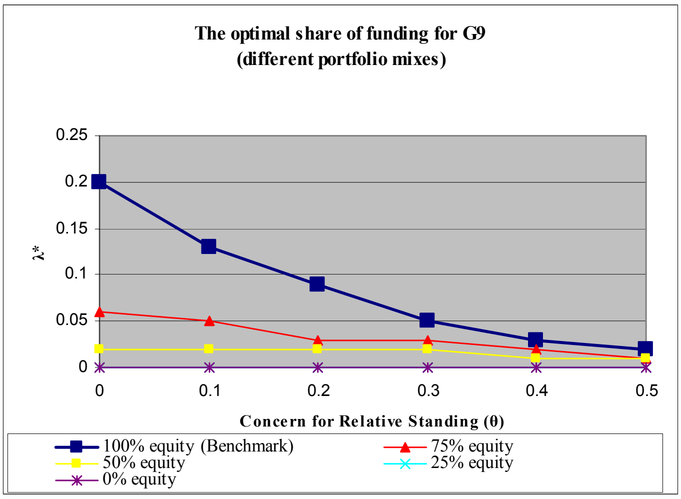


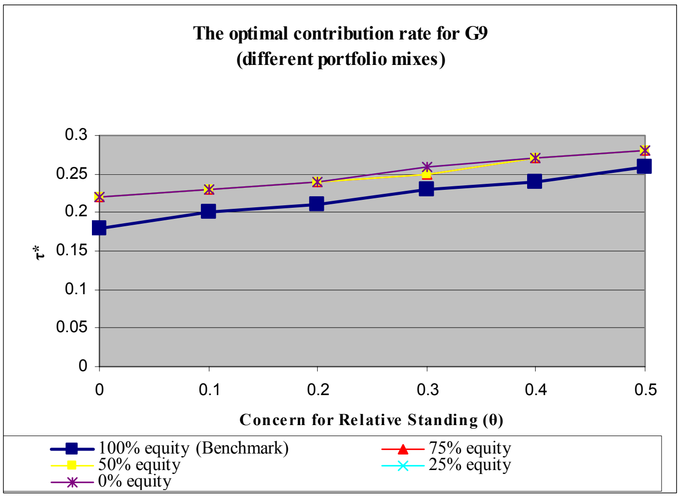
Note: Figure continued on the next page.

Figure 3 (continued)

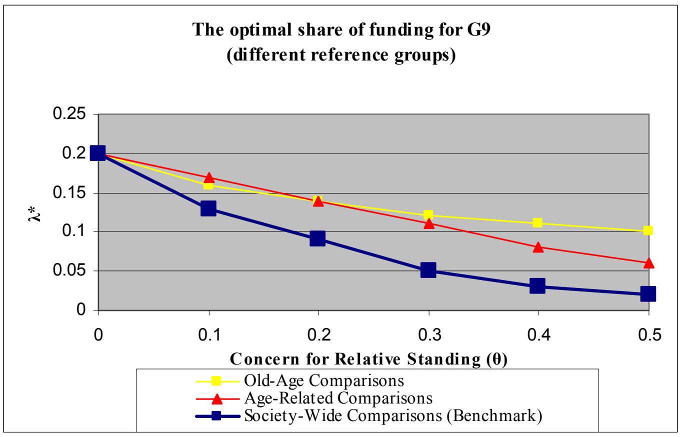
The optimal share of funding (λ)*	*The optimal contribution rate (τ*)*

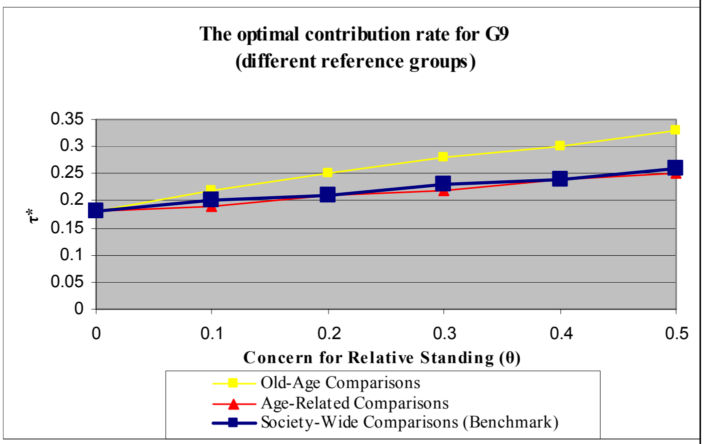
(d) Different portfolio mixes between equity and bonds





(e) Different reference groups





Note: The optimal values are derived as explained in the note to Figure 1. The alternative assumptions are explained in the text. The data are based on the sample period 1950-1999.

Index of Working Papers:

January 23, 2006	James Foreman-Peck (comment by Ivo Maes)	113	Lessons from Italian Monetary Unification
February 9, 2006	Stefano Battilossi (comments by Patrick McGuire and Aurel Schubert)	114	The Determinants of Multinational Banking during the First Globalization, 1870-1914
February 13, 2006	Larry Neal	115	The London Stock Exchange in the 19th Century: Ownership Structures, Growth and Performance
March 14, 2006	Sylvia Kaufmann, Johann Scharler	116	Financial Systems and the Cost Channel Transmission of Monetary Policy Shocks
March 17, 2006	Johann Scharler	117	Do Bank-Based Financial Systems Reduce Macroeconomic Volatility by Smoothing Interest Rates?
March 20, 2006	Claudia Kwapil, Johann Scharler	118	Interest Rate Pass-Through, Monetary Policy Rules and Macroeconomic Stability
March 24, 2006	Gerhard Fenz, Martin Spitzer	119	An Unobserved Components Model to forecast Austrian GDP
April 28, 2006	Otmar Issing (comments by Mario Blejer and Leslie Lipschitz)	120	Europe's Hard Fix: The Euro Area
May 2, 2006	Sven Arndt (comments by Steve Kamin and Pierre Siklos)	121	Regional Currency Arrangements in North America
May 5, 2006	Hans Genberg (comments by Jim Dorn and Eiji Ogawa)	122	Exchange-Rate Arrangements and Financial Integration in East Asia: On a Collision Course?
May 15, 2006	Petra Geraats	123	The Mystique of Central Bank Speak
May 17, 2006	Marek Jarociński	124	Responses to Monetary Policy Shocks in the East and the West of Europe: A Comparison

June 1, 2006	Josef Christl (comment by Lars Jonung and concluding remarks by Eduard Hochreiter and George Tavlas)	125	Regional Currency Arrangements: Insights from Europe
June 5, 2006	Sebastian Edwards (comment by Enrique Alberola)	126	Monetary Unions, External Shocks and Economic Performance
June 9, 2006	Richard Cooper Michael Bordo and Harold James (comment on both papers by Sergio Schmukler)	127	Proposal for a Common Currency among Rich Democracies One World Money, Then and Now
June 19, 2006	David Laidler	128	Three Lectures on Monetary Theory and Policy: Speaking Notes and Background Papers
July 9, 2006	Ansgar Belke, Bernhard Herz, Lukas Vogel	129	Are Monetary Rules and Reforms Complements or Substitutes? A Panel Analysis for the World versus OECD Countries
August 31, 2006	John Williamson (comment by Marc Flandreau)	130	A Worldwide System of Reference Rates
September 15, 2006	Sylvia Kaufmann, Peter Kugler	131	Expected Money Growth, Markov Trends and the Instability of Money Demand in the Euro Area
September 18, 2006	Martin Schneider, Markus Leibrecht	132	AQM-06: The Macroeconomic Model of the OeNB
November 6, 2006	Erwin Jericha and Martin Schürz	133	A Deliberative Independent Central Bank
December 22, 2006	Balázs Égert	134	Central Bank Interventions, Communication and Interest Rate Policy in Emerging European Economies

May 8, 2007	Harald Badinger	135	Has the EU's Single Market Programme fostered competition? Testing for a decrease in markup ratios in EU industries
May 10, 2007	Gert Peersman	136	The Relative Importance of Symmetric and Asymmetric Shocks: the Case of United Kingdom and Euro Area
May 14, 2007	Gerhard Fenz and Martin Schneider	137	Transmission of business cycle shocks between unequal neighbours: Germany and Austria
July 5, 2007	Balázs Égert	138	Real Convergence, Price Level Convergence and Inflation Differentials in Europe
January 29, 2008	Michał Brzoza-Brzezina, Jesus Crespo Cuaresma	139	Mr. Wicksell and the global economy: What drives real interest rates?
March 6, 2008	Helmut Stix	140	Euroization: What Factors drive its Persistence? Household Data Evidence for Croatia, Slovenia and Slovakia
April 28, 2008	Kerstin Gerling	141	The Real Consequences of Financial Market Integration when Countries Are Heterogeneous
April 29, 2008	Aleksandra Riedl and Silvia Rocha-Akis	142	Testing the tax competition theory: How elastic are national tax bases in Western Europe?
May 15, 2008	Christian Wagner	143	Risk-Premia, Carry-Trade Dynamics, and Speculative Efficiency of Currency Markets
June 19, 2008	Sylvia Kaufmann	144	Dating and forecasting turning points by Bayesian clustering with dynamic structure: A suggestion with an application to Austrian data.
July 21, 2008	Martin Schneider and Gerhard Fenz	145	Transmission of business cycle shocks between the US and the euro area
September 1, 2008	Markus Knell	146	The Optimal Mix Between Funded and Unfunded Pensions Systems When People Care About Relative Consumption