Introduction
The use of complex and powerful risk management methods has been one of the key innovations in the banking sector over the past two decades. One of the factors driving this development is certainly that since the early 1970s banks have had to cope with a significantly more volatile and dynamic environment compared to the years following World War II. In the immediate post-war period, currency crises were largely insignificant, market interest rates fluctuated only negligibly, competition was limited by cartels and interest rate regulation, and competition by financial intermediaries outside the banking sector was insignificant. Once the Bretton Woods system had collapsed, the situation changed drastically, however. Exchange rate risks started to play a role, interest rate fluctuations reached previously unknown dimensions, the lifting of capital controls resulted in a considerable internationalization of the financial system, and competition by nonbanks increased strongly. New technologies and means of communications rendered barriers to competition, such as distance and national borders, obsolete. In addition, financial innovations abounded. Against this background, regulators started to exert more pressure on banks. As capital adequacy provisions were continuously extended and refined, regulators relied heavily on individual risk management models. In the public these regulatory measures were invariably justified by pointing out the need to attenuate systemic risks and strengthen financial stability. The question remains, however, whether improving risk management models and implementing capital adequacy guidelines at the level of individual banks automatically leads to more efficient risk control at the level of the banking system.

There are reasons to doubt this argument. One of them lies in the fact that risks may arise from complex interbank lending transactions in the course of liquidity management and derivatives trading, which cannot be captured at the level of the individual institution. It is difficult to assess e.g. the counterparty risk of a bank in an isolated manner, because this approach fails to disentangle the interdependencies of interbank liabilities. It could therefore go unnoticed that a single institution figures in a cascade of interbank liabilities in which risks are highly correlated. Another problem pointed out by Hellwig (1997) is that a complex network of interbank debtor/creditor relations may result in sophisticated maturity transformation, which, in turn, at the level of the individual institution, may mask interest rate exposures of the banking system. Since it is hardly possible to assess the risk of a banking system based on the evaluation of individual banks, a “system approach” is called for. While risk management methods may certainly be suitable for individual credit institutions, regulators, concerning themselves primarily with the stability of the whole banking system, have to get a clear idea of the risk borne by the banking system. This is important since a systemic banking crisis, i.e. a situation in which financial inter-
mediation collapses at a large scale, translates into substantial costs to the real economy.

In what way does systemic risk assessment differ from risk assessment at the level of individual institutions, and how can we put such a method into practice? A research project conducted by the Economic Studies Division of the Oesterreichische Nationalbank (OeNB) and the Center for Business Studies of the University of Vienna was aimed at finding answers to the following questions: 1) How can we assess the risk of interbank loans at a system level, accounting explicitly for complex credit chains/interdependencies? How can we, to this effect, make optimal use of the data sources as they normally exist in central banks? The following sections briefly present the salient results of the said joint research project.

An Overview of the Model

The basic framework consists in a network model of the interbank market. Based on specific assumptions about the resolution of insolvencies, the model endogenously explains the possible payment flows among banks in different future states of the world (scenarios) for a given structure of interbank liabilities and for a given structure of other bank assets and liabilities. The states of the world are described by the impact interest rate changes, exchange rate and stock price fluctuations as well as credit defaults have on the banking business. The network model explicitly determines the possible interbank payments for each state of the world. Based on these results, we can calculate the expected default frequencies and the expected loss of interbank loans. The model is also capable of differentiating between insolvencies that are traceable to shocks resulting directly in the insolvency of a bank (fundamental insolvencies) and insolvencies that are triggered by the insolvency of another institution within the system (contagious defaults). This allows for an evaluation of the relative significance of fundamental insolvencies against insolvencies set off by chain reactions. We assess the risk of interbank loans on the basis of this analysis.

The main data sources are bank balance sheet data reported monthly to the OeWB and data of the Major Loans Register of the OeNB and a credit rating association, Kreditschutzverband of 1870. In addition, we use market data from Datastream. From the bank balance sheet data we estimate bilateral interbank positions and derive additional information about the claims and liabilities of individual institutions. The market data as well as Major Loans Register and Kreditschutzverband of 1870 data feed into the description of states of the world.

We use a cross-section of Austrian banks as at September 2001. According to the model calculation, the Austrian banking system is very stable and the likelihood of systemic banking crises is extremely low. In line with the September 2001 results, the median default probability of Austrian banks was below 1%. Only a very small percentage of all insolvencies of the model calculation are attributable to contagion. The frequency of contagious defaults is clearly correlated to the strength of negative developments in the fundamental risk factors.

For an overview of the basic structure of our model, see figure 1.

1 For first results of this project, see Elsinger, Lehar and Summer (2002).
The Network Model

The network model we use to analyze the system of interbank loans was introduced to the literature by Eisenberg and Noe (2001), who present an abstract, static analysis of a clearing problem in their paper. We extended this model to include uncertainty. To illustrate the key concepts of this approach to modeling the interbank network, let us take a look at a highly simplified example. The banking system in this case consists of three banks whose interbank loans are known. Here, the structure of claims and liabilities may be shown as a matrix, which could look as follows:

\[
L = \begin{pmatrix}
0 & 0 & 2 \\
3 & 0 & 1 \\
3 & 1 & 0
\end{pmatrix}
\]

The rows of this matrix refer to the liabilities of bank 1, bank 2 and bank 3 vis-à-vis the other banks in the system. Bank 2, for instance, has liabilities of 3 against bank 1 and liabilities of 1 against bank 3. The columns of the matrix demonstrate which claims the individual banks have on the other banks within the system. Since banks do not incur liabilities against themselves, the diagonal shows only zeros. We may illustrate the total liabilities of each bank using a list or a vector \( d = (2, 4, 4). \)
Let us assume that the net income of banks 1, 2, 3, which derives from their other activities, may be shown by the income flow \( e = (1, 3, 2) \). We may now ask: Can the banks fulfill all their interbank liabilities? In this particular case the answer is yes. Given the income flows in this example, all three banks can meet their liabilities simultaneously. Figure 2 depicts the payments effected between the individual institutions.

Let us assume that exchange rate fluctuations, interest rate changes or credit defaults affect the positions on the assets and liabilities sides which do not fall into the interbank category in such a way that \( e = (1, 3, 2) \) turns into \( e = (1, 1, 1) \). If we ask now whether the banks can fulfill all their interbank liabilities, the answer is a clear no.

To better understand this, it is useful to alter the matrix \( L \) of interbank liabilities by normalizing the individual entries with the total liabilities, which produces the following matrix:

\[
\begin{pmatrix}
0 & 0 & 1 \\
\frac{3}{4} & 0 & \frac{1}{4} \\
\frac{3}{4} & \frac{1}{4} & 0
\end{pmatrix}
\]

If all banks met all their liabilities, the net value of all banks can be derived as follows:

\[
\begin{pmatrix}
0 & \frac{3}{4} & \frac{3}{4} \\
0 & 0 & \frac{1}{4} \\
1 & \frac{1}{4} & 0
\end{pmatrix}
\begin{pmatrix} 2 \\ 4 \\ 4 \end{pmatrix}
+ \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix}
- \begin{pmatrix} 2 \\ 4 \\ 4 \end{pmatrix}
= \begin{pmatrix} 5 \\ -2 \\ 0 \end{pmatrix}
\]

Note that it is necessary in the above calculation to transpose the normalized liabilities matrix in order to calculate each bank’s income from interbank transactions. Assuming that all banks meet all their obligations, we arrive at a negative value for bank 2; in other words, this bank would be insolvent. Let us
assume therefore that the debt owed to bank 1 and bank 2 is serviced proportionately, while these two banks meet all their obligations. We thus arrive at the following:

\[
\begin{pmatrix}
0 & \frac{3}{4} & \frac{3}{4} & 2 \\
0 & 0 & \frac{1}{4} & 2 \\
1 & \frac{1}{4} & 0 & 1 \\
\end{pmatrix}
\begin{pmatrix}
1 \\
1 \\
4 \\
\end{pmatrix}
\begin{pmatrix}
1 \\
2 \\
4 \\
\end{pmatrix}
- \begin{pmatrix}
2 \\
2 \\
4 \\
\end{pmatrix}
= \begin{pmatrix}
\frac{7}{2} \\
-2 \\
-\frac{1}{2} \\
\end{pmatrix}
\]

The insolvency of bank 2 results in an interesting consequence. It reduces the interbank claims of bank 3 to such an extent that bank 3 fails to meet its obligations. Subsequently, bank 3 defaults as well. Triggering a *chain reaction*, the insolvency of bank 2 results in the insolvency of bank 3.

When we repeat the insolvency resolution rule of this example through proportionate debt servicing, we arrive at a payment vector which makes all claims consistent. In our case, this payment flow reads \( p^* = (2.28, 15.52, 15) \). It is evident from this vector that bank 2 and bank 3 are insolvent. We can furthermore infer how big their defaults are. In addition, the method used for calculating this solution reveals that the insolvency of bank 2 triggers the insolvency of bank 3. Figure 3 demonstrates the consistent payment flows.

Eisenberg and Noe (2001) proved that this example may be generalized. It is in particular possible to show that vectors making reciprocal claims consistent, so-called clearing payment vectors, always exist. Moreover, these vectors are unique under very weak regularity assumptions about the network. The algorithm used in the example to calculate the vector converges after a finite number of steps, namely at the most after as many steps as there are banks in the system.

This outcome enables us to perform a *scenario analysis* since we know that there is a unique clearing vector for each state of the world. We collect the bank balance sheet data for a given observation date and then identify \( L \) and \( e \). Subsequently, we define states of the world for a clearing date in the future, say, in one year’s time. For each scenario, the network model determines the payment
flows and thus the default frequencies, the loss given default and the contagious insolvencies. Using the relative frequencies of the individual events across the various scenarios, we may then conduct probability estimations. Figure 4 illustrates this method for the above example.

**Estimation of Bilateral Interbank Liabilities**

The bank balance sheet data reported monthly to the OeNB show both the claims and the liabilities vis-à-vis other banks. This facilitates our analysis of the data; the information does not, however, provide much insight into the structure of bilateral claims and liabilities. According to the reporting requirements, banks must break down interbank claims and liabilities also by joint stock banks, savings banks, state mortgage banks, Raiffeisen credit cooperatives, building and loan associations, Volksbank credit cooperatives, special purpose banks, foreign banks and the OeNB. In sectors with one or two tiers of central institutions, i.e. the savings bank, Raiffeisen and Volksbank sectors, banks furthermore must indicate claims and liabilities positions vis-à-vis the central institution. Since the interbank liabilities of the bulk of Raiffeisen credit cooperatives, savings banks and Volksbank credit cooperatives are almost exclusively vis-à-vis the central institution, we can observe some 80% of the entries in matrix $L$ directly from the data. To arrive at the remaining entries, we apply a specific estimation method.

To illustrate this method, let us take another look at our example. In the light of the available data, it is not possible to fill out the entire matrix $L$. From the data, we know the row and column totals for the individual sectoral subgroups. We also know that the diagonal must be zero. We derive individual entries based on what we know about the positions banks hold vis-à-vis the respective central institution. As many banks have only one interbank position, namely that against the central institution of their sector, it is clear from the sectoral row and column totals that the remaining row and column entries must be zero. In our example, we make the following observation for the matrix $L$: 

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure4.png}
\caption{Flow diagram illustrating the estimation of bilateral interbank liabilities.}
\end{figure}
As is evident from this table, we know the column and row totals as well as the diagonal entries. We cannot as yet say anything about the other entries. This problem of reconstructing data from tables is a frequent problem in applied mathematics and occurs in various contexts. The best known example from the field of economics is the calculation of the input-output table. In that case, the new table must be estimated based on the previous input-output table and current aggregated information.

The method we use to this end is called entropy optimization. This method attempts to distribute the mass of the row and column totals in such a way across the cells that the sum conditions are fulfilled and that as much consistency as possible is preserved with the a priori information about the unknown cell entries. For a more in-depth description of the formal details, see Elsinger, Lehar and Summer (2002). In this illustration, we only show the result this method generates for the matrix in our example.

\[
L = \begin{pmatrix}
0 & 0.443637 & 1.55456 \\
2.55452 & 0 & 1.445441 \\
3.44548 & 0.556363 & 0 \\
\end{pmatrix}
\]

In contrast to the given example the data derived from the monthly bank balance sheet data are not consistent. This is not surprising since the accounting identities are not exact, as reporting institutions may interpret items differently, make mistakes, etc. To estimate the matrix, it is of course necessary to strictly adhere to these identities, since it must not make any difference in which order we add up the matrix entries. At present, we are testing various methods to cope with these discrepancies. For the calculation presented in this study, we introduced a fictitious bank into each sector to account for any discrepancies in the accounting identities.

**The Creation of Scenarios**

The scenarios we use are created by exposing various balance sheet items to risk factors. In each scenario banks face gains and losses derived from market and credit risks. While shocks which affect all non-interbank balance sheet items are exogenous, the interbank credit risk is modeled endogenously using the network model. Table 1 shows the balance sheet items and illustrates which risks the individual items are exposed to in our analysis.

We choose a standard risk management framework to model exogenous shocks. We use historical simulation to model scenario losses and gains that derive from market risks and a credit risk model to capture losses from loans to nonbanks. For historical simulations, past realizations of interest, foreign
exchange rates and stock prices are treated as an empirical distribution from
which market scenarios are created. This method calls for a number of implicit
considerations and the use of several approximations since not all the informa-
tion can be read directly from the monthly return data. This applies, for
instance, to estimations of changes in the term structure. For a more in-depth
description, see Elsinger, Lehar and Summer (2002).

While we may use time series from Datastream for market risk data, this is
not possible for modeling credit defaults. For this reason we attempt to capture
loan losses via a standard credit risk model. In our analysis, we use CreditRisk+
(Credit Suisse, 1997). Since we are dealing with a system of credit portfolios
and not just with the credit portfolio of a single bank, we have to adapt this
model.

In simplified terms, the credit risk model works as follows: It considers that
all banks are affected by both aggregate and idiosyncratic shocks to their credit
portfolios. Input as to the average default frequency of each bank’s individual
credit portfolio and the standard deviation of this frequency need to be fed into
the credit risk model. Based on these parameters, we may calculate a distribution
of default losses for each bank. From this distribution we may in turn deduce the
loan losses under each scenario.

We can, of course, only approximate these data. First, we decompose the
balance sheet item “claims on nonbanks” in line with the Major Loans Register
data into several exposures to industries. Second, we assign the remaining credit
volume to a general item. Since we also know the number of large exposures in
the individual industries, we have industry-specific information about the num-
ber and average volume of loans. On the basis of the credit rating data provided
by the Kreditschutzverband of 1870, we may assign an estimated default
frequency and its standard deviation to each loan recorded for the various
industries. For the remainder of the credit volume which cannot be assigned
on the basis of the Major Loans Register information, we deduce approxima-
tions from averages of the data available. In this way we can define the necessary
parameters for the individual credit portfolios and calculate a distribution of
default losses for each bank. The distribution then yields default loss scenarios.

### Table 1

<table>
<thead>
<tr>
<th>Risk of Balance Sheet Items</th>
<th>Interest rate/stock price risk</th>
<th>Credit risk</th>
<th>FX risk</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Assets</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Short-term government bonds and receivables</td>
<td>yes)</td>
<td>no</td>
<td>yes)</td>
</tr>
<tr>
<td>Loans to other banks</td>
<td>yes)</td>
<td>no</td>
<td>yes)</td>
</tr>
<tr>
<td>Loans to nonbanks</td>
<td>yes)</td>
<td>no</td>
<td>yes)</td>
</tr>
<tr>
<td>Bonds</td>
<td>yes)</td>
<td>no</td>
<td>yes)</td>
</tr>
<tr>
<td>Equity</td>
<td>yes)</td>
<td>no</td>
<td>yes)</td>
</tr>
<tr>
<td>Other assets</td>
<td>no</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td><strong>Liabilities</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Liabilities to other banks</td>
<td>yes)</td>
<td>no</td>
<td>yes)</td>
</tr>
<tr>
<td>Liabilities to nonbanks</td>
<td>yes)</td>
<td>no</td>
<td>yes)</td>
</tr>
<tr>
<td>Securitized liabilities</td>
<td>yes)</td>
<td>no</td>
<td>yes)</td>
</tr>
<tr>
<td>Other liabilities</td>
<td>no</td>
<td>no</td>
<td>no</td>
</tr>
</tbody>
</table>

1) Historical simulation.
2) Primarily general government.
For an in-depth description of the formal details, see Elsinger, Lehar and Summer (2002).

As a next step, scenarios may be generated by combining historical simulation and the credit risk model. Under each scenario the network model determines the impact of the shocks on the possible interbank payments.

**Results for Austria**

The results presented here derive from a model calculation for the observation date September 2001. We generated 10,000 scenarios for this calculation and attained the following outcome.

**Default frequencies**

Table 2 shows various quantiles of the probabilities of default resulting from the model calculation. For each of the 908 banks in our data set, the probability of default is calculated for the 10,000 simulation scenarios. After that the banks are sorted by their probability of default in an ascending order. From this we compute the measures shown in table 2. The last row refers, for instance, to the entire banking system. In the column “10% quantile” we see that the probability of default of the “best” 10% of banks comes to 0%. In other words, these banks do not default under any of the 10,000 scenarios. According to the column “Median,” 50% of banks default in fewer than 0.73% of the scenarios. The right-most column “90% quantile” shows that the probability of default of only 10% of the banks is higher than 5.52%. Table 2 also indicates these measures for the individual sectors. All in all, it is evident that a predominant share of the banks is very sound.

**Severity of losses**

When assessing credit risk, it is, of course, not only important to determine default frequencies, but also the severity of losses. The network model produces endogenous recovery rates. We calculate for each bank the share of debt it could still service in the case of default. We then average these shares for each bank and sort the results in ascending order. Let us again look at the last row, which refers to the entire banking system (see table 3). For 10% of the defaulting banks the recovery rate amounts to zero, while on average 50% of the banks would, in case of default, be only be able to meet less than 53.31% of their interbank obligations. Finally, the recovery rate of 10% of the banks exceeds 90.8%. Table 3 also shows the recovery rates per sector.

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Table 2

<table>
<thead>
<tr>
<th>Probability of Default by Sectors</th>
<th>10% quantile</th>
<th>median</th>
<th>90% quantile</th>
</tr>
</thead>
<tbody>
<tr>
<td>Joint stock banks</td>
<td>0.00</td>
<td>0.06</td>
<td>2.39</td>
</tr>
<tr>
<td>Savings banks</td>
<td>0.00</td>
<td>0.19</td>
<td>2.34</td>
</tr>
<tr>
<td>State mortgage banks</td>
<td>0.00</td>
<td>0.17</td>
<td>0.61</td>
</tr>
<tr>
<td>Raiffeisen credit cooperatives</td>
<td>0.09</td>
<td>0.98</td>
<td>6.33</td>
</tr>
<tr>
<td>Volksbank credit cooperatives</td>
<td>0.12</td>
<td>0.48</td>
<td>7.16</td>
</tr>
<tr>
<td>Building and loan associations</td>
<td>1.21</td>
<td>3.35</td>
<td>7.18</td>
</tr>
<tr>
<td>Special purpose banks</td>
<td>0.00</td>
<td>0.00</td>
<td>0.61</td>
</tr>
<tr>
<td>Entire banking system</td>
<td>0.00</td>
<td>0.73</td>
<td>5.52</td>
</tr>
</tbody>
</table>

Source: OeNB, authors' calculations.

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A regulator who needs to assess the risk of banks at the system level may learn several interesting details from the simulation results. Banks may default as a direct consequence of shocks (fundamental insolvencies), but also due to a chain reaction, i.e. because other banks defaulted in the first place. The algorithm we use to compute the clearing vector allows us to distinguish between these two types of insolvencies.

Table 4 presents the results of the simulation calculation. The value 0.075 in the row “11–20” and the column “1–10” shows e.g. that for between 11 and 20 failed banks, 0.075 of them are contagious insolvencies.

### Fundamental and Contagious Insolvencies

<table>
<thead>
<tr>
<th>Number of fundamental insolvencies</th>
<th>Contagious insolvencies</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>1–10</td>
<td>11,784</td>
</tr>
<tr>
<td>11–20</td>
<td>46,877</td>
</tr>
<tr>
<td>21–30</td>
<td>17,357</td>
</tr>
<tr>
<td>31–40</td>
<td>7,838</td>
</tr>
<tr>
<td>41–50</td>
<td>4,795</td>
</tr>
<tr>
<td>51–60</td>
<td>2,441</td>
</tr>
<tr>
<td>61–70</td>
<td>1,484</td>
</tr>
<tr>
<td>71–80</td>
<td>1,365</td>
</tr>
<tr>
<td>81–90</td>
<td>0,892</td>
</tr>
<tr>
<td>91–100</td>
<td>0,516</td>
</tr>
<tr>
<td>101–110</td>
<td>0,398</td>
</tr>
<tr>
<td>111–120</td>
<td>0,204</td>
</tr>
<tr>
<td>121–130</td>
<td>0,065</td>
</tr>
<tr>
<td>131–140</td>
<td>0,194</td>
</tr>
<tr>
<td>141–150</td>
<td>0,323</td>
</tr>
<tr>
<td>151–160</td>
<td>0,258</td>
</tr>
<tr>
<td>161–170</td>
<td>0,065</td>
</tr>
<tr>
<td>171–180</td>
<td>0,108</td>
</tr>
<tr>
<td>over 180</td>
<td>0,183</td>
</tr>
</tbody>
</table>

Total 97,945 2,322 0.129 0.194 0.108

Source: OeNB, authors’ calculations.
20 banks the probability to default because of a shock and in turn cause 1 to 10 banks to default is 0.075%. The last row of the table reveals that the bulk of all insolvencies, namely 97%, may be classified as fundamental, and only a small share, 3% to be precise, may be ascribed to contagion in the system.

The first column of table 4, giving the number of fundamental insolvencies, may also be shown in a histogram (see figure 5). When we look at the frequency of bank insolvencies under all scenarios, we see that a larger banking crisis is highly unlikely.

![Figure 5: Fundamental Insolvencies](chart)

Source: DeNBe authors’ calculations.

![Figure 6: Bank Insolvencies by Total Assets](chart)

Source: DeNBe authors’ calculations.
Figure 5 does, however, not indicate the size of the insolvent banks. It could be much more problematic for a banking system if only a few major banks become insolvent than if many smaller banks default. To analyze this question, we finally calculate the size of the balance sheets of all insolvent banks under each scenario. According to figure 6, the balance sheets of the insolvent banks are relatively small; smaller banks thus have a comparatively higher probability of default.

Concluding Remarks
We have presented a new approach to assessing the risk of interbank loans and applied it to a set of Austrian bank data. The approach is innovative in so far as risk is assessed at the system level instead of the level of individual institutions and that it demonstrates how the data sources usually available to central banks may be used to this effect. The advantages of such an approach are threefold.

First, assessment at the system level uncovers the exposure to aggregate risks which traditional banking supervision, focusing on the individual institutions, fails to detect and account for. The method allows for a distinction between risks emanating from fundamental shocks and risks resulting from the threat of chain reactions. Second, our approach may help redirect the debate about regulatory issues, which currently centers on the refinement of capital adequacy provisions, to the more fundamental question of risk allocation in the overall economy and specifically the question of which share of aggregate risk is actually borne by the banking system. Our model could further this discussion in particular because it lends itself to the analysis of many if-then scenarios. Last but not least the model is designed to draw as much as possible on existing data sources. Even though such data might not be perfect, we hope that our work shows that systemic risk assessment is feasible. As we go along, experience is likely to help us pinpoint the truly essential information for assessing the stability of the banking system.

We hope that these ideas will benefit regulators and central bankers by pointing out ways how to use existing data sources to analyze systemic risks. Furthermore, we hope our work will make a valuable contribution to the academic debate about a system approach to banking supervision.

References