

# Modeling and Predicting the EUR/USD Exchange Rate: The Role of Nonlinear Adjustments to Purchasing Power Parity

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*Reliable medium-term forecasts are essential for forward-looking monetary policy decision-making. Traditionally, predictions of the exchange rate tend to be linked to the equilibrium concept implied by the purchasing power parity (PPP) theory. In particular, the traditional benchmark for exchange rate models is based on a linear adjustment of the exchange rate to the level implied by PPP. In the presence of aggregation effects, transaction costs or uncertainty, however, economic theory predicts that the dynamics of the nominal exchange rate around the equilibrium value implied by PPP are nonlinear. This paper presents some of the shortcomings of the traditional linear exchange rate models and assesses whether alternative nonlinear formulations outperform them for forecasting purposes. We find that the theory of nonlinear adjustment to PPP is supported by the data in a threshold cointegration framework for the monthly EUR/USD exchange rate between 1990 and 2010. Furthermore, sizeable gains in terms of medium-term forecast accuracy can be obtained using nonlinear specifications.*

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*Keywords: exchange rates, purchasing power parity, nonlinearity, threshold models*

Interest rate policy has a lagged, medium-term impact on inflation, which implies that monetary policy decisions must be forward looking, i.e. based on assessments of future price developments. The Eurosystem's monetary policy strategy, in particular, has a medium-term perspective: Due to the time lag of the monetary policy transmission mechanism, the ECB's interest rate decisions are based on regular economic and monetary analyses, which serve to identify risks to price stability in the short to medium term and in the medium to long term, respectively. Since exchange rates have a direct impact on price developments through their effect on import prices, they are crucial to the Eurosystem's economic assessments and projections. However, exchange rates are among the macroeconomic variables that are most difficult to forecast. Since model-based forecasts are often unsatisfactory, the Eurosystem's published economic projections are based on the technical

assumption that bilateral exchange rates remain unchanged over the projection horizon.

This paper presents some of the shortcomings of the traditional benchmark for medium-term exchange rate forecasts, namely the linear adjustment of the nominal exchange rate to the level implied by purchasing power parity (PPP), and assesses whether an alternative nonlinear formulation outperforms the traditional model in terms of forecasting accuracy.

PPP is an economic concept which postulates that international goods arbitrage will, in the long run, equate price levels across countries. If the national price level in country A is lower than in country B, international goods arbitrage will cause the currency of country A to appreciate with respect to the currency of country B, such that the price levels are equalized if expressed in one currency, and no arbitrage opportunities remain. It follows that the exchange rate – the relative

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price of two currencies – must equal the relative price levels of the respective countries. Letting  $S_t$  denote the spot exchange rate in price notation (the price of the foreign currency in terms of domestic currency) and letting  $P_t$  and  $P_t^*$  denote the domestic and foreign price levels, respectively, PPP requires that  $S_t = P_t/P_t^*$  or, letting lowercase letters denote natural logarithms

$$s_t = p_t - p_t^* \quad (1)$$

From the definition of the real exchange rate as the nominal exchange rate adjusted for relative price levels,  $q_t = s_t - (p_t - p_t^*)$ , it follows that PPP holds as long as the real exchange rate is constant. Empirically, if PPP is a relevant equilibrium concept, the real exchange rate would be expected to fluctuate closely around its constant mean, keeping the nominal exchange rate and the relative price levels together in the long run. Hence, in this setting real exchange rate fluctuations are assumed to be caused mainly by transitory shocks.

Although a growing body of literature finds some evidence for mean reversion in real exchange rates, the estimated degree of persistence remains extremely high, with half-lives of shocks ranging between three and five years (Rogoff, 1996). These long half-lives are not justifiable through price rigidities. “The purchasing power parity puzzle then is this,” writes Rogoff, “How can one reconcile the enormous short-term volatility of real exchange rates with the extremely slow rate at which shocks appear to damp out?” (Rogoff, 1996, p. 647).

Taylor (2001) shows that two pitfalls – namely sampling (temporal aggregation) and model specification (linear specification) – could cause an overestimation of half-lives and thus explain much of the persistence of real exchange rates. The idea that real exchange rates might exhibit nonlinear mean reversion dates back to 1916, when Heckscher proposed that trading costs could create a “band of inaction” in which no international goods arbitrage occurs, even though purchasing power differs between two countries. Only when arbitrage opportunities outweigh trading costs, international goods arbitrage takes place and mean reversion occurs.<sup>2</sup> Other approaches (O’Connell and Wei, 1997) consider variable costs of trade in addition to fixed costs. While there is mean reversion outside some band of inaction, real exchange rates will rarely cross their mean because of proportional trading costs. An alternative explanation for nonlinear adjustment was developed by Kilian and Taylor (2001), who focus on the determination of exchange rates on financial markets rather than on goods markets. In their model, on the one hand traders take the advice of economic fundamentalists, who differ in their opinions about exchange rate misalignments with respect to the equilibrium exchange rate, and on the other hand of technical analysts, who focus on trend projection in order to obtain exchange rate forecasts. The more uncertain economic fundamentalists are about the equilibrium exchange rate, the more weight is given to the forecast by technical analysts, which implies that the exchange rate persistently deviates from its equi-

<sup>2</sup> *The transaction costs argument holds, in a purer form, for exchange rate determination during the gold standard. Since deviations of nominal exchange rates from the gold parity opened arbitrage opportunities, exchange rate fluctuations were restricted through the cost of shipping gold from one country to another, including freight, insurance and opportunity costs as well as a risk premium (Mooslechner, 2008).*

librium value. As the deviations become greater, however, the degree of agreement between economic fundamentalists on the misalignment increases, and more weight is given to their increasingly conclusive consensus forecasts, so that mean reversion sets in.

Based on this theoretical discussion, in this contribution we formulate an empirical model for the EUR/USD exchange rate which allows for nonlinear reversion toward the PPP equilibrium. Focusing on the original “band of inaction” theory, the real exchange rate will be modeled as a nonstationary process within certain thresholds and as mean reverting outside the band defined by those thresholds. We analyze whether the “band of inaction” theory can explain the persistence of the EUR/USD real exchange rate (i.e. whether there is a threshold effect within the adjustment of the nominal exchange rate to the PPP equilibrium) and whether the underestimation of the speed of adjustment can be traced to the above rigidities (i.e. whether the speed of mean reversion of real exchange rates found outside the thresholds is significantly higher than the speed determined over the whole sample). We also investigate whether modeling the nonlinear adjustment with the aid of threshold specifications improves the out-of-sample predictive ability for the nominal exchange rate.

This paper is structured as follows: Section 1 provides an overview of the EUR/USD exchange rate developments between 1990 and 2009 and relates major deviations from the PPP equilibrium to the evolution of the monetary policy framework in the euro area. Section 2 presents the framework for empirical analysis, i.e. the cointegrated

vector autoregressive (VAR) model, the estimation of the band of inaction and the resulting model with nonlinear adjustment. Next, section 3 compares the out-of-sample forecasting properties of the linear and the nonlinear model. Finally, section 4 summarizes the main findings and provides an outlook for further research.

## 1 Twenty Years of PPP and the EUR/USD Exchange Rate

Since the beginning of the process toward the Economic Monetary Union (EMU) in 1990, the dynamics of the exchange rates between the legacy currencies of the euro and the U.S. dollar can be understood more properly by making reference to the evolution of the institutional framework of European monetary integration. The first decade – from the liberalization of capital transactions in the European Economic Community in 1990 to the introduction of the common currency in 1999 – was marked by persistent deviations of the synthetic EUR/USD exchange rate<sup>3</sup> from the PPP equilibrium. While the price differential remained relatively constant during the decade, the nominal exchange rate experienced extended periods of sustained appreciation and depreciation. The EUR/USD exchange rate and the price differential are plotted in charts 1 and 2, respectively.

In particular, the exchange rate dynamics in the early 1990s were closely related to the European Exchange Rate Mechanism (ERM), with pairwise fluctuation bands of  $\pm 2.25\%$ . When the Deutsche Bundesbank strongly increased its interest rates in the beginning of the 1990s to counteract inflationary

<sup>3</sup> The calculation of the synthetic EUR/USD exchange rate is based on a weighted average of the bilateral exchange rate of the former national currencies against the U.S. dollar and was sourced from Thomson Reuters.

pressures (which stemmed at least partly from German reunification), this caused an appreciation vis-à-vis the U.S. dollar not only of the Deutsche mark but also of the remaining ERM currencies. Markets considered some European currencies overvalued and high interest rates were perceived as unsustainable for countries with weaker economic fundamentals (e.g. United Kingdom and France). Eventually, speculation against some currency pegs led to the ERM crisis between 1992 and 1993, in which the pound sterling had to leave the ERM and the fluctuation margins for the remaining currencies had to be widened to  $\pm 15\%$ . This crisis caused a sharp depreciation of the synthetic EUR/USD exchange rate from 1992 to 1993, even though the PPP level remained roughly unchanged for this period.

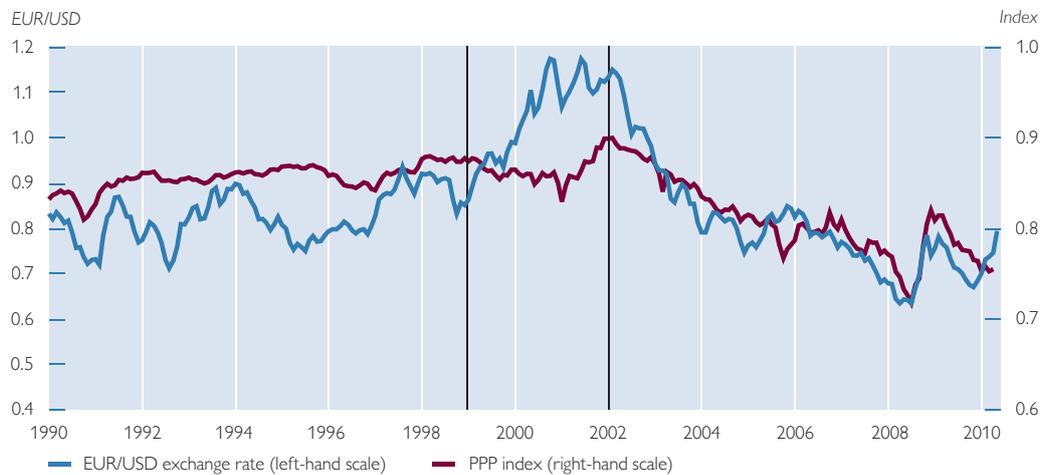
In order to promote economic convergence between the future euro area economies, the Treaty on European Union of 1992 – which established the completion of EMU by 1999 as a formal objective – stated a number of economic criteria concerning inflation, long-term interest rates, exchange rates and public finances. At the same time, the Clinton administration proclaimed a “strong dollar” policy in the United States. The political will to guard the value of the U.S. dollar resulted in the coordinated intervention of the Federal Reserve System (Fed), the Bank of Japan and the Deutsche Bundesbank against the U.S. dollar depreciation in 1995. This intervention marked the beginning of a sustained nominal appreciation trend of the U.S. dollar. Since the inflation differential remained relatively constant until the introduction of the euro, the weakening of the synthetic euro against the U.S. dollar recorded from 1995 onward was not in line with PPP fundamentals.

In January 1999 the conversion rates of the legacy currencies against the euro were irrevocably fixed, the euro was introduced as the noncash single currency and the single monetary policy entered into force. The handing over of monetary policy responsibility to the ECB might have contributed to the relative decline of European producer prices between 1999 and 2001. The declining price differential between 1999 and 2001 would have suggested a strengthening of the euro with respect to the U.S. dollar for this period. Instead, the phase between the introduction of the euro in January 1999 and the cash changeover in January 2002 was characterized by a strong weakening trend (from about 0.85 EUR/USD in January 1999 to almost 1.20 EUR/USD in 2001). In autumn 2000, several concerted foreign exchange interventions were carried out by the ECB and the Fed (the only official concerted interventions involving the ECB hitherto) to counteract the euro depreciation. In 2001, the recessive period in the U.S.A. and the resulting disinflationary period contributed to bringing the nominal exchange rate closer toward the PPP equilibrium rate.

In January 2002, euro banknotes and coins became the sole legal tender in the euro area countries. Since then, producer price inflation has been substantially lower in the euro area than in the U.S.A., leading to a strong decline in the producer price differential (which had until then been relatively constant). This process was accompanied by a strengthening of the euro (from over 1.10 EUR/USD in January 2002 to around 0.60 EUR/USD in summer 2008), such that the magnitude of exchange rate misalignments with respect to the PPP equilibrium seems to have decreased since the cash turnover in 2002. Since then, the euro

Chart 1

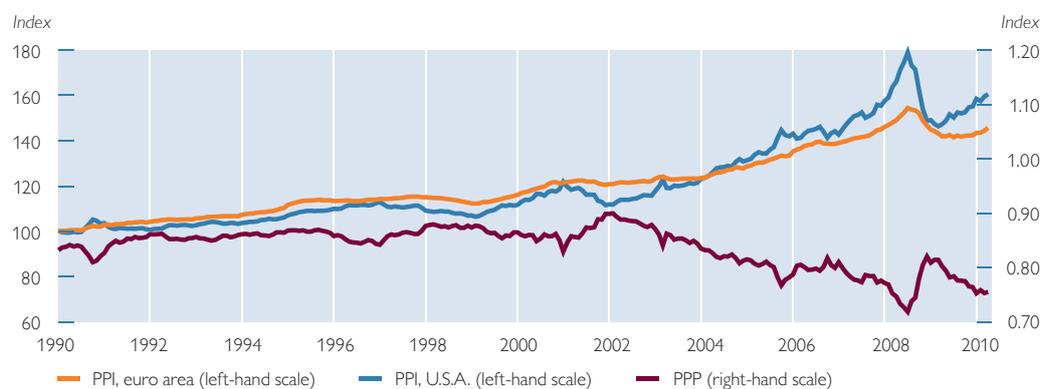
### EUR/USD Exchange Rate and Purchasing Power Parity (PPP)



Source: Eurostat, Fed, OeNB.

Chart 2

### Producer Price Index (PPI) and Purchasing Power Parity (PPP)



Source: Eurostat, Fed, OeNB.

has somewhat weakened, with USD 1 trading for EUR 0.70 on average in 2009.

Against the background of the substantial exchange rate fluctuations over the past twenty years, the recent EUR/USD depreciation associated with the outbreak of the Greek sovereign debt crisis in winter 2009/2010 cannot be judged a major deviation from PPP equilibrium at this point in time. At the cutoff date (June 2010), the level of the EUR/USD exchange rate lied slightly above the historical average.

## 2 PPP, Nonlinearities and the Exchange Rate: An Empirical Analysis

### 2.1 Data

For our empirical analysis, we use monthly data on exchange rates and producer prices for the euro area and the U.S.A. between January 1990 and December 2009. The EUR/USD exchange rate was specified as the nominal exchange rate in price notation, such that an increase in the exchange rate is interpreted as a nominal depre-

ciation of the euro (until December 1998: synthetic EUR/USD exchange rate). Price levels were specified as PPIs rather than CPIs. On the one hand, due to the strong weight of services in the CPI, the proportion of tradable and traded goods is higher in the former than in the latter; on the other hand, producer prices might also suffer fewer distortions through taxes than consumer prices.<sup>4</sup> The producer price series were seasonally adjusted for the estimation.

Empirically, PPP theory holds when the real exchange rate  $s_t - (p_t - p_t^*)$  is stationary. More generally, the PPP holds as long as  $s_t$  and  $(p_t - p_t^*)$  are integrated of order one ( $I(1)$ ) and a linear combination of  $s_t$  and  $(p_t - p_t^*)$  is stationary, i.e. if  $s_t$  and  $(p_t - p_t^*)$  are cointegrated. Augmented Dickey-Fuller tests on the null hypothesis of a unit root suggest that the nominal exchange rate and the price differential are both  $I(1)$ , such that they fulfill the prerequisites for further cointegration analysis.

## 2.2 Cointegration and the PPP Equilibrium

The presence of a long-run equilibrium of nonstationary time series implies that the natural modeling framework is given by vector error correction (VEC) specifications (Engle and Granger, 1987). The basic linear specification used in this study to model the relationship between exchange rate and price differential dynamics is thus given by

$$\Delta x_t = \Gamma(0) + \sum_{j=1}^k \Gamma(j) \Delta x_{t-j} + \Pi x_{t-1} + \varepsilon_t$$

$$x_t = [s_t, p_t - p_t^*]' \quad (2)$$

$$\varepsilon_t = [\varepsilon_t^1, \varepsilon_t^2]': NID(0, \Sigma)$$

where  $\Gamma(0)$  is a vector of constants, the matrices  $\Gamma(j)$  contain the short-run coefficients and the reduced rank matrix  $\Pi = \alpha\beta'$  summarizes the long-run elasticities ( $\beta$ ) and the adjustment parameters associated with the equilibrium condition ( $\alpha$ ). Adjustment of the nominal exchange rate toward PPP implies that the first element of  $\alpha$  is negative. The Johansen test for cointegration is inconclusive concerning the statistical evidence for cointegration between the exchange rate variable and relative prices. In particular, the results change strongly depending on the set of deterministic elements included in the model and the lag length of the short-run adjustment part. To the extent that nonlinearities play an important role in the adjustment, the lack of evidence for cointegration in the linear setting is not surprising. We thus stick to the VEC setting despite the weak empirical support in terms of cointegration test results. The estimates of model (2) are presented in the first column of table 1.

A further simplification of (2) can be achieved if relative PPP is assumed to hold in its “pure” theoretical form, namely with the restriction that the long-run parameter vector is  $\beta = (1, -1)'$ . The result of the likelihood ratio test in table 1 indicates that the restrictions imposed by the pure relative PPP model cannot be rejected by the data.

The estimated adjustment coefficients for the exchange rate are negative and marginally significant in both model specifications, which indicates that there is some equilibrium correction of the nominal exchange rate toward its PPP fundamentals. However, the adjustment speed is very low, with half-lives of 3.2 years and 2.6 years for the

<sup>4</sup> The monthly average nominal EUR/USD exchange rate was provided by Thomson Reuters (until December 1998: synthetic EUR/USD exchange rate). Seasonally adjusted monthly PPIs were provided by the Fed for the United States and by the OECD for the euro area in its respective composition.

unrestricted and the restricted model, respectively.<sup>5</sup> This finding is consistent with the vast majority of half-life estimates in the empirical literature dealing with PPP in the framework of linear models, which tend to range between three and five years (Rogoff, 1996).

Interestingly, there is no evidence for a robust exchange rate pass-through between the euro area and the U.S.A., since the hypothesis that price levels adjust toward the exchange rate is not backed up by the data. The fact that both the euro area and the U.S.A. are large and relatively closed economies intuitively supports the lack of pass-through from exchange rates to price differentials. From a modeling point of view, the price differential is thus weakly exogenous to the exchange rate. Hence, the result concerning the weak exogeneity of relative prices may be seen as a justification to use univariate specifications to model exchange rates. However, we stick to a bivariate model for

several reasons. On the one hand, our model allows for differential short-run effects of relative price and nominal exchange rate dynamics on the current exchange rate and thus nests the case where only the real exchange rate depends on its own past. On the other hand, since we are interested in exchange rate predictions, a specification for relative price dynamics is necessary for our out-of-sample exercise.

The high persistence of deviations from the long-run relationship, which materializes itself in a low speed of adjustment – the PPP puzzle – motivates the use of nonlinear model specifications to tackle the issue of inaction bands and regime-specific speeds of adjustment to PPP.

### 2.3 Threshold Effects and Nonlinear Adjustments to PPP

Following the arguments of Heckscher (1916) and Taylor (2001), according to which the mean reversion of real exchange rates might depend on the

Table 1

#### Cointegrating Vectors, Adjustment Coefficients and Half-lives of Long-run Deviations – Linear Models

	(1)	(2)
Long-run elasticity	2.576 (1.148)	1.000 imposed as $\beta = (1, -1)$
Exchange rate adjustment parameter ( $\alpha_s$ )	-0.018 (0.013)	-0.021 (0.013)
Price differential adjustment parameter ( $\alpha_{p-p}$ )	0.006 (0.005)	0.002 (0.004)
Half life of PPP deviation (in years)	3.2	2.6
LR-test for $\beta = (1, -1)$	1.061	
p-value	0.302	

Source: Author's calculations.

Note: Numbers in parenthesis are standard errors. Number of observations: 238.

<sup>5</sup> Half life estimates, which correspond to the estimated time it takes for the exchange rate to correct half of the deviation to the equilibrium, are estimated as  $\hat{h} = \frac{\log(2)}{-\hat{\alpha}_s}$

extent to which the nominal exchange rate deviates from the PPP equilibrium, we expect that model specifications with such a property would reduce the downward bias of the overall speed of adjustment toward the price fundamentals. Particularly, theories predicting a band of inaction in the dynamics of the nominal exchange rate around the equilibrium value implied by PPP theory would justify the use of empirical models which explicitly take into account this feature of the adjustment process. A simple approach would consist of dividing the deviations of the equilibrium level implied by PPP into regimes and allowing for potentially different adjustment processes in each one of them.

Threshold models have been used extensively to model asymmetric adjustment in macroeconomic series. General threshold autoregressive models were first introduced by Tong (1993), and the special case of threshold cointegration was formulated by Balke and Fomby (1997). In order to capture the nonlinear properties of the long-run equilibrium, we introduce the following threshold VEC (T-VEC) model

$$\begin{aligned} \Delta x_t = & \Gamma(0) + \sum_{j=1}^k \Gamma(j) \Delta x_{t-j} + \\ & + \alpha_1 \beta' x_{t-1} I(\beta' x_{t-1} < \theta_1) + \\ & + \alpha_2 \beta' x_{t-1} I(\beta' x_{t-1} > \theta_2) + \varepsilon_t \end{aligned} \quad (3)$$

where  $\theta_1$  and  $\theta_2$ ,  $\theta_1 < \theta_2$ , are the thresholds delimiting three regimes in terms of the lagged level of the PPP deviation and  $I(\cdot)$  is the indicator function, taking the value of one if its argument is true and zero otherwise. In the central regime ( $\theta_1 < \beta' x_{t-1} < \theta_2$ ) no adjustment to the PPP equilibrium takes place, and the model representing the joint dynamics of the nominal exchange rate and relative prices is a vector autoregression in first differences. When the deviation from equilibrium surpasses the thresh-

old, the adjustment process (to the central regime) sets in, with a speed given by the size of  $\alpha_1$  for corrections that are triggered if the exchange rate falls below the band of inaction and  $\alpha_2$  for adjustments that set in if the exchange rate exceeds the band.

This specification is able to model the asymmetric dynamics implied by theoretical settings in which corrective behavior is only expected to be relevant when the deviation from equilibrium is large enough, as in PPP models assuming transaction costs and aggregation effects in the basket of goods included in the price indices.

We fix the constrained cointegration relationship supported by the linear model, so that  $\beta = (1, -1)'$  is maintained for the estimation of the nonlinear specification. The thresholds can then be estimated by carrying out a grid search over all pairs of realized values of the deviation from PPP and choosing the pair that minimizes the sum of squared residuals for equation system (2). In order to avoid regimes with too few observations and to identify a relevant band of inaction, we impose that at least 10% of the observations lie in the outer regimes and 30% in the central band.

The results of the estimation of the T-VEC model are presented in table 2 and the estimated band of inaction, together with the deviations from the PPP equilibrium, is depicted in chart 3. The central regime implied by the estimates is large compared to the regimes where adjustment takes place, thus deviations from PPP equilibrium are not corrected for most of the estimation period. The estimated upper and lower regime thresholds are symmetric with respect to the PPP equilibrium (as defined by the mean of the real exchange rate over the full period under study). Furthermore, the point estimate

of the adjustment speed is higher in both regimes than in the linear alternatives presented above, although the speed of downward adjustment seems not statistically significant on a 5% level. Strong differences can be observed across adjustment regimes, with short-lived deviations in the lower regime and a single longer period of downward

adjustment spanning the period from 2000 to 2002.

We also perform a nonlinearity test in the spirit of Hansen (1996) and Hansen and Seo (2002), bootstrapping the distribution of the likelihood ratio statistic for testing the linear VEC model against the threshold alternative. The result of the test gives evidence against

Table 2

### Adjustment Coefficients, Thresholds and Half-lives of Long-run Deviations – T-VEC Model

	$\beta x_{t-1} < \theta_1$	$\beta x_{t-1} > \theta_2$
	Lower regime	Upper regime
Exchange rate adjustment parameter ( $a_\$$ )	-0.038 (0.014)	-0.054 (0.062)
Price differential adjustment parameter ( $a_{p-p^*}$ )	-0.002 (0.006)	0.008 (0.020)
Half life of PPP deviation (in years)	1.5	1.1
$\theta_1$		-0.129
$\theta_2$		0.131
LR-test for linearity		4.999
Bootstrap p-value		0.050

Source: Authors' calculations.

Note: Numbers in parenthesis are standard errors. Number of observations: 238.

Chart 3

### Real Exchange Rate and Bands of Inaction

Index (standardized around the mean)



Source: Eurostat, Fed, Thomson Reuters, authors' calculations.

the linear specification and supports the threshold adjustment model.

The half-lives of the deviations, which are now to be interpreted as the speed of adjustment to the central inaction band, are significantly reduced in comparison with the linear models and indicate that half of the gap between the realized exchange rate and the band of inaction closes in around 1 to 1½ years.

Alternative estimations of single-threshold models with different adjustment speeds for positive and negative deviations confirm that there is significant adjustment of negative deviations from the PPP fundamentals, while such correction is not statistically significant for positive deviations. Since a positive error correction term corresponds to an undervaluation of the euro relative to its PPP fundamentals, the asymmetry of the adjustment process implies that exchange rate dynamics should be more predictable in periods when the euro is overvalued (in terms of PPP equilibrium). We explicitly test for this hypothesis in our out-of-sample prediction exercise below.

### 3 The Role of Nonlinear Adjustment to PPP for Exchange Rate Prediction

The question arises whether the non-linearity found can be used to improve policy advice by anticipating exchange rate dynamics more efficiently than if linear models are used. For this purpose, we carry out an out-of-sample prediction exercise aimed at comparing the forecasting accuracy of the T-VEC model proposed for specifying exchange rate dynamics.

The design of the prediction exercise is as follows. Using data up to December 1999, we estimate the thresholds of the T-VEC model and the linear specification. We predict the EUR/USD

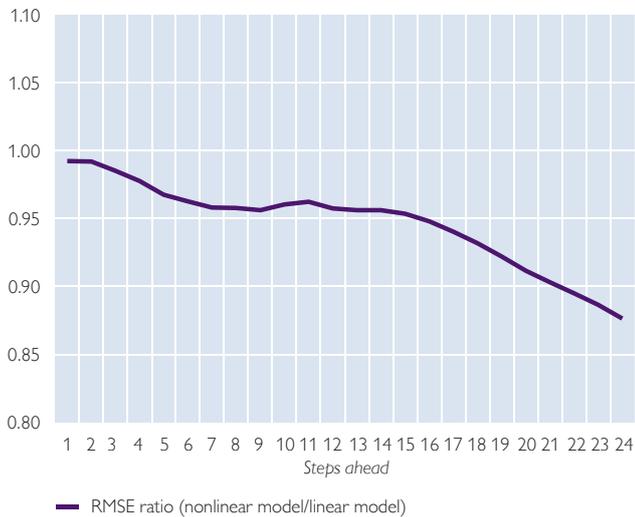
exchange rate for the period from January 2000 to December 2001 using this T-VEC model and the linear VEC alternative. For the forecast evaluation, the root mean squared prediction error (RMSE) is calculated for forecasting horizons up to two years ahead. We then add to the estimation sample the observation corresponding to January 2000 and repeat the prediction exercise. This is repeated until we reach the end of the sample, and the average RMSE for each forecasting horizon is obtained both for the linear and the nonlinear model. We also evaluate the performance of the models in terms of whether the direction of exchange rate movements corresponds to the prediction in the direction-of-change (DOC) statistic.

Chart 4 shows the RMSE-ratio, i.e. the RMSE of the T-VEC divided by the RMSE of the linear VEC. For all forecasting horizons the RMSE-ratio is smaller than one, implying that the forecasting accuracy of the T-VEC model is higher than the forecasting accuracy of the linear VEC model. Moreover, the RMSE-ratio decreases with the length of the forecasting horizon, implying that the forecasting advantage of the T-VEC model increases further for longer-term forecasts. The gains in terms of forecasting accuracy are of around 5% at the one-year horizon and 15% at the two-year horizon. We also performed Diebold-Mariano tests of significance in the difference of predictive accuracy, which gave strong evidence of statistically significant improvements in the forecasting ability of the T-VEC model after a horizon of 18 months. At prediction horizons between four and ten months ahead, the T-VEC improvement in forecast accuracy is marginally significant.

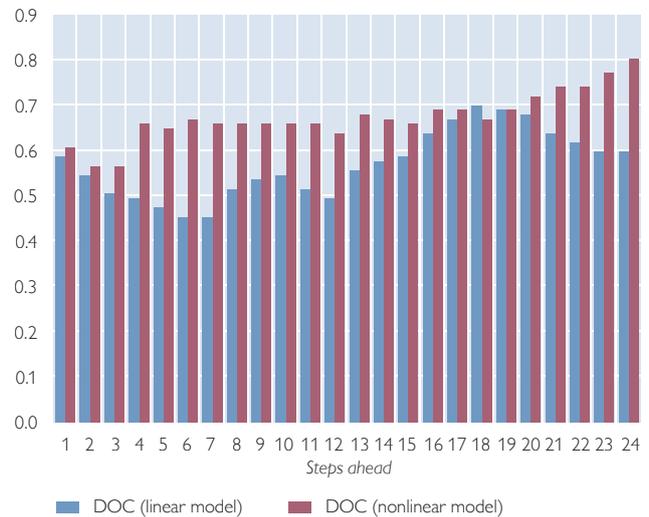
Chart 4

### Out-of-Sample Predictive Performance

#### Root Mean Squared Error (RMSE)



#### Direction of Change (DOC) Statistics

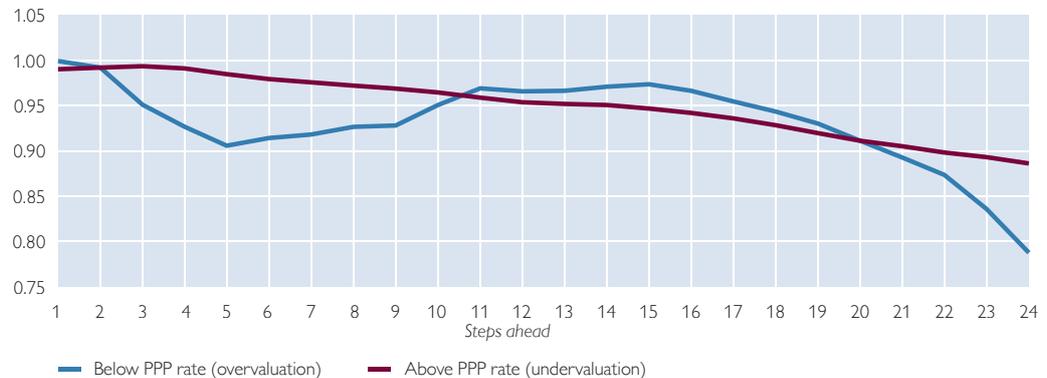


Source: Authors' calculations.

Chart 5

### Out-of-Sample Predictive Performance by Subsample

#### Root Mean Squared Error (RMSE) Ratio, (nonlinear model to linear model)



Source: Authors' calculations.

Very sizeable improvements are found in terms of predicting the direction of change in the exchange rate. In chart 4, the DOC statistics is shown for both models over the 24 forecasting horizons. With the exception of the forecasting horizons around 1½ years, where the performance of both models is very similar, the T-VEC model clearly outperforms the linear specification, in

particular for horizons of 4 to 12 months and of more than 20 months.

In order to understand the nature of the improvement of the predictive ability of the nonlinear model, we calculate the RMSE ratio in the two subsamples defined by whether the nominal exchange rate appears overvalued or undervalued with respect to the PPP equilibrium. The results are presented

in chart 5 and indicate that there are interesting differences across subsamples. The advantage of the T-VEC model in terms of forecasting accuracy is independent of the subsample analyzed. Improvements are particularly large for the sample of observations below the PPP rate (overvaluation) for forecasts ranging from three to nine months ahead, as well as for very long-run forecasts. Since deviations in the lower regime tend to be corrected toward the band of inaction at a relatively high speed, these results indicate that the nonlinear specification is able to replicate and predict adjustment dynamics better when the euro is relatively overvalued with respect to the PPP equilibrium.

#### 4 Conclusions and Further Paths of Research

In this study we propose a simple threshold error correction specification for the EUR/USD exchange rate aimed at modeling nonlinear adjustments to the PPP equilibrium. Our results show that the data strongly support the pres-

ence of nonlinearities related to the adjustment of the nominal exchange rate to the equilibrium level given by PPP. Furthermore, we present empirical evidence concerning the fact that sizeable out-of-sample forecast gains for exchange rate dynamics can be obtained when nonlinearities are explicitly modeled. Our results imply that monetary policy makers' potential to anticipate exchange rate movements would be improved if they based their assumptions on model specifications that take into account the dynamics of nonlinear adjustment to PPP as the long-run equilibrium.

Admittedly, the specification proposed in this study is only one of various possible ways how to specify nonlinear adjustment to the equilibrium in the framework of exchange rate models. In particular, comparing threshold adjustment models with other nonlinear models based on smooth transition specifications or nonparametric methods is a promising avenue for further research in this topic.

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