The Transmission of Foreign Shocks in a Networked Economy

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Monetary policy trade-offs in a heterogeneous currency area

53rd OeNB and SUERF Annual Economics Conference

June 18–19, 2026

Motivation

- Recent wave of supply-side **macroeconomic shocks with sectoral and international origins:**
  - energy price increases
  - supply bottlenecks
  - Covid-19
  - risks of trade fragmentation, import tariffs ...

- Despite their different underlying causes, share key characteristic
  - originate in specific sectors
  - but quickly spread through complex production networks and international supply chains

- **Aim:** understand how these shocks are transmitted through input-output (IO) linkages and spillover across countries and sectors

This Paper

- Investigate the **transmission of supply-side shocks** through production networks
- Particular focus on their impact on **inflation dynamics**
  - increasing energy prices, key determinant of EA recent headline inflation
- Build a **model that accommodates the disaggregated and international nature of shocks**:
  - global economy with multiple countries
  - **multi-sectoral** productive structure with **national and international production linkages**
  - sectors subject to nominal price and wage rigidities
- Explore how production networks reshape the monetary-policy trade-off
- Calibrate the model to the main Euro-Area countries and their trade partners

Findings

- Focus on the **effects and transmission of imported energy price shocks** in the Euro Area
  1. **Analytical results**: international networks steepen the Phillips curve and sectoral ToT gaps prevent the Divine Coincidence
  2. **Contribution of input-output (IO) linkages**:
     - Without IO: cumulative headline inflation is $\approx$ 60% of baseline, shorter lived, with smaller pass-through to core
  3. **Cross-country heterogeneity**:
     - DE: longer production chains generate larger transmission to core and more persistent headline inflation
     - ES: higher CPI energy weights increase headline inflation on impact, but the response is shorter lived
  4. **Implications for monetary policy**:
     - IO worsens the policy trade-off: stabilizing inflation is costlier in terms of output-gap

Model

Model

1. **Model Overview**
2. Households
3. Firms
4. Monetary Authority
5. Market Clearing

Model Overview

- Global economy with **K countries**
  - International financial markets incomplete
  - Monetary arrangements:
    - $\mathbf{K}^{\mathrm{MU}} \subset \mathbf{K}$ countries form part of **monetary union** with a common central bank
    - Remaining countries have monetary autonomy
- Within each country $\mathbf{k} \in \mathbf{K}$
  - **I sectors**: multi-sector productive structure with **national and international networks**
  - **Nominal rigidities** on *prices* (heterogeneous across sectors) and *wages* (homogeneous across sectors)

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Households

Intertemporal Decision

Problem

- Representative household chooses consumption, labor supply, and portfolios of international nominal bonds
- Budget constraint links consumption and bond purchases to bond payoffs, labor income, profits, transfers, and adjustment costs
- Portfolio adjustment costs close the incomplete-markets block and stabilize net foreign assets (Schmitt-Grohé and Uribe, 2003)

Consumption Basket

Quantitative specification

- Country-k consumption combines sector-country varieties:

$$C_{k,t} = \mathscr{C}_k\left(\left\{C_{kli,t}\right\}_{l\in K,\, i\in I}\right)$$

- $\mathscr{C}_k(\cdot)$ is a general aggregator, allows for heterogeneity across sectors and countries of origin

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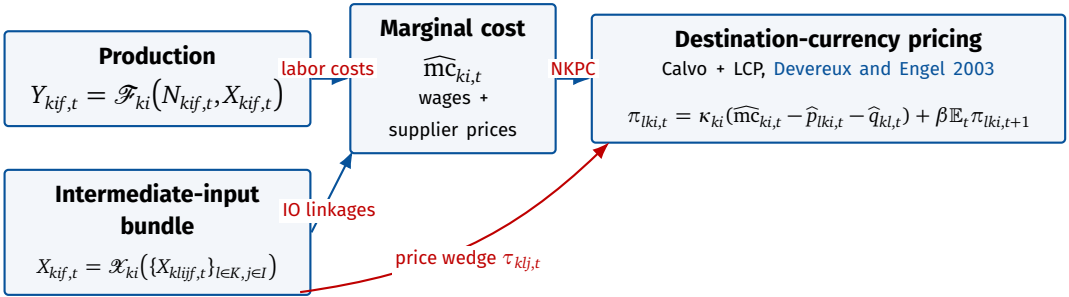
Firms: Production and Price Setting



- Heterogeneous $\mathscr{F}_{ki}(\cdot)$ and $\mathscr{X}_{ki}(\cdot)$ make sectoral marginal costs depend on domestic wages and domestic/foreign supplier prices.
- Price wedges move import prices independently of exporters' activity, feeding into firms' costs through the input network.

Model

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5. Market Clearing

Monetary Authority

- **Countries not members of a currency union**: Taylor rule that targets *domestic* headline inflation and domestic GDP:

$$i_{k,t} = \rho_{kr} i_{k,t-1} + \left(1-\rho_{kr}\right)\phi_{k\pi}\pi_{k,t} + \left(1-\rho_{kr}\right)\phi_{ky}\widehat{y}_{k,t} + \varepsilon^{r}_{k,t} \tag{1}$$

- **Member countries of a currency union** (K^{MU}):
  - Country $k^{MU} \in K^{MU}$ sets interest rates responding *union-wide* headline inflation and GDP:

$$i_{k^{MU},t} = \rho_{\text{MU}r} i_{k^{MU},t-1} + \left(1-\rho_{\text{MU}r}\right)\phi_{\text{MU}\pi}\pi^{\text{MU}}_{t} + \left(1-\rho_{\text{MU}r}\right)\phi_{\text{MU}y}\widehat{y}^{\text{MU}}_{t} + \varepsilon^{r}_{\text{MU},t}$$

  - Remaining countries in K^{MU} *peg* their nominal exchange rate to k^{MU}:

$$\mathcal{E}_{k,k^{MU},t} = \mathcal{E}_{k,k^{MU}} \quad \forall k \in K^{MU}$$

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5. **Market Clearing**

Market Clearing & GDP

- Labor market clears:

$$N_{k,t} = \sum_{i=1}^{I} N_{ki,t}$$

- Goods' market clears:

$$Y_{ki,t} = \sum_{l=1}^{K} \left(C_{lki,t} + \sum_{j=1}^{I} X_{lkji,t} \right) \quad \forall i \in I$$

- Bonds' market clears:

$$\sum_{l} B^{k}_{l,t} = 0, \quad \forall k \in K$$

- Nominal GDP $\mathscr{Y}_{k,t}$:

$$\mathscr{Y}_{k,t} = P_{kC,t} C_{k,t} + P_{k\mathsf{EXP},t} \mathsf{EXP}_{k,t} - P_{k\mathsf{IMP},t} \mathsf{IMP}_{k,t}$$

- Real GDP:

$$Y_{k,t} = \frac{\mathscr{Y}_{k,t}}{P_{kY,t}}$$

Analytical Results

Analytical Results – A Tractable SOE

Simplify the theoretical model to isolate the mechanisms:

- Two countries: one small open economy (EA) and the rest of the world
- Complete international financial markets
- Nominal rigidities on prices, flexible wages
- Cobb-Douglas aggregators for consumers and firms

The sectoral Phillips curves are

$$\pi_t^H = \underbrace{\mathscr{B}\left(1+\varphi+\varphi\boldsymbol{\lambda}^{\intercal}\operatorname{diag}\left(\boldsymbol{\Omega}_F\mathbf{1}\right)\left(\boldsymbol{I}-\boldsymbol{\Lambda}^{-1}\boldsymbol{\Omega}_H^{\top}\boldsymbol{\Lambda}\right)^{-1}\boldsymbol{\alpha}\right)}_{\text{Slope w.r.t. output gap}}\widetilde{y}_t + \underbrace{\boldsymbol{\Psi}\widetilde{\boldsymbol{s}}_t}_{\text{ToT gaps}} - \underbrace{\boldsymbol{\mathcal{V}}\boldsymbol{\chi}_t}_{\text{Price wedges}} + \beta(\boldsymbol{I}-\boldsymbol{\mathcal{V}})\mathbb{E}_t\pi_{t+1}^H.$$

pass-through

input demand

Setup | Objects | Slope objects

Result #1: Foreign Networks Steepen the Phillips Curve

Proposition #1. The slope of the Phillips curve with respect to the output gap is strictly steeper than its closed-economy counterpart:

$$\varphi \boldsymbol{\lambda}^{\intercal} \operatorname{diag}\left(\boldsymbol{\Omega}_F \mathbf{1}\right)\left(I - \boldsymbol{\Lambda}^{-1}\boldsymbol{\Omega}_H^{\top}\boldsymbol{\Lambda}\right)^{-1}\boldsymbol{\alpha} > 0$$

- With cross-sector production networks, producing a given increase in domestic output requires more domestic labor once imported intermediates and sectoral trade imbalances are taken into account.
- Consumption therefore rises less than output, requiring a larger real-wage response and steepening the Phillips curve.

In the one-sector economy, the open- and closed-economy slopes coincide (Galí and Monacelli, 2005). Derivation

Result #2: ToT Gaps Pass Through to Inflation

$$\Psi \equiv \underbrace{\Delta(I-\Omega\Delta)^{-1}\Omega_F}_{\text{(1) Direct \& network propagation}} + \underbrace{\mathscr{B}\beta^{\intercal}\Delta(I-\Omega\Delta)^{-1}\Omega_F}_{\text{(2) CPI-induced wage response}} + \underbrace{\mathscr{B}\lambda^{\intercal}\left[\operatorname{diag}\left(\Omega_F\mathbf{1}\right)\left(I-\Lambda^{-1}\Omega_H^{\top}\Lambda\right)^{-1}(I-\Omega_H)-\Omega_F\right]}_{\text{(3) Cross-sector trade-imbalance channel}}$$

- **(1)** Imported inputs raise marginal costs directly; IO linkages amplify pass-through through domestic suppliers
- **(2)** Higher domestic prices raise CPI; maintaining the real wage requires a common nominal-wage response
- **(3)** Sectoral ToT gaps reallocate demand, employment, and imports across sectors, affecting wages

Proposition #2. The matrix governing (3) is hollow: this channel cancels in the one-sector economy and is absent with diagonal production matrices.

Details

Result #3: Why Divine Coincidence Does Not Return

In the multisectoral open economy, the output gap is proportional to a *combination* of sectoral ToT gaps:

$$\widetilde{y}_t = \frac{\boldsymbol{\beta}^\intercal + \boldsymbol{\lambda}^\intercal \left[\boldsymbol{\Omega}_F - \operatorname{diag}\left(\boldsymbol{\Omega}_F \mathbf{1}\right) \mathcal{N}_s \right]}{1 + \varphi \boldsymbol{\lambda}^\intercal \operatorname{diag}\left(\boldsymbol{\Omega}_F \mathbf{1}\right) \mathcal{N}_y} \widetilde{\boldsymbol{s}}_t.$$

- In Galí and Monacelli (2005), the ToT gap is proportional to the output gap: closing one closes the other
- Here, $\widetilde{\boldsymbol{s}}_t$ is a **vector**: closing the aggregate output gap does not close every sectoral ToT gap

Proposition #3. no constant-weight linear inflation index achieves the Divine Coincidence in the open economy.

DCI

Quantitative Model

Quantitative Model – Specialization and Calibration

- To study imported energy-price shocks, we specialize the general aggregators to distinguish **energy** and **non-energy** goods
- The different layers of the consumption and production aggregators are **nested CES**
- We allow for sectoral price rigidities and country-level **wage rigidities** à la Erceg *et al.* (2000)

Consumption Production Wages

Calibration

- Calibrate the model to **6 countries** ($K = 6$) and **44 sectors** within each country ($I = 44$):
  - Euro-Area: Spain (ES), Germany (DE), Italy (IT), France (FR), Rest of the Euro Area (REA)
  - Rest of the World
- Sector and Country **price frequency adjustments** using data from Gautier *et al.* (2024)
- We linearize the model *by hand* so we can **read input-output matrix, labor shares, and consumption shares directly from the data** (ICIO Input-Output Tables, OECD; and Figaro, Eurostat)

Figure D1 Figure D2

Calibration – II

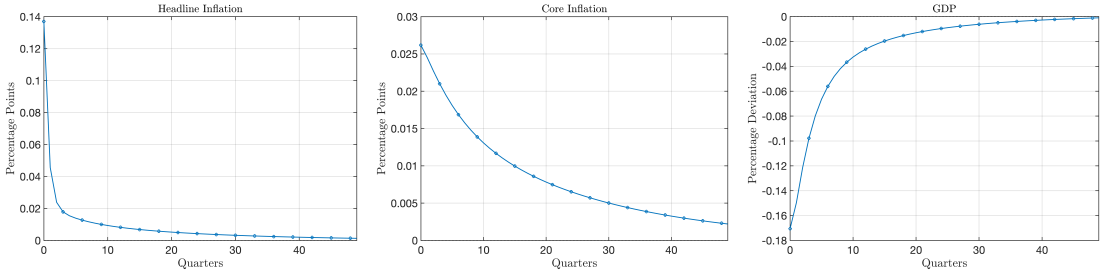
Parameter	Description	Value	Target / Source
Households			
β	Discount factor	0.99	$R = 4.5\%$ p.a.
σ	Inv. Intertemp. Elast. Subs.	1	Standard Value
φ	Inv. Frisch Elasticity	1	Chetty *et al.* (2011)
γ	Elast. Subst. E and M	0.4	Böhringer and Rivers (2021)
η	Elast. Subst. E	0.9	Atalay (2017)
ι	Elast. Subst. M	0.9	Atalay (2017)
δ	Trade Elasticity	1	Standard value
$\{\widetilde{\beta}_k, \widetilde{\nu}_{ki}, \widetilde{\upsilon}_{ki}, \widetilde{\zeta}_{kli}\}$	Quasi-shares consumption		ICIO tables (OECD)
θ_k^w	Calvo wage prob.	0.75	Christoffel *et al.* (2008)
Monetary Policy			
$\rho_{k,r}$	Interest Rate Smoothing	0.7	Standard Value
$\phi_{k,\pi}$	Reaction to Inflation	1.5	Galí (2015)
$\phi_{k,y}$	Reaction to real GDP	0.125	Galí (2015)

Calibration – III

Parameter	Description	Value	Target / Source
Firms			
ψ_{ki}	Returns to scale	1	Constant returns to scale
ψ	Elast. Subst. N and X	0.5	Atalay (2017)
ϕ	Elast. Subst. E and M	0.4	Böhringer and Rivers (2021)
χ	Elast. Subst. M	0.2	Atalay (2017)
ξ	Elast. Subst. E	0.2	Atalay (2017)
μ	Trade Elasticity	1	Standard value
$\{\widetilde{\alpha}_{ki}, \widetilde{\vartheta}_{ki}, \widetilde{\beta}_{ki}, \widetilde{\nu}_{kij}, \widetilde{\upsilon}_{kij}, \widetilde{\zeta}_{kij}\}$	Quasi-shares production		ICIO tables (OECD)
$\mathscr{M}_{ki}$	Markups		Labor shares (Eurostat)
θ^{p}_{ki}	Calvo price prob.		Gautier *et al.* (2024)
Foreign Shock Processes			
$\rho^{\tau}_{1,kli}$	Persistence price wedge shock	1.17	Brent crude oil
$\rho^{\tau}_{2,kli}$	Persistence price wedge shock	-0.2	Brent crude oil
σ^{τ}_{kli}	Std. Dev. price wedge shock	1	Standard Value

Results

The Transmission of an Increase of Energy Prices



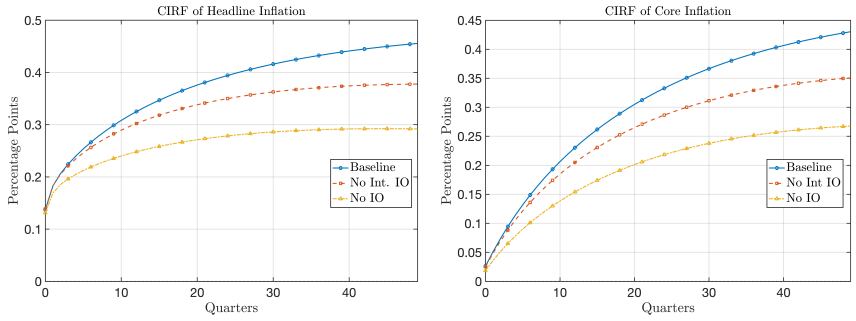
Notes: IRFs to a 10% increase in import energy prices for Euro-Area wide variables.

- Energy shock: large spike of headline inflation on impact which *dies out slowly*
- Significant **pass-through to core inflation** ($\approx$ 20% of headline) responsible for inflation persistence (Adolfsen *et al.*, 2024)

The Role of Production Networks

The Role of Production Networks - I

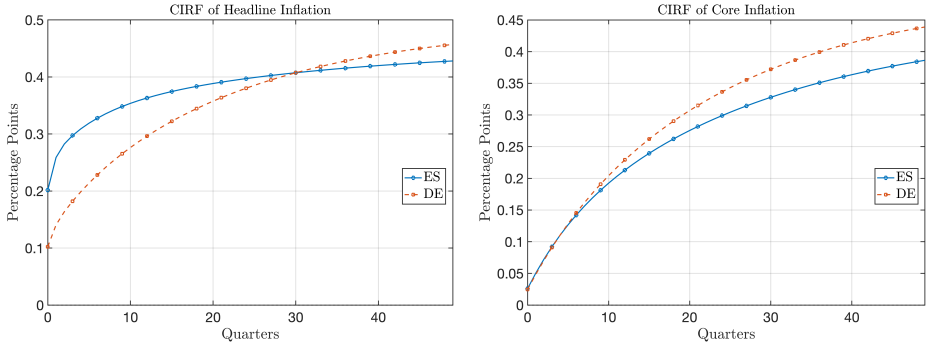
Intuition



Notes: Cumulative IRF of headline (left panel) and core (right panel) inflation for the baseline and turning off the full, international, or national input-output structure. When turning off the IO structure, we always keep the use of energy as an intermediate input.

- **Without IO**: inflation dies much faster, responding ≈ 60% (cumulative) of baseline with full IO
- Both *national* and *international* production networks important in driving the effects

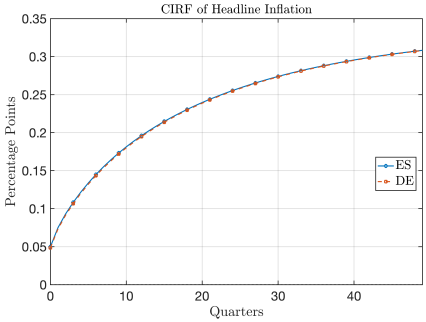
Cross-Country Heterogeneity - I



Notes: Cumulative IRF of headline (left panel) and core (right panel) inflation for Spain (ES) and Germany (DE).

- **ES** strongest headline response on impact but dies out the quickest: *large weight of energy in CPI* but with more *upstream production structure* core inflation raises the least
- **DE** shows the opposite: less weight of energy in CPI but with *downstream and long production chains*

Cross-Country Heterogeneity - II

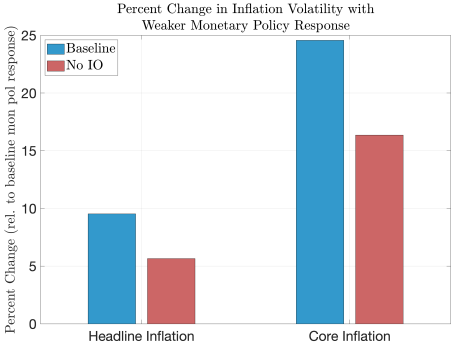


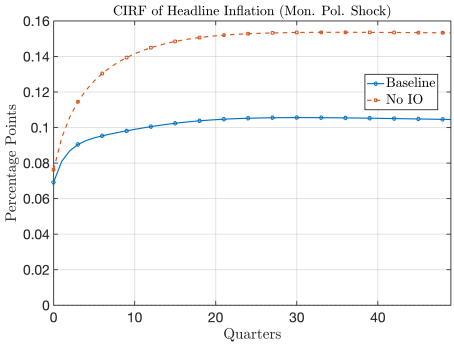
Notes: Cumulative IRF of headline inflation under common IO structure for Spain (ES) and Germany (DE).

- **Homogenizing** the IO structure reduces the gap between headline inflation dynamics in ES vs. DE
- Remaining difference explained through (slightly) **more flexible prices** in DE

Monetary Policy Implications

The Interaction Between Monetary Policy and Production Networks





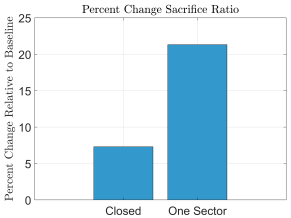
Notes: Left panel: percent change in inflation volatility (conditional on energy price shocks) with a lower coefficient on inflation in the Taylor rule. Right panel: CIRF of headline inflation to a monetary policy shock (easing) in the baseline and without IO.

- **Production networks amplify the increase of inflation volatility** from a weaker mon. pol. response
- Despite production networks generating *more* monetary non-neutrality (Nakamura and Steinsson, 2010)

Monetary Policy Trade-offs - Sacrifice Ratio

Differential sacrifice ratio: output gap cost per unit of inflation reduction when moving from Looking Through to Leaning Against the Wind

- Tighter monetary policy induces larger output gap losses relative to a one-sector economy or a "closed-economy" with IO and only imported energy
- **Takeaway:** sectoral heterogeneity and international IO spillovers make strict inflation stabilization costlier



Notes: Change in the sacrifice ratio in the one-sector and closed-economy frameworks relative to the baseline.

Conclusions

Conclusions

- We provide an international multi-sector macroeconomic model with input-output linkages that can accommodate the disaggregated and international nature of macroeconomic shocks
- We analyze the impact and transmission of rising imported energy prices:
  - Production networks important transmission channel to core inflation, increasing persistence of inflation dynamics
  - Heterogeneity in production structures gives rise to cross-country heterogeneity in inflation developments
  - Production networks amplify the increase in inflation volatility resulting from a weaker systematic response of monetary policy to inflation and worsen the trade-off faced by monetary policy

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Figure D1: Calibration Heatmaps

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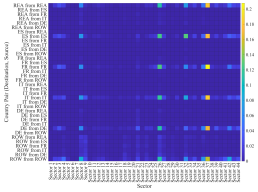
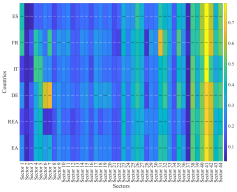
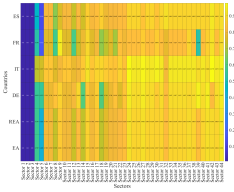


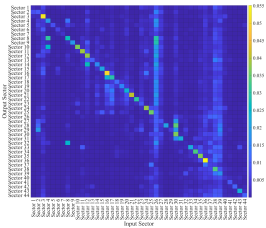
Consumption shares



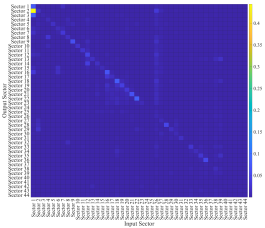
Labor shares



Price rigidities



EA domestic IO



EA foreign IO

Figure D2: Country IO Heatmaps – I

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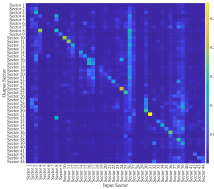
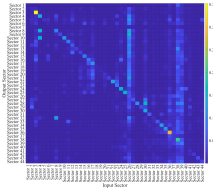
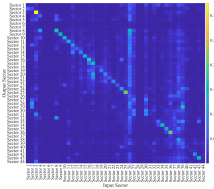


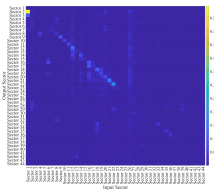
ES domestic IO



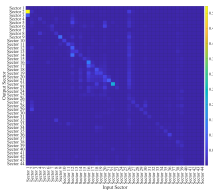
FR domestic IO



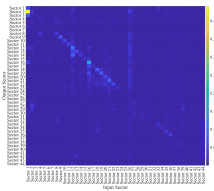
IT domestic IO



ES foreign IO



FR foreign IO



IT foreign IO

Figure D2: Country IO Heatmaps – II

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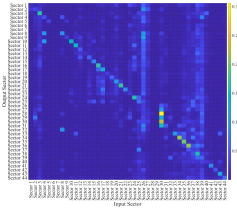
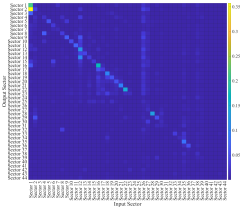
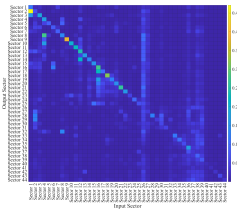


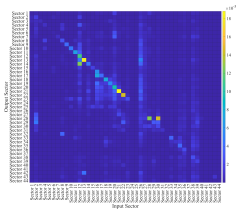
DE domestic IO



DE foreign IO



ROW domestic IO



ROW foreign IO

Analytical Setup: What Is Being Shut Down?

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- Two countries: a small open economy and the rest of the world
- Log utility, complete markets, PCP, flexible wages
- Cobb–Douglas consumption, input, and production aggregators

Why these assumptions? They make the one-sector benchmark transparent: the Divine Coincidence holds, and openness does not change the Phillips-curve slope. Any new object in the multisector model must therefore come from sectoral production networks and sectoral ToT gaps.

Analytical Objects: From Marginal Costs to Gaps

Back

- Sectoral inflation starts from a standard NKPC:

$$\pi_t^H = \kappa \left(mc_t - p_t^H \right) + \beta \mathbb{E}_t \pi_{t+1}^H.$$

- Production networks enter through sectoral nominal marginal costs:

$$mc_t = \alpha w_t + \Omega p_t^H + \Omega_F s_t, \qquad s_t = \tau_t + p_t^F - p_t^H.$$

- Flexible prices define the efficient domestic-price response:

$$p_t^{H,n} = (I - \Omega_H)^{-1} \Omega_F \tau_t, \qquad \chi_t \equiv p_{t-1}^H - (I - \Omega_H)^{-1} \Omega_F \tau_t.$$

Analytical Objects: Slope Terms

Back

$$\mathscr{B} = \frac{\Delta(I-\Omega\Delta)^{-1}\alpha}{1-\beta^{\intercal}\Delta(I-\Omega\Delta)^{-1}\alpha}, \qquad \Delta = (I+\kappa)^{-1}\kappa.$$

- $\mathscr{B}$: wage-to-price pass-through. Labor costs enter through labor shares and ripple through the rigidity-adjusted input-output network.
- The denominator captures CPI feedback: higher domestic prices partially offset the real-wage response.

$$y_t = \left(1+\varphi\lambda^{\intercal}\mathrm{diag}(\Omega_F\mathbf{1})\mathscr{N}_y\right)^{-1}\{c_t + \text{ToT terms} - \text{wedge terms}\}.$$

- $\mathscr{N}_y$: output-to-input-demand map obtained from goods-market clearing. It maps an aggregate output gap into sectoral employment and imported-input demand.
- Thus $\lambda^{\intercal}\mathrm{diag}(\Omega_F\mathbf{1})\mathscr{N}_y$ is the open-economy amplification term: GDP and domestic absorption need not move one-for-one when production uses foreign inputs.

Result #1 Backup: Why the Slope Changes

Back

- One-sector SOE with intermediates:

$$\pi_t^H = \kappa(1+\varphi)(1-\omega_H)\widetilde{y}_t + \beta\,\mathbb{E}_t\pi_{t+1}^H.$$

- Multisector slope:

$$\mathscr{B}\left(1+\varphi+\varphi\boldsymbol{\lambda}^{\intercal}\mathrm{diag}(\boldsymbol{\Omega}_F\mathbf{1})\mathscr{N}_y\right).$$

- The new term is positive because higher output raises imported-input needs and sectoral labor requirements
- Output rises more than domestic absorption, so households must supply labor without a proportional consumption increase
- Real wages and marginal costs become more sensitive to the output gap

Result #2 Backup: Three ToT Channels

Back

$$\Psi \equiv \underbrace{\Delta(I-\Omega\Delta)^{-1}\Omega_F}_{\text{(1) imported-input cost + network pass-through}} + \underbrace{\mathscr{B}\beta^{\intercal}\Delta(I-\Omega\Delta)^{-1}\Omega_F}_{\text{(2) CPI-induced nominal-wage response}} + \underbrace{\mathscr{B}\lambda^{\intercal}[\operatorname{diag}(\Omega_F\mathbf{1})\mathscr{N}_s-\Omega_F]}_{\text{(3) sectoral trade-imbalance channel}}.$$

- Channels (1)–(2) are entrywise non-negative when sectors are directly or indirectly exposed to foreign inputs
- Channel (3) is governed by a hollow matrix:

$$\operatorname{diag}(\Omega_F\mathbf{1})\mathscr{N}_s-\Omega_F.$$

- In one sector, expenditure switching and imported-input demand cancel under Cobb–Douglas
- With cross-sector linkages, other domestic sectors reallocate demand too; the cancellation breaks

Result #3 Backup: Why a DCI Index Fails

Back

- Closed-economy DCI logic weights sectoral inflation by sales and inverse price flexibility:

$$\pi_t^{DC} = \left(\boldsymbol{\lambda}^{\intercal}\boldsymbol{\kappa}^{-1}\mathbf{1}\right)^{-1}\varphi\widetilde{y}_t + \left(\boldsymbol{\lambda}^{\intercal}\boldsymbol{\kappa}^{-1}\mathbf{1}\right)^{-1}\boldsymbol{\lambda}^{\intercal}(\boldsymbol{I}-\boldsymbol{\Omega}_H)\widetilde{\boldsymbol{s}}_t + \beta\mathbb{E}_t\pi_{t+1}^{DC}.$$

- But in the open economy:

$$\widetilde{y}_t = \frac{\boldsymbol{\beta}^{\intercal} + \boldsymbol{\lambda}^{\intercal}\left[\boldsymbol{\Omega}_F - \operatorname{diag}(\boldsymbol{\Omega}_F\mathbf{1})\mathcal{N}_s\right]}{1+\varphi\boldsymbol{\lambda}^{\intercal}\operatorname{diag}(\boldsymbol{\Omega}_F\mathbf{1})\mathcal{N}_y}\widetilde{\boldsymbol{s}}_t.$$

- Closing the scalar output gap need not close every sectoral ToT gap
- Hence no constant-weight linear inflation index restores the Divine Coincidence

Households – Intertemporal Problem

Back

- Representative households' problem in country $k \in K$:

$$\max_{C_{k,t},\{B^l_{k,t}\}^K_{l=1}} \sum_{t=0}^{\infty} \beta^t \left(\frac{C_{k,t}^{1-\sigma}-1}{1-\sigma} - \int_0^1 \frac{\mathcal{N}_{gk,t}^{1+\varphi}}{1+\varphi} dg \right) Z_{k,t} \quad \text{s.t.}$$

$$P_{kC,t}C_{k,t} + \sum_{l=1}^{K} B^l_{k,t}\left[1-\Gamma(\mathscr{B}^l_{k,t})\right]^{-1} \mathscr{E}_{kl,t} \leq \sum_{l=1}^{K} B^l_{k,t-1}\mathscr{E}_{kl,t}(1+i_{l,t-1}) + \int_0^1 W_{gk,t}\mathcal{N}_{gk,t}dg + \Pi_{k,t} - T_{k,t} - \Xi_{k,t}$$

- $B^l_{k,t}$: Household holdings in country k of bonds issued by country l
- $\mathscr{E}_{kl,t}$: nominal exchange rate between country k and country l
- $\Gamma(\mathscr{B}^l_{k,t})$: portfolio adjustment costs to stabilize the model (Schmitt-Grohé and Uribe, 2003)

Households – Consumption Baskets

General model | Quantitative model

- Consumption C_k: aggregator of energy ($C_{kE,t}$) and non-energy ($C_{kM,t}$) baskets:

$$C_{k,t} = \left[\widetilde{\beta}_k^{\frac{1}{\gamma}} C_{kE,t}^{\frac{\gamma-1}{\gamma}} + (1-\widetilde{\beta}_k)^{\frac{1}{\gamma}} C_{kM,t}^{\frac{\gamma-1}{\gamma}} \right]^{\frac{\gamma}{\gamma-1}}$$

- Energy and non-energy: goods produced by energy and non-energy industries

$$C_{kE,t} = \left[\sum_{i \in I_E} \widetilde{\nu}_{ki}^{\frac{1}{\eta}} C_{ki,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \quad C_{kM,t} = \left[\sum_{i \in I_M} \widetilde{\upsilon}_{ki}^{\frac{1}{\iota}} C_{ki,t}^{\frac{\iota-1}{\iota}} \right]^{\frac{\iota}{\iota-1}}$$

- Each energy and non-energy good combines varieties produced by different countries:

$$C_{ki,t} = \left[\sum_{l=1}^{K} \widetilde{\zeta}_{kli}^{\frac{1}{\delta}} C_{kli,t}^{\frac{\delta-1}{\delta}} \right]^{\frac{\delta}{\delta-1}}$$

Wage Setting

Quantitative model

- In each country k, nominal wages are infrequently reset by a union (Erceg *et al.*, 2000)
  - Wages are different across countries
  - But common across sectors within a country
- **Wage NKPC** for country k (log-linearized):

$$\pi_{wk,t} = \kappa_{wk}\left(\sigma\widehat{c}_{k,t} + \varphi\widehat{n}_{k,t} - \widehat{w}_{k,t}\right) + \beta\mathbb{E}_t\pi_{wk,t+1} + u^w_{k,t}$$

- Where $u^w_{k,t}$ is a wage cost-push shock that follows an AR(1) process

Firms - Production I

General model | Quantitative model

- Firm f (sector i, country k) combines labor and intermediates:

$$Y_{fki,t} = A_{ki,t}\left[\widetilde{\alpha}_{ki}^{\frac{1}{\psi}} N_{fki,t}^{\frac{\psi-1}{\psi}} + \widetilde{\vartheta}_{ki}^{\frac{1}{\psi}} X_{fki,t}^{\frac{\psi-1}{\psi}}\right]^{\frac{\psi}{\psi-1}}$$

- Intermediates X_{fki} combines energy $X_{fkiE,t}$ and non-energy baskets $X_{fkiM,t}$:

$$X_{fki,t} = \left[\widetilde{\beta}_{ki}^{\frac{1}{\phi}} X_{fkiE,t}^{\frac{\phi-1}{\phi}} + (1-\widetilde{\beta}_{ki})^{\frac{1}{\phi}} X_{fkiM,t}^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}}$$

- Energy and non-energy baskets: goods produced by energy and non-energy industries

$$X_{fkiE,t} = \left[\sum_{j\in I_E} \widetilde{\nu}_{kij}^{\frac{1}{\chi}} X_{fkij,t}^{\frac{\chi-1}{\chi}}\right]^{\frac{\chi}{\chi-1}}, \quad X_{fkiM,t} = \left[\sum_{j\in I_M} \widetilde{\upsilon}_{kij}^{\frac{1}{\xi}} X_{fkij,t}^{\frac{\xi-1}{\xi}}\right]^{\frac{\xi}{\xi-1}}$$

- Energy and non-energy goods combines varieties produced by different countries:

$$X_{kij,t} = \left[\sum_{j=1}^{I} \widetilde{\zeta}_{klij}^{\frac{1}{\mu}} X_{klij,t}^{\frac{\mu-1}{\mu}}\right]^{\frac{\mu}{\mu-1}}$$

The Role of Production Networks – Intuition

Back

- For intuition: domestic currency pricing, fixed nominal exchange rates, and $\tau_{klijt} = \tau_{ljt}$.
- Sticky prices make intermediate goods' prices inherit persistence from past inflations and shocks; IO linkages amplify it:

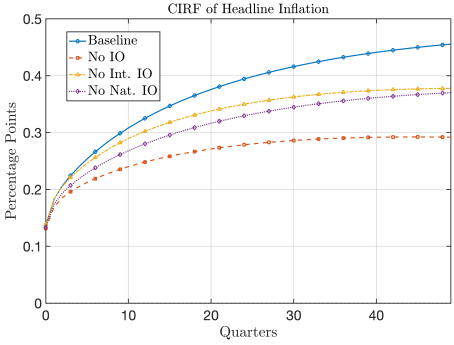
$$\boldsymbol{p}_{t+h} = (I - \Delta\Omega)^{-1}\left[\Delta\Omega R^h \tau_t + (I-\Delta)\sum_{s=1}^{h}\pi_{t+h-s} + \Delta\alpha\sum_{s=0}^{h}\pi^w_{t+h-s} + \Delta K^{-1}\beta\mathbb{E}_t\pi_{t+h+1}\right]$$

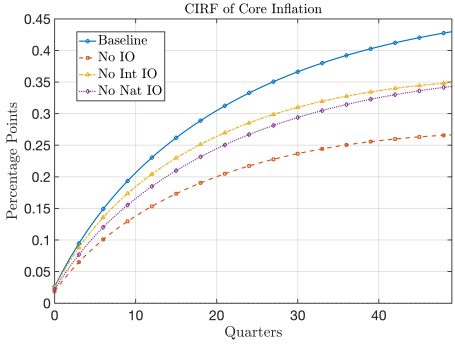
$\Omega \equiv$ input-output, $K \equiv$ Slope PC, $\Delta = (I-K)^{-1}K$, $\alpha \equiv$ labor share, $R \equiv$ Shock persistence,

- The rigidity-adjusted Leontief inverse $(I - \Delta\Omega)^{-1}$ **amplifies**
  - The impact and **exogenous persistence** of shock $\Delta\Omega R^h \tau_t$
  - ...and the **intrinsic persistence** induced by staggered price– and wage–setting
- More persistent prices feed into marginal costs and hence inflation:

$$\widehat{\mathbf{mc}}^n_t = \alpha w_t + \Omega\left(\tau_t + p_t\right)$$

The Role of Production Networks - AI





Notes: Cumulative IRF of headline (left panel) and core (right panel) inflation for the baseline and turning off the full, international, or national input-output structure. When turning off the IO structure, we always keep the use of energy as an intermediate input.