Credit Risk Models and Credit Derivatives

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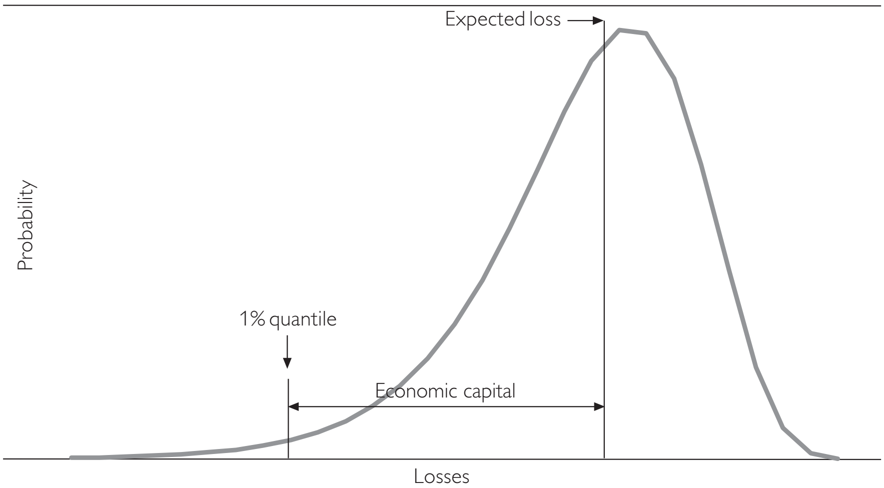
1 Introduction

In the past few years it has become ever more obvious that, generally speaking, the regulatory capital regime offered by the 1988[1]) Basle Capital Accord does not adequately address credit risks. A case in point is regulatory capital arbitrage, which allows for a reduction of the regulatory capital, even though the credit risk incurred by an institution has not really changed. The same financial instrument must be backed with differing amounts of capital depending on whether it concerns the trading book or the bank book. Faced with this discrepancy between the regulatory capital charge and economically sound capital allocation, many large banks have in recent years developed complex mathematical-statistical models for quantifying credit risks. They currently use such models to determine in-house the economic capital for covering credit risks as well as to manage credit risks in a more effective way. After all, these credit risk models do not yet lend themselves to ascertaining the regulatory capital charge to be applied to credit risks.

Although the various modeling approaches differ significantly, all of them must invariably consider default probabilities and expected loss ratios. Furthermore they must be capable of estimating the loss distributions, on whose basis the expected loss and the economic capital are determined. These models allow for the first time to set limits for concentrations of individual counterparty risk, industrial sectors or geographical regions in a sound way. To do just that, the loss distribution of the credit portfolio is determined either analytically or via numerical simulations. As opposed to the distributions in connection with market risks, this distribution is not symmetric, but negatively skewed. The economic capital corresponds to the 1% quantile of the loss distribution.

Loss Distribution of a Credit Portfolio



Source: OeNB.

The input data requirements pose one of the main problems of modeling credit risks. Given the limited availability of such data, credit institutions are frequently compelled to make numerous simplistic assumptions, such as the independence of various sectors or the time-independence of statistical parameters. Such misspecifications may, however, totally distort the tail of the loss distribution, which is of central interest. Furthermore, it is difficult to determine the statistical quality of such models by means of backtesting, as this testing method necessitates random sampling covering at least one credit cycle, for which the available data are usually insufficient.

Using credit risk models allows for quantifying credit risks and hedging them via appropriate financial instruments. Small wonder that the development of credit risk models goes hand in hand with the design of credit derivatives. Credit derivatives for the first time allow for an active management of credit risks of both individual credits and entire credit portfolios and they significantly boost the market liquidity of credits. The following chapter introduces two commonly used approaches to credit risk modeling, draws a comparison between the two models and deals with the possible uses of credit derivatives for actively managing credit risk.

2 CreditMetrics

CreditMetrics[2]) was designed by J. P. Morgan and serves to measure credit portfolio losses. It takes into account credit quality rating migration, credit defaults, recovery rates and obligor correlations derived from the respective stock price correlations. Unlike CreditRisk+[3]), CreditMetrics is capable of modeling changes in credit ratings, recovery rates and obligor correlations. Due to this additional modeling it is no longer possible to present the loss distribution of the portfolio in an analytically closed form. Instead, Monte Carlo simulations are used to approximate the loss distribution. For each scenario of the simulation, the present value of the portfolio is calculated as the sum total of the present values of the individual instruments and then weighted by the given scenario's probability of occurring. The downside of this approach is that a broad data basis is necessary to parameterize this model: in particular, credit default probabilities, credit quality migration likelihoods, credit spreads[4]), recovery rates[5]), stock prices and industry indices.

2.1 Computing the Portfolio Value

For a credit portfolio consisting of n instruments, each of which may have k different ratings (including insolvency), the number of possible scenarios comes to k^n at the end of the observation period (e.g. one year). The value V_i^j $(i=1, \ldots k^n, j=1,\ldots,n)$ of each instrument is calculated for each of these scenarios. To determine the present values, you need to know the cash flows generated by the instrument and the discount factors of a given scenario, which are derived from the credit spreads and the associated reference interest rates.

When calculating the present value, a basic distinction is made between changes in the rating category and the obligor's default. In the former case, the calculation is reduced to revaluating the instrument by means of the respective discount factors; in the latter case, the recovery rate is estimated based on the seniority class of the liability.

At the end of the observation period the value V_i of the portfolio for a given scenario is the sum total of the individual values of the instruments under this scenario:

$$V_i = \sum_{j=1}^{n} V_i^j, \ i = 1,\ldots,k^n.$$

These portfolio values must then be weighted for the respective scenario probabilities to obtain the loss distribution of the portfolio.

2.2 Computing the Scenarios' Probabilities of Occurring

To determine the distribution of the portfolio values, the probability of occurring of each scenario is required. Empirical research has shown that changes in the credit ratings of individual obligors are correlated, as they are in part influenced by the same macroeconomic variables. Therefore the probability of occurring of a scenario may not be calculated as the product of individual changes in an obligor's credit quality rating. Instead, it is necessary to pinpoint these probabilities indirectly, i.e. via measures observed on the market. In this context, CreditMetrics assumes that there is a relationship between changes in credit ratings and changes in an enterprise's asset value. As changes in asset values are not directly observable either, they are approximated via the returns of the stock prices for which observed market prices are available on a continuous basis.

First, each rating is assigned an interval of stock returns, with the interval bounds S_i to be calculated by means of the individual probabilities of the credit rating changes:

$$P(R^j = i) = P(S_{i-1} < X_j < S_i).$$

R^j denotes the rating of the jth obligor and X_j designates the respective stock return. The probability of occurring of a scenario (i.e. a joint change in the obligors' credit ratings) is derived from the joint distribution of the returns. This distribution is assumed to be a multivariate normal distribution with the density function $f(x_1, \ldots, x_n; \Sigma)$:

$$P(\text{Scenario} = (i_1, i_2, \ldots, i_n)) = P(R^1 = i_1, R^2) = i_2, \ldots R^n = i_n) =$$
$$= \int_{S_{i_1 - 1}}^{S_{i_1}} \ldots \int_{S_{i_n - 1}}^{S_{i_n}} f(x_1, \ldots, x_n; \Sigma) dx_1 \ldots dx_n.$$

To characterize this distribution completely, the correlation matrix Σ of the stock returns[6]) is necessary. CreditMetrics calculates this matrix from the stock prices of the enterprises, not for each pair though, but only across the industries of individual countries. The calculation is based on historical weekly returns, and the observations are equally weighted. Each enterprise is assigned various industry indices, which has the advantage that only correlations between industry indices need to be computed. The correlations between individual enterprises are calculated via the correlations between the industry indices, factoring in company-specific data.[7])

2.3 Computing the Standard Deviation and the Quantile

The standard deviation is one of the most commonly used risk measures. Here, it shows the average deviation from the expected loss and is calculated for a portfolio which consists of n instruments as follows:

$$\sigma_p = \sqrt{\sum_{i=1}^{n}\sigma^2{}_i + 2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\sigma_i\sigma_j\ {}_{ij}}.$$

As the distribution of portfolio losses of a credit portfolio is not symmetric, the standard deviation is not an adequate risk measure. If, however, the value of each portfolio and the corresponding probability of occurring are known, it is possible to determine any quantile of the distribution of the portfolio values and use it as a measure of risk. When it comes to the economic capital, the 1% quantile is, as a rule, applied.

2.4 Simulation

In the case of a portfolio which is made up of a large number of instruments it would take up an enormous amount of computer time to calculate all individual values and the corresponding probabilities of occurring. If there are merely ten instruments in the portfolio, with eight rating categories (including insolvency) this already results in $8^{10} \approx 10^9$ scenarios. In light of the computer time needed an analytical calculation is no longer feasible, which is why simulations are used to approximate the distribution of the portfolio values.

Since the multivariate normal distribution was assumed for the joint distribution of returns, multivariate normally distributed random variables need to be generated. This is achieved via Cholesky factorization or eigenvector decomposition.[8]) Both the correlation matrix of the returns and the individual probabilities of changes in the obligors' credit ratings, which are used to calculate the interval bounds, serve as input for the simulation. These interval bounds are employed to assign rating categories to the simulated returns.

In case the rating category changes, the portfolio values are calculated by revaluating the instruments according to this scenario. However, if a default event occurs, the recovery rate is simulated as an additional random variable (beta distribution[9])), since empirical research has shown that the recovery rate is not constant.[10])

Approximation of the loss distribution increases in quality with the number of generated scenarios. This, of course, means that computer time also mounts.

3 CreditRisk+

CreditRisk+ was developed by Credit Suisse Financial Products, and unlike J. P. Morgan's CreditMetrics, it employs an actuarial approach to present, in terms of probabilities, the losses of a bond or credit portfolio resulting from loan defaults. Here, only the credit default risk is modeled, under the assumption that the enterprise's capital structure is independent of default risk. The risk arising from the change in the company's rating or from credit spreads is not considered. As is the case with the commonly used market risk models, the factors causing loan defaults are not explicitly modeled. On the one hand, this reduces the risk of modeling errors, on the other, this has the added advantage of allowing for a closed-form model approach. Consequently, the data required for parameterizing the model may be cut to the minimum. Fewer simulations, in turn, make for greater computer speed. This way it becomes possible to determine the credit risk of very sizeable bond and credit portfolios and to analyze the marginal effects of incorporating additional products into the portfolio.

3.1 Basic Model

Credit default losses are caused by the default of one or more obligors, yet at the time of the credit default any expected offsetting payments[11]) must be subtracted from the claims outstanding. In CreditRisk+ defaults represent the main risk to which everything else may be traced. It is, above all, possible to determine the distribution of a portfolio's default losses, which is of central interest, from the probabilistic distribution of the number of defaults. The theory of generating functions[12]) is an especially useful and efficient tool for carrying out the necessary mathematical manipulations.

In practice, the credit portfolio is first divided into several independent sectors S_k, with each sector comprising loans which have about the same credit risk. The sectors are typically grouped by country and by rating group, i.e. where obligors come from and the group they belong to. Once the individual sectors' loss distribution is known, it is possible to calculate the entire portolio's loss distribution by multiplying the individual sectors' loss distributions on the assumption that they are independent of each other. The upside of this method is that it allows focusing on a given sector in a further analysis without losing sight of the overall situation. The downside, however, is that correlations between the individual sectors are not considered.

3.1.1 Distribution of the Number of Credit Defaults within a Sector

With credit portfolios, it is impossible to precisely predict the number of credit defaults or the point in time when they will occur. Experience shows, however, that it can, for a start, be assumed that credit defaults occur very seldom and that the likelihood of their occurrence is not contingent on the time period. This statistical property of being a "rare event" represents the formal link between credit risk and the typical risks inherent in insurance exposures. Due to this property, it is possible to describe the number of credit defaults X within a fixed period (e.g. one year) in probabilistic terms via a Poisson distribution[13]), whose probability function is

$$P(X = n) = \frac{\mu^n e^{-\mu}}{n!},$$

where μ designates the expected number of defaults within the selected time frame.

Since, according to the model, a one-to-one mapping exists between credit default losses and the number of credit defaults, it is possible to infer the distribution of the default losses from the statistical law of the number of credit defaults.

3.1.2 Distribution of Credit Default Losses within a Sector

To minimize the data that have to be incorporated into the calculation, it makes sense to rank the number of all potential default losses within a sector by size and to group them in exposure bands, or classes, with a predefined exposure with L (e.g. ATS 1 million). All default losses within a band j are represented by their upper band bound L·j. A default in the first class thus, for instance, corresponds to a loss of ATS 1 million, a default in the second class designates a loss to the amount of ATS 2 million, etc. In general, the expected number of credit defaults does not only depend on the sector, but also on the given band within a sector. When μ_j denotes the expected number of default events in the jth band, X_j refers to the number of defaults in the jth class, and V_j designates the amount of the default losses in the jth band in units of L, then it follows:

$$P(V_j = n \cdot j) = P(X_j = n) = \frac{e^{-\mu_j} \mu_j^{n}}{n!} .$$

The generating function $G_j(z)$ of the loss probabilities of the jth band is therefore given by

$$G_j(z) = \sum_{n=0}^{\infty} P(V_j = n \cdot j) z^{n \cdot j} = \sum_{n=0}^{\infty} P(X_j = n) z^{n \cdot j} = \sum_{n=0}^{\infty} \frac{e^{-\mu_j} \mu_j^{n}}{n!} z^{n \cdot j} = e^{-\mu_j (1-z^j)}.$$

As a consequence of assumed independence between the losses of the individual classes, the probability generating function for the respective sector S_k is the product of the individual classes' probability generating functions:

$$G_{s_k}(z) = \Pi G_j(z) = e^{-\Sigma \mu_j + \Sigma \mu_j z^j}.$$

The probability function of the default losses of the sector S_k is achieved by successively differentiating the generating function. The probability function of the credit default loss of sector S_k may be expressed as the following recurrence relation[14]):

$$P(V_{s_k} = n \cdot L) = \sum_{j=1}^{n} \frac{\mu_j j}{n} P(V_{s_k} = (n-1) \cdot L), \text{ where } P(V_{s_k} = 0) = e^{-\Sigma \mu_j}.$$

Thanks to this recurrence relationship, the loss distribution in any given sector – and therefore also the loss distribution of the entire portfolio – is known in an analytically closed form. This, in turn, allows for the computation of any statistical variable of the distribution, such as the expected loss and the losses assigned to certain predefined percentiles.

3.2 Adjustment of the Basic Model to the Available Data Set

To employ the approach described above to forecasting potential credit default losses at a specified confidence level, it is necessary to parameterize the model with the available input data. The data required concern recovery rates and expected default rates per sector class. As recovery rates are not statistically modeled in CreditRisk+, the multi-year average of recovery rates, as periodically issued by Standard & Poor's or Moody's[15]), serves as exogenous model input.

The number of credit defaults within a sector class were modeled by means of a Poisson distribution necessitating a calibration of the distribution. Poisson distributions are characterized by one single parameter which corresponds to the expected value and the variance of the distribution. Statistics on the incidence of default events as periodically published by Standard & Poor's[16]) or Moody's demonstrate that the variance of the number of credit defaults is significantly higher than their expected value. This empirical evidence implies that the parameter of the Poisson distribution should not be presumed to be constant but should be modeled as a stochastic value. The randomness of expected credit defaults is due to macroeconomic factors, such as economic growth or the interest rate policy pursued by central banks, which observedly influence the fortunes of obligors. CreditRisk+ assumes in this context that the average yearly defaults may be described by a Gamma distribution[17]). The two parameters α and β of this distribution are computed from the empirically derived mean and the variance of the number of default events. By using the probability generating functions it is possible to show that the number of credit defaults X within a sector class are no longer reflected by a Poisson distribution but a Negative Binomial distribution[18]). The associated probability function is given as

$$P(X = n) = \binom{n+\alpha-1}{n} p^n (1-p)^\alpha, \ \ p = \frac{\beta}{1+\beta} \ .$$

Due to stochastic modeling of the default rates the distribution function becomes significantly positively skewed, which is why the risk of a high number of credit defaults grows. The probability function for the number of credit defaults given stochastic default rates forms, as is the case with constant default rates, the input for computing the probability function of the credit default losses via the associated generating function. The concrete computations again lead to a recurrence relation[19]) of the portfolio probabilities.

Even after adjusting the basic model as required, the portfolio loss distributions may be presented as a closed-form solution. This renders the model approach especially attractive for large portfolios and allows for the analysis of marginal credit risks.

4 Comparison of the Two Models

The two models outlined above quantify credit risk and produce a credit value at risk. While in CreditRisk+ only a credit default qualifies as a credit event, CreditMetrics also considers a credit rating change as a credit event. In CreditMetrics correlations are explicitly modeled, whereas the CreditRisk+ approach takes account of obligor correlations in an implicit way only. The recovery rate, which is an exogenous input variable in

CreditRisk+, is simulated as a random variable in CreditMetrics. While the data requirements are very low for CreditRisk+, CreditMetrics relies on a broad data basis. Correlations can, however, only be considered between listed obligors, which is why it is difficult to capture the credit risk of unlisted obligors. CreditRisk+ allows for the closed-form presentation of the portfolio's loss distribution. CreditMetrics, by contrast, requires simulations, thus stepping up computer time. In CreditMetrics and CreditRisk+ alike, it is very difficult to integrate nonlinear products, such as options and forex swaps.

The following Table again summarizes the most important characteristics of both models.

Feature	Credit Risk Model	
	CreditMetrics	CreditRisk+
Modeling of credit event correlations	yes	partly
Rating category change classified as a credit event	yes	no
Modeling of credit rating migration	yes	no
Modeling of recovery rate	yes	possible
Consideration of industry- and country-specific correlations	yes	partly
Consideration of unlisted obligors possible	difficult	yes
Outcome may be interpreted as credit VaR	yes	yes
Use of simulations required	yes	no
Consideration of derivatives possible	partly	partly
High computer power requirements	yes	no
Data-intensive	yes	no

Source: OeNB.

5 Credit Derivatives

Credit risk models for the first time allow for the identification and quantification of the typcial risks inherent in credit portfolios. Actively managing credit risk is, however, not comparable to buying and selling market risks, as it is at present quite difficult, if not impossible, to buy or sell many types of credit risks on financial markets. The reasons for this are manifold. On the one hand, the market liquidity of credits is low, and on the other hand, it is frequently impossible to get a credit instrument with the desired maturity and the desired risk profile on the market. Apart from the fiscal, accounting and regulatory hurdles another factor lies at the root of the low market liquidity of credits: selling credits generally has an adverse impact on the business relations with the obligors, as a credit sale necessitates the disclosure of information on the client some of which needs to be treated confidentially. This is why credit derivatives present a simple, efficient and, at least, partial solution to these problems. They enable portfolio managers to actively manage credit portfolios in an effective way, who may use credit derivatives

– to reduce the concentration risk of a credit portfolio through active country and industry risk management;
– to diversify the credit portfolio with the help of new credit risks without possessing the underlying security;
– to actively manage the credit risks of individual large credits while maintaining the existing client relations;

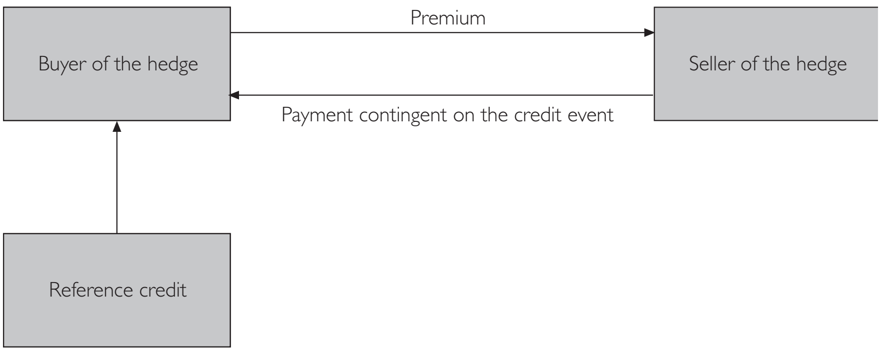
– to actively manage the credit risk of on-balance-sheet instruments without impacting the balance sheets;
– to create the desired cash flow and risk profiles;
– to hedge the dynamic credit risks, such as the counterparty risk in an interest rate swap, whose size is determined by market moves;
– to protect against credit defaulting and undesired credit spread changes;
– to open speculative positions at low refinancing costs.

5.1 Active Portfolio Management Using Credit Derivatives – Select Examples

Credit derivatives[20]) refer to OTC contracts which are tailored to meet existing customer demands and serve to pass on credit risks of individual credits or entire credit portfolios to one or more contract partners either in part or completely. The following describes the use of credit default options, total return swaps and credit spread options in active portfolio management.

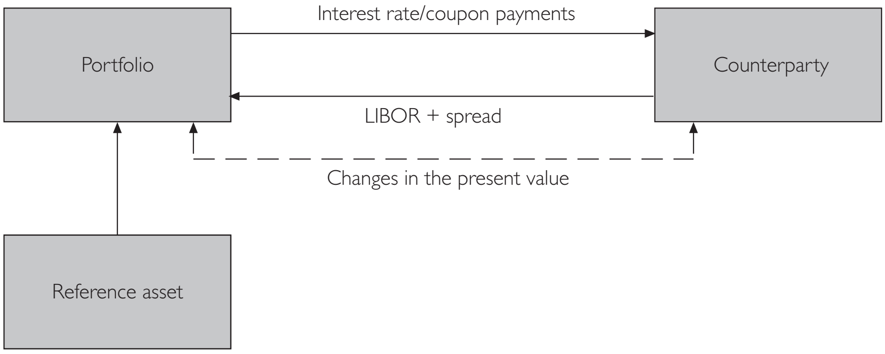
5.1.1 Credit Default Option/Swap

A credit default option/swap is a transaction where one party to the contract makes an advance or periodical payment and receives a contingent payment from the other party in turn, provided a certain credit event as defined at the time when the contract was closed occurs within a predefined period of time. Such instruments may be used to hedge the credit default risk of a reference credit without influencing the existing relations with the client and altering the bank's balance sheet. The definition of credit events must be agreed on in the contract in advance. Such events usually refer to the default of a reference credit or of a specific payment. The following figure illustrates the key components of a credit default option/swap.



5.1.2 Total Return Swap

A total return swap designates a bilateral contract which allows the parties to the contract to cede any returns on a credit or credit portfolio, such as coupon payments and changes in the present value, to or accept them from one another in exchange for receiving or making periodical cash flow payments. Such payments are, as a rule, linked to a reference interest rate, e.g. the LIBOR. Portfolio managers may take advantage of total return swaps to diversify their portfolios without actually possessing the underlying assets. Total return swaps lend themselves to protecting against a company's credit defaults and credit ratings migration and to reducing concentrations in portfolios.



5.1.3 Credit Spread Options

Credit spread options allow for fixing the spread of a corporate bond at a future point in time with regard to a benchmark interest rate agreed upon in the contract. A credit spread put option, for instance, enables the buyer to sell a given corporate bond at a later point in time at a spread on a reference interest rate which is fixed at the present time. Credit spread options may be used in portfolio management to hedge against undesired spread movements. They are not only suitable for hedging default risk but also risks associated with changes in the company's credit rating; besides, they may be used to contain the maximum possible loss of a corporate bond.

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1 *See Basle Committee on Banking Supervision (1988).*

2 *See J. P. Morgan (1997).*

3 *See Credit Suisse Financial Products (1997) and section 3.*

4 *The credit spread refers to the premium on the risk-free interest rate which the borrower or issuer has to pay.*

5 *The recovery rate refers to the share of the exposure repaid in case of default.*

6 *The mean return is assumed to be zero.*

7 *For details on the computation of the covariance matrix see J. P. Morgan (1997): 100f.*

8 *See G. Strang (1988).*

9 *For details on the beta distribution see Rohatgi (1976): 213.*

10 *It is assumed that each obligor's recovery rate is independent of the values of the instruments making up the portfolio.*

11 *In CreditRisk+ offsetting payments are assumed to be given exogenously, which is why they are among the required model input data.*

12 *The generating function G(z) of a discrete random variable X with the probability function P(X=i) is expressed as*

$$G(z) = \sum_{i=0}^{\infty} P(X = i) z^i$$

Successive differentiating of the generating function of a discrete random variable results in the probability function, to which the following relationship applies:

$$P(X = i) = \left. \frac{d^i G(z)}{i!\, dz^i} \right|_{z=0}$$

For more details see Rohatgi (1976): 93f.

13 *For more details see Rohatgi (1976): 194.*

14 *For more details see Credit Suisse Financial Products (1997): 38.*

15 *See e.g. Moody's Investor Service Global Credit Research.*

16 *See e.g. Standard and Poor's Ratings Performance.*

17 *For more details see Rohatgi (1976): 206.*

18 *The Negative Binomial distribution is also sometimes referred to as Pascal distribution. For details see Rohatgi (1976): 186f.*

19 *See Credit Suisse Financial Products (1997): 46f.*

20 *See J. P. Morgan (1998b) and J. P. Morgan (1998a).*