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AN UNOBSERVED COMPONENTS

MODEL TO FORECAST AUSTRIAN GDP

GERHARD FENZ AND MARTIN SPITZER

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Editorial

This paper deals with forecasting quarterly Austrian GDP growth using monthly conjunctural indicators and state space models. The latter provide an efficient econometric framework to analyse jointly data with different frequencies. Based on a Kalman filter technique the authors estimate a monthly GDP growth series as an unobserved component using monthly conjunctural indicators as explanatory variables. From a large data set of more than 150 monthly indicators the following six explanatory variables were selected on the basis of their in-sample fit and out of sample forecast performance: the ifo-index, credit growth, vacancies, the real exchange rate, the number of employees and new car registrations. Subsequently, quarterly GDP figures are derived from the monthly unobserved component using a weighted aggregation scheme. Several tests for forecasting accuracy and forecasting encompassing indicate that the unobserved components model (UOC-model) is able to outperform simple ARIMA and Naïve models.

March 24, 2006

An Unobserved Components Model to forecast Austrian GDP¹

Gerhard Fenz and Martin Spitzer²

February 2005

Abstract

This paper deals with forecasting quarterly Austrian GDP growth using monthly conjunctural indicators and state space models. The latter provide an efficient econometric framework to analyse jointly data with different frequencies. Based on a Kalman filter technique we estimate a monthly GDP growth series as an unobserved component using monthly conjunctural indicators as explanatory variables. From a large data set of more than 150 monthly indicators the following six explanatory variables were selected on the basis of their in-sample fit and out of sample forecast performance: the ifo-index, credit growth, vacancies, the real exchange rate, the number of employees and new car registrations. Subsequently, quarterly GDP figures are derived from the monthly unobserved component using a weighted aggregation scheme. Several tests for forecasting accuracy and forecasting encompassing indicate that the unobserved components model (UOC-model) is able to outperform simple ARIMA and Naïve models.

¹ We would like to thank Robert Kunst, Gerhard Rünstler and Martin Schneider for helpful comments and discussions.

² The authors are economists at the Oesterreichische Nationalbank, Economic Analysis Division. The views expressed in this paper are those of the authors and not necessarily of the institutions with which they are affiliated.

1. Introduction

The forward looking nature of monetary policy requires an assessment of economic developments and inflationary risks on a regular basis. In this context, projections of the state of economic activity in form of GDP growth figures play a prominent role. Moreover, a careful assessment of the current conjunctural situation and of the short term growth prospects is a crucial starting point of the OeNB's biannual projection exercise of the Austrian economy. As national accounts data for Austria are usually published at the end of the subsequent quarter (i.e. with a lag of around 90 days) one has to stick to short term forecasts in assessing the current conjunctural situation. The aim of this paper is to develop a model to forecast Austrian GDP growth in the current and the subsequent quarter, i.e. two quarters ahead.

Contrary to national account GDP figures several conjunctural indicators are already available at the end of the relevant observation period or within a few days later. Moreover many of these indicators are published at a higher frequency, typically on a monthly basis. As these indicators usually describe only certain parts of the economy one has to extract the information from several monthly indicators to draw a complete picture of the conjunctural situation. In order to link data of different frequencies (i.e. monthly indicators and quarterly GDP figures) we use an unobserved components model based on a Kalman filter technique. A monthly GDP series is estimated as the unobserved component using an ARMA structure and a small set of monthly indicators as exogenous explanatory variables. To select the exogenous variables, the in sample estimation fit and the out of sample forecasting performance of more than 150 variables from various sectors and markets was analysed. Variables tested cover inter alia the labour market, external trade data, confidence indicators, prices, financial variables, whole and retail sale, industrial production and exchange rates. According to statistical tests and economic intuition but also with respect to the quality of the data and their timeliness, the following time series were selected as explanatory variables: the ifo-index, credits to the non-financial sector, vacancies, a real exchange rate index, changes in the number of employees and new car registrations. A weighted aggregation scheme was used to derive the quarterly GDP figures from the monthly unobserved component. Out of sample forecasting performance tests show, that the model clearly outperforms simple ARIMA or Naïve models.

The paper is organized as follows: Chapter 2 describes briefly the method used. Estimation results are presented in chapter 3. In chapter 4 the out of sample forecast performance is assessed. Finally we summarize and draw some conclusion in chapter 5.

2. The model

In this section we briefly describe the econometric method used. The main challenge consists in finding an efficient econometric framework to analyse jointly data of different frequencies (i.e. monthly conjunctural indicators and quarterly National Account GDP figures). A straightforward solution would be the aggregation of monthly to quarterly data. But this method is associated with a considerable loss of information as the dynamics within a quarter are no longer explicit.³ Furthermore, it does not solve the problem how the latest information from monthly indicators can be used if observations are available only until the first or second

³ Under the assumption that the mean value theorem holds, one third of a quarterly growth rate is determined by the monthly dynamics within the previous quarter. In the limit, for an infinitely high frequency of observations, the ratio approaches one half (see appendix B for further details).

month within a quarter. Finally, the method should allow for a quick update of the forecast to incorporate new monthly information.

State space models provide an efficient way of dealing with the problems mentioned before. The basic idea of state space models is that an observable time series (Y_t) under study can be explained by a vector of unobserved components (α_t) (see Harvey (1989)). The unobserved components are linked to the observed variable via a measurement equation, i.e. conclusions about the unobserved components using the observed variable can be drawn. A typical example from time-series econometrics is the decomposition of GDP in a trend-, a cyclical-, a seasonal- and an irregular-component. In contrast, the present paper aims to extract unobservable monthly GDP growth rates from the quarterly GDP figures and exogenous monthly conjunctural indicators (see Rünstler (2000)). The estimation of this unobserved component (i.e. monthly GDP growth rates) is based on an ARMA part and exogenous monthly indicators like labour market figures or confidence indicators. This relation is formulated in the so called transition equation. The evaluation of the estimation takes place in the measurement equation. Using this framework, each third month, at the end of a quarter, a quarterly GDP growth rate (derived from the estimated monthly GDP figures) can be estimated.

A special feature of the model is the application of a weighted aggregation scheme to derive a quarterly GDP growth rate from monthly GDP growth rates. The aggregation procedure makes clear that quarterly growth rates are not independent from the monthly dynamics within the previous quarter (see footnote 3 and Appendix B). This phenomenon is closely associated with the well known carry-over effect in other macroeconomic forecast exercises.

For the complete formal set up of the state space model we refer to the appendix. In this section only the central elements are discussed. In general, a state space model consists of a measurement equation (or sometimes called observation equation) and a transition equation (or sometimes called state equation). In the present model the measurement equation which compares the actual and the estimated quarterly GDP growth rates takes the simple form:

$$\Delta \ln y_{\tau}^{Q} = \Delta \ln y_{\tau}^{Qe} \qquad \tau = 1...T/3; \quad t = 1,2,3...T \quad \text{(Measurement equation)} \tag{1}$$

 $\Delta \ln y_{\tau}^{Q}$ denotes the actual growth rate of real GDP, $\Delta \ln y_{\tau}^{Qe}$ the estimated growth rate of real GDP and t and τ the index of months and the index of quarters, respectively. The estimated quarterly growth rate of real GDP is a weighted sum of the present and the past four estimated monthly growth rates of real GDP where the weights are given by the vector $(1/3 \ 2/3 \ 1 \ 2/3 \ 1/3)$.⁴ Thus, the estimated growth rate of real GDP is given by:

$$\Delta \ln y_{\tau}^{Qe} = \frac{1}{3} \Delta \ln y_{t}^{m} + \frac{2}{3} \Delta \ln y_{t-1}^{m} + \Delta \ln y_{t-2}^{m} + \frac{2}{3} \Delta \ln y_{t-3}^{m} + \frac{1}{3} \Delta \ln y_{t-4}^{m}$$
(2)
$$\tau = 1...T/3; \quad t = 1,2,3...T$$

where $\Delta \ln y_t^m$ denotes the unobserved monthly GDP growth rates.

The transition equation describes the path of the unobserved components over time. The unobserved component we are most interested in is the monthly GDP growth rate. As is generally the case in state space models the unobserved component is assumed to follow a

⁴ For a derivation of the weights see appendix A.

first order Markov process. Monthly GDP growth additionally depends on a number of stationary exogenous variables (i.e. explanatory monthly indicators, $x_{n,t}^m$, with n = 1...N).

$$\Delta \ln y_t^m = \zeta \cdot \Delta \ln y_{t-1}^m + \beta_1 \cdot x_{1,t}^m + \dots + \beta_N \cdot x_{N,t}^m + e_t \qquad \text{(Transition equation)} \tag{3}$$

The error term e_t follows an AR(1)-process. Depending on the leading indicator properties of each single indicator and the time lags of data releases the available observations of monthly indicators may not be sufficiently long to cover the whole forecasting horizon. In the case of missing observations monthly indicators have to be forecast themselves using ARIMA models.

To summarize, we estimate an unobservable series of monthly GDP growth rates using monthly indicators and evaluate these forecasts each third month using actual quarterly GDP growth rates. One main advantage of this technique is that the variables and parameters of the state space model have a straight forward economic interpretation. The effect of each single explanatory variable as well as the importance of autoregressive processes can be stated explicitly. This renders the interpretation of the results comparatively easy. Furthermore a new time series, monthly GDP, is calculated which explicitly describes the conjunctural situation at a higher frequency. On the other hand, the number of explanatory variables used to estimate monthly GDP growth is limited by the criterion that they are required to pass certain statistical tests implying a loss of potentially important information contained in excluded indicators.

3. Indicator selection criterions and estimation results

The selection of explanatory variables is based on the following four principles: First, the estimation and forecasting performance of the monthly indicators is evaluated applying the usual statistical tests. Second, the lag/lead structure of explanatory variables is determined. Variables with good leading indicator properties with respect to GDP growth are used in favour. Third, the availability/timeliness (i.e. release date) of the indicator is assessed. For example labour market indicators are typically assumed to follow the economic cycle with some time lag. Nevertheless time series like employment figures or vacancies are almost contemporaneously available and therefore may improve the estimation and forecasting performance. In contrast, industrial production which typically plays a prominent role in many leading indicator models is released with a time lag of 60 days which limits the usefulness for short-term estimates. Fourth, size and frequency of potential revisions are taken into account. Once again, industrial production figures for Austria are a typical example. As latest information from these indicators has to be interpreted with caution, they are only chosen if their leading indicator properties are excellent.

The estimation period ranges from the first quarter / first month 1988 to the second quarter / sixth month of 1999. The remaining periods until the end of 2001 were used for the assessment of the out of sample forecasting performance. From more than 150 variables and according to the above described criterions the following six monthly indicators were selected as exongenous explanatory variables: The ifo-index (*ifo*), credit growth (*loans*), the number of vacancies (*vac*), a real exchange rate index (*exrate*), the number of employees (*empl*) and new car registrations (*cars*). All explanatory variables are in logarithm and enter the equations in first differences with the exception of the number of employees for which we used second differences. The time subscript indicates the leading indicator properties of the explanatory variables.

$$\Delta \ln y_t^m = \zeta \cdot \Delta \ln y_{t-1}^m + \beta_1 \Delta \ln i f o_{t-1} + \beta_2 \Delta \ln loans_{t-4} + \beta_3 \Delta \ln vac_t + \beta_4 \Delta \ln exrate_{t-3} + \beta_5 \Delta \Delta \ln empl_{t-1} + \beta_6 \Delta \ln cars_{t-2} + e_t$$

The error term e_t is assumed to follow an AR(1) process: $e_t = \rho_1 \cdot e_{t'-1} + \sigma^2 u_{t-1}$. The inclusion of the parameter σ^2 is due to computational convenience. u_t denotes the innovations of the equation system which are calculated each third month via the measurement equation. The estimation results are summarized in table 1.

aber 11 Estima	nion results for			
	$\Delta \ln / \Delta \Delta \ln$	Lag	Coefficient	t-value
ifo	Δln	1	0.17	2.34
loans	Δln	4	0.17	2.25
vac	Δln	0	0.13	2.29
exrate	$\Delta \ln$	3	-0.19	-2.63
empl	$\Delta\Delta \ln$	1	0.45	2.65
cars	Δln	2	0.61	3.26
dummy94 4			-3.87	-5.02
dummy95 ¹			3.18	4.13
dummy97 ¹			-2.09	-2.77
ζ			-0.40	-2.36
ρ_{l}			-0.63	-3.04
σ			0.81	18.34

Tabel 1: Estimation results for monthly GDP growth rates

All variables standardized.

 $\Delta \ln / \Delta \Delta \ln$ indicate first and second differences, respectively.

Lag: number of lags of the exogenous variable - indicates leading indicator properties

Ifo: ifo-index business climate.

cred: outstanding credits to domestic non financial sector.

vac: Number of vacancies.

exrate: real exchange rate index.

empl: number of employees.

cars: new car registrations.

ifo-index (ifo): Most monthly confidence indicators for Austria do not start before 1996. Indicators which cover a sufficiently long time period for econometric analysis – especially indicators published by the European Commission (EC) - are clearly outperformed in terms of the out of sample forecasting performance and the in-sample fit by the ifo business climate index for Germany. One reason may be the higher volatility of the EC-indicators in the first half of the sample period. On the other hand Austrian confidence indicators are closely linked to European, especially German, indicators reflecting strong economic linkages to the European Union. Overall, the ifo-index seems to be a good indicator of business confidence in Austria and, in addition, seems to mirror the latest developments on Austria's most important export market. The ifo-index enters the forecasting equation for monthly GDP growth in first differences and with a lead of one month.

Outstanding loans to the domestic non financial sector (loans): This variable captures financing conditions and credit standards in the banking sector as well as monetary policy decisions. We preferred to include credit growth in the forecasting equation instead of a measure of money supply growth as the latter was more volatile in the recent past. According to the estimation results, credit growth leads the economic cycle by four months.

Number of vacancies (vac): Private consumption, the largest GDP component with a share of more than 50%, depends strongly on disposable income which itself is closely related to labour market developments. As the labour market is typically lagging the economic cycle, employment figures or the unemployment rate are poor leading indicators. But empirical evidence suggests that the number of vacancies is an early indicator for the labour market. This is confirmed by our estimations. The growth of vacancies is positively linked to monthly GDP growth. It is the only contemporaneous indicator used in the UOC-model.

Real exchange rate index (exrate): Besides private consumption exports are the largest GDP component. The export share in GDP reached almost 55% in 2002 and is expected to increase further in the course of globalization and the further integration into and the enlargement of the European Union. Export performance depends on the one hand on the import-demand of our trading partners and on the other hand on the price competitiveness of Austrian exports. The latter is partly determined by domestic factors like unit labour costs and partly by "exogenous" determinants like nominal exchange rates. Both, domestic price developments and nominal exchange rates, are captured by the real exchange rate index. Empirical and theoretical evidence suggests that an appreciation of the euro (i.e. in our case an increase of the real exchange rate index) affects exports negatively with a time lag of several months. Our results support these considerations: Changes in the real exchange rate index enters the equation of monthly GDP growth with a significantly negative coefficient and a time lag of three months.

Number of employees (empl): The labour market is known to follow the economic cycle with a time lag of some months. Although this time lag has become smaller in Austria in the recent past, changes in employment are still a "lagging" indicator for economic activity. Thus we used second differences which usually indicate new developments significantly earlier. Changes in employment growth lead monthly GDP growth by one month. The coefficient has the expected positive sign.

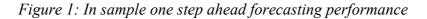
Car registrations (cars): Private consumption, the most important demand-component of GDP, includes non-durable and durable goods. The latter are known to be especially sensitive to the economic cycle. Car sales, measured in terms of new car registrations, are an important component of durable consumption goods and data become available right at the end of the respective month. The variable has the expected positive coefficient and is leading GDP growth by two months.

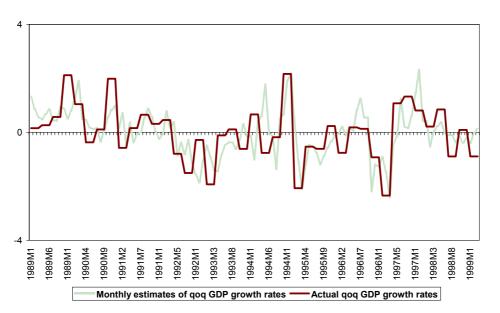
Variables with a more volatile growth patterns in the past (*cars, empl*) tend to enter the estimation equation for month-on-month (mom) GDP growth rates with higher coefficients than the other remaining variables (*ifo, loans, vac, exrate*). Since the model is evaluated along actual quarter-on-quarter (qoq) GDP-growth figures, which are per definition a weighted average of current and past mom growth rates, the high impact of more volatile monthly indicators on the mom GDP growth rates partly cancels out in case of qoq GDP growth rates. Less volatile variables that follow the economic cycle rather closely (*vac, ifo*) have typically the lowest coefficients. Whether the high coefficients of more volatile indicators cause problems in real time forecasting has to be carefully monitored in the future.

Three dummies were introduced to control for outliers in the GDP time series. The high qoq growth rate in the fourth quarter 1994 followed by a sharp decline in economic activity in the first quarter 1995 were interpreted as typical outliers with no economic rational behind while the low growth performance in the fourth quarter 1997 is known to be a pure statistical artefact. The introduction of dummies is not straightforward in this equation system where only monthly indicators are used to explain quarterly growth rates. But since qoq GDP growth

estimates are derived from the weighted sum of the current and the past four mom GDP growth estimates, there always exists one mom GDP growth rate that enters solely one single qoq GDP growth rate estimate and no other. This mom growth rate is that of the first month of the quarter we are interested in; i.e. if we want to set a dummy for the first quarter of 1997 we can use a monthly dummy variable with zeros everywhere except for January 1997. Albeit present, the impact of this dummy on future estimates of qoq GDP growth rates via autoregressive terms tends to be negligibly small. All three dummies used are highly significant.

The model has an excellent in sample estimation fit. The coefficient of determination R^2 amounts to 0.74. Figure 1 shows the actual quarter-on-quarter GDP growth rates for Austria published in the National Account data and the in sample estimation results from the unobserved-components-model. Since the estimated GDP series is available at a monthly frequency one can calculate a quarter-on-quarter GDP growth estimate for each month. Thus instead of just four qoq growth rates we can calculate twelve estimates. The qoq growth rate for the month January, for example, denotes the growth in the period November to January relative to the period August to October. As can be seen from figure 1 the estimated monthly qoq growth rates follow the actual outcomes very closely and show a somewhat higher volatility.



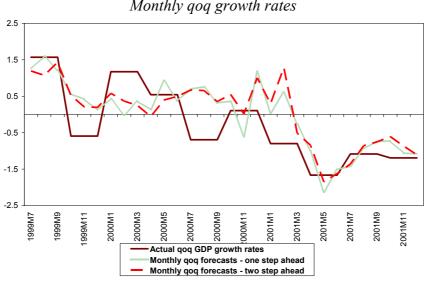


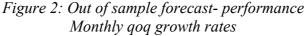
4. Out of sample forecasting performance

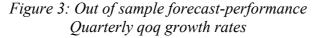
Due to data constraints the whole data set covers only the time period from the first quarter of 1989 to the fourth quarter of 2001. Data for the years 2002 and 2003 have not bee included as the latest National Account figures are often subject to large revisions. Observations until the first half of 1999 (10 years or 40 quarters) have been used for estimation purposes. The remaining 10 quarters (1999Q3 to 2001Q4) where retained to evaluate the out of sample forecasting performance. The number of out of sample forecasts is obviously at the lower bound for running the common tests for forecasting accuracy and forecast accuracy. But the

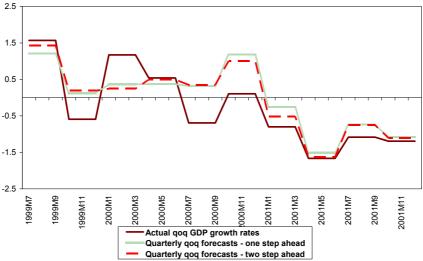
out-of-sample forecast performance is just one of several model selection criterions and we decided to put a high weight on estimation criterions.

Figures 2 and 3 show the actual quarter-on-quarter GDP growth rates and the out-of-sample forecast results from the UOC-model one-step- and two-steps-ahead. Since GDP is forecast as an unobserved component at a monthly frequency we present in figure 2 again the monthly quarter-on-quarter GDP growth forecasts. A visual inspection indicates a good performance of the out of sample forecasts, both one-step and two-steps-ahead. The economic slowdown at the beginning of 2001 has been anticipated quite well at both forecast horizons. Overall the relative performance of the two-steps-ahead forecast is surprisingly good. Figure 3 shows the quarterly forecast performance on which the OeNB's economic indicator actually rests upon. These forecast-figures are identical to the forecast of each third month and therefore naturally much smoother than the monthly forecast.









The assessment of the forecasting performance is based on a comparison of the out-of-sample forecasts of the UOC-model with forecasts produced by an ARIMA model and a Naïve

model. The number of AR and MA terms and the use of a constant in the ARIMA model are determined by the minimum RMSE criterion while the Naïve model is a simple random walk forecast. The forecast accuracy is measured by the root mean squared error (RMSE) and the mean absolute error (MAE). With regard to these measures, the UOC-model clearly outperforms the other models (see table 2). This holds for both forecasting horizons – one and two steps ahead – and when both horizons are assessed together.

		RMS	E		MAE	
	1step	2step	1 and 2steps	1step	2step	1 and 2steps
UOC-model	0.63	0.60	0.61	0.53	0.46	0.49
ARIMA	1.03	1.05	1.03	0.93	0.94	0.94
Naïve Forecast	1.12	1.32	1.21	0.95	1.06	1.01

Table 2: Out of sample forecast performance

Finally, three statistical tests for relative predictive accuracy have been performed. The Diebold-Mariano (1995) test and the Wilcoxon signed rank test (see Siegel and Castellan (1988)) have been used to test for equal forecasting accuracy of the UOC-model relative to the two rivalling models. The test for multiple forecast encompassing proposed by Harvey, Leybourne and Newbold (1998) has been used to test the null hypothesis whether the UOC-model forecast encompasses the forecast of the ARIMA model or the Naïve model. Detailed explanations of these tests can be found in Appendix C. Table 3 summarizes the results.

	1step	2step	1 and 2steps
UOC-model compared to ARIMA Model			
RMSE gain in %	0.39	0.43	0.41
MAE gain in %	0.43	0.51	0.47
Diebold-Mariano RMSE	0.045*	0.036*	0.006**
Diebold-Mariano MAE	0.047*	0.035*	0.006**
Wilcoxon RMSE	0.045*	0.016*	0.002**
Wilcoxon MAE	0.045*	0.035*	0.006**
Harvey et. al. Model encompassing	0.475	0.414	0.417
UOC-model compared to Naïve Model			
RMSE gain in %	0.44	0.55	0.49
MAE gain in %	0.45	0.57	0.51
Diebold-Mariano RMSE	0.036*	0.029*	0.004**
Diebold-Mariano MAE	0.010**	0.032*	0.003***
Wilcoxon RMSE	0.020*	0.016**	0.028*
Wilcoxon MAE	0.015*	0.010**	0.020**
Harvey et. al. Model encompassing	0.4819	0.4581	0.4790

Table 3: Out of sample forecast tests

The most important finding is that the relative forecasting performance of the UOC-model is significantly better than of the ARIMA and the Naïve model. This holds for both forecast horizons and for the RMSE and the MAE criterion. A second important finding is that compared to the one-step-ahead forecast - the relative forecast accuracy of the two-step-ahead forecast is somewhat better.

The RMSE of the UOC-model forecast is around 40% smaller than the ARIMA forecast and around 50% smaller than the Naïve forecast. A similar picture arises in case of the MAE gain. According to the Diebold-Mariano test, the gains of the UOC-model are significant at the 5% level (in case of the pooled forecast set at the 1% level). With respect to the Wilcoxon test gains are significance at either the 5% or the 1% level.

5. Conclusions

In order to use monthly indicators in the estimation of quarterly GDP growth rates we use an unobserved component model based on a Kalman filter technique. As unobserved component a monthly GDP growth rate is estimated. This unobserved component is then used to compute an estimate of quarterly GDP. The model was selected along the criterions of in sample estimation properties and out of sample forecasting performance. For the selection of explanatory variables, the leading indicator properties of the monthly indicators, their availability/ timeliness with respect to the release date and whether they have been revised frequently in the past where taken into account.

Six conjunctural indicators published at a monthly frequency which cover various aspects of the economy enter the unobserved components model as explanatory variables. The ifo-index seems to be a good indicator of business confidence in Austria and additionally mirrors the latest developments on Austria's most important export market. Credit growth captures financing conditions and credit standards in the banking sector as well as monetary policy decisions. The number of vacancies and changes in employment growth are leading indicators for the labour market. The international competitiveness is captured by the real exchange rate index. And finally, car registrations are used as a proxy for durable goods which are known to be especially sensitive to the economic cycle.

In real time forecasting⁵ the specification of the UOC-model, i.e. the exogenous variables and the ARMA structure of the state space model, are typically fixed for a longer period of time. In contrast, the parameter values are recalculated each forecasting round as is typically the case in Kalman filter applications.

One main advantage of the UOC-model is that the variables and parameters of the state space model have a straight foreword economic interpretation. The effect of each single explanatory variable as well as the importance of autoregressive processes can be stated explicitly. This renders the interpretation of the results comparatively easy. Furthermore a new time series, monthly GDP, is calculated explicitly describing the conjunctural situation at a higher frequency.

⁵ The UOC-model is one of two models on which the OeNB's economic indicator rests upon (see Fenz et al. (2004)). The OeNB economic indicator is published four times per year. Its aim is to forecast growth of real GDP in Austria in the current and the consecutive quarter. The second model is a dynamic factor model (see Schneider and Spitzer (2004)).

Appendix A

The General State Space Model:

The general state space form adapted for different frequencies in the observation equation (quarters) and the transition equation (months) and exogenous variables in the transition equation can be written as:

$$y_{\tau} = Z \cdot a_{\tau} + \varepsilon_{\tau} \qquad \tau = 1 \dots T / 3 \qquad \text{(Observation equation)}$$
$$a_{t+1} = T_t \cdot a_t + W_t \cdot X_t + H_t \cdot u_t \qquad t = 1, 2, 3 \dots T \qquad \text{(Transition equation)}$$

where y_{τ} denotes the 1×1 vector of observations and a_{τ} is a $m \times 1$ vector of unobserved components called state vector. t and τ are the indices of months and quarters, respectively. The first quarter ranges from month 1 to 3, the second quarter from month 4 to 6.... X_t denotes the $N \times 1$ vector of monthly indicators (explanatory variables). X_t, T_t, W_t, H_t are matrixes of parameters which are either known or have to be estimated using Kalman filter techniques. The error terms ε_{τ} and u_t are assumed to be serially independent and independent of each other for all t.

The Specific State Space Model for estimating the Short Term Indicator for the Austrian Economy

The Transition equation takes the form:

$$a_{t+1} = T_t \cdot a_t + W_t \cdot X_t + H_t \cdot u_t$$
 $u_t \approx N(0,1)$ $t = 1,2,3...T$

 u_t , the estimation-errors, are called innovations and assumed to be normally distributed with mean zero and variance one. The unknown variance of the innovations enters the H_t matrix and has to be estimated. The standardisation is due to computational convenience. The explicit presentation of the transition equation takes the form:

In the first row the estimation equation of monthly GDP growth rate $(\Delta \ln y_{t+1}^m)$ (w.r.t. the previous month not the corresponding month in the previous quarter) is stated. It depends on its own lag $(\zeta \cdot y_t^m)$, an error term e_{t+1} that follows an AR process and N exogenous monthly indicators $(\beta_1 \cdot x_{1,t+1}^m + ... + \beta_N \cdot x_{N,t+1}^m)$.

$$\Delta \ln y_{t+1}^m = \zeta \cdot \Delta \ln y_t^m + \beta_1 \cdot x_{1,t+1}^m + \dots + \beta_N \cdot x_{N,t+1}^m + e_{t+1} \qquad \text{(Row 1 of transition equation)}$$

In row 5 the AR process of the error term is defined. The AR-structure of the error term was determined by statistical tests. It turned out that an AR(1) specification fits best: $e_{t+2} = \rho_1 \cdot e_{t+1} + \sigma^2 u_{t+1}$. Rows 2-4 and 6-7 are identities. In row 8 the estimated quarterly GDP growth rate ($\Delta \ln y_{\tau}^{Qe}$) is calculated as a weighted sum of the three monthly GDP growth rates of the current quarter and the last two monthly GDP growth rates of the previous quarter. The weights are given by: 1/3, 2/3, 1, 2/3, 1/3.⁶

/3,

The Observation equation takes the form:

$$\Delta \ln y_{\tau}^{Q} = Z \cdot a_{\tau} = \Delta \ln y_{\tau}^{Qe} \quad \text{where } \tau = 1, 2, 3 \dots T/3; \quad t = 1, 2, 3 \dots T; \quad \tau = t$$

$$\Delta \ln y_{\tau}^{m} = (0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1) \cdot \begin{pmatrix} \Delta \ln y_{t}^{m} \\ \Delta \ln y_{t-1}^{m} \\ \Delta \ln y_{t-2}^{m} \\ \Delta \ln y_{t-2}^{m} \\ \Delta \ln y_{t-4}^{m} \\ e_{t+1} \\ e_{t} \\ \Delta \ln y_{t}^{Qe} \end{pmatrix} = \Delta \ln y_{\tau}^{Qe}$$

where $\Delta \ln y_{\tau}^{Q}$ denotes the actual quarterly GDP growth rate, $\Delta \ln y_{\tau}^{Qe}$ the forecast of the quarterly GDP growth rate. Each third month, at the end of a quarter, the estimation of quarterly GDP is evaluated on the basis of actual outcomes. In fact standard multivariate time series methods could be used in estimating the above system. But due to the serial correlation in the observation variable the complexity of the computation increases rapidly with the number of observations. The Kalman filter provides a very efficient tool for estimating the unknown parameters ζ , ρ_1 , σ^2 and β_1 to β_N . Once the parameters are estimated it is straightforward to calculate the unobserved components. For forecasting purposes the measurement equation is no longer needed. As soon as new observations for the monthly indicators are available, an update of the forecast can be calculated using the transition equation.

Appendix B

Aggregating monthly to quarterly growth rates⁷

A quarterly GDP growth rate can be expressed as the weighted sum of the current and the past four monthly growth rates where the weights are given by $(1/3 \ 2/3 \ 1 \ 2/3 \ 1/3)$. The

⁶ see Appendix B.

⁷ Appendix B follows Rünstler and Sédillot (2002).

weights are correct under the assumption that deviations of the levels of monthly GDP (y_t^m) from the mean level (\bar{y}_{τ}^m) in the corresponding quarter are negligibly small:

$$y_t^m - \bar{y}_\tau^m = 0 \qquad (Assumption 1)$$

where t = 1...T is the index of months and $\tau = 1...T/3$ the corresponding index of quarters and $\bar{y}_{\tau}^{m} = \frac{1}{3} \sum_{i=0}^{2} y_{t+i}^{m}$.

The mean value theorem and Taylor expansion state:

$$f(y_t^m) = f(\bar{y}_{\tau}^m) + (\bar{y}_{\tau}^m - y_t^m)f'(y_t^m).$$

Under assumption 1 it therefore follows that

$$\sum_{i=0}^{2} \ln y_{t+1}^{m} = 3 \ln \bar{y}_{t}^{m} = 3 \ln \left(\sum_{i=0}^{2} y_{t+1}^{m} \right) - 3 \ln(3)$$

This holds also for the previous quarter. The difference of the current and the previous quarter is given by:

$$\sum_{i=0}^{2} \ln y_{t+i}^{m} - \sum_{i=0}^{2} \ln y_{t-3+i}^{m} = 3 \left(\ln \left(\sum_{i=0}^{2} y_{t+i}^{m} \right) - \ln \left(\sum_{i=0}^{2} y_{t-3+i}^{m} \right) \right) \,.$$

The right hand side of this equation is three times the quarterly GDP growth rate in quarter τ (months t to t+2) while the left hand side is the sum of the GDP growth rates of each month in quarter τ with respect to the corresponding month of the previous quarter. Simple arithmetic manipulation shows that

$$\frac{1}{3} \left(\sum_{i=0}^{2} \ln y_{t+i}^{m} - \sum_{i=0}^{2} \ln y_{t-3+i}^{m} \right) = \frac{1}{3} \left((\ln y_{t+3}^{m} - \ln y_{t+2}^{m}) + 2(\ln y_{t+2}^{m} - \ln y_{t+1}^{m}) + 3(\ln y_{t+1}^{m} - \ln y_{t}^{m}) + 2(\ln y_{t-1}^{m} - \ln y_{t-1}^{m}) + (\ln y_{t-1}^{m} - \ln y_{t-2}^{m}) + (\ln y_{t-1}^{m} - \ln y_{t-2}^{m}) \right)$$

holds. Thus the GDP growth rate for the current quarter τ (months t to t+2) is equal to the weighted sum of the five monthly GDP growth rates (i.e. growth rates with respect to the previous month and not with respect to the corresponding month of the previous quarter) of the months t+2 to t-2. The weights are given by $(1/3 \ 2/3 \ 1 \ 2/3 \ 1/3)$.

Appendix C

Tests for equal forecasting accuracy and forecast encompassing

C.1 Wilcoxon's signed rank test

The non-parametric Wilcoxon signed rank test tests the null hypothesis of equal forecast accuracy. It is an alternative to the t-test in situations, where the assumption of a normal

distribution is violated, which is typically the case in small samples. The test assumes that both forecasting errors have identical distributions. Hence the distribution of the differences d_t between the loss functions $g(e_t^A)$ and $g(e_t^B)$ of the forecasting errors e_t^A and e_t^B is symmetric around zero. The null hypothesis is that the loss differential $\{d_t\}_1^T = e_t^A - e_t^B$ has median value zero. The test is illustrated for a squared loss function, although it can be applied for an absolute loss function as well. The following steps are necessary to perform the test. Begin with calculating the loss differential series d_t

$$d_{t} = \begin{cases} (e_{t}^{A})^{2} - (e_{t}^{B})^{2} & , \text{ if } RMSE^{A} < RMSE^{B} \\ (e_{t}^{B})^{2} - (e_{t}^{A})^{2} & , \text{ otherwise} \end{cases}$$
(C1)

and remove all zero elements from d_t . Next, compute a 0/1 vector with ones for all elements of d_t which are greater than zero.

$$l_{+}(d_{t}) = \begin{cases} 1 & \text{, if } d_{t} > 0 \\ 0 & \text{, otherwise} \end{cases}$$
(C2)

Determine the rank numbers of all elements of d_t , disregarding the sign of d_t . Assign the rank number 1 to the smallest and T to the highest element. If tied values occur, than rank all elements with the mean of the rank numbers that would have been assigned if they would have been different. Compute the test statistic W as the sum of the positive ranks only.

$$W = \sum_{t=1}^{T} l_{+}(d_{t}) * rank(|d_{t}|)$$
(C3)

Critical values for W are tabled. If W is smaller than the critical value, reject the null of equal forecasting accuracy. Asymptotically, W converges to a normal distribution

$$W \sim N(\mu, \sigma^2)$$
, with $\mu = \frac{T(T+1)}{4}$ and $\sigma^2 = \frac{T(T+1)(2T+1)}{24}$

Therefore, if T > 20, one can compute the transformed test statistic

$$W^* = \frac{W - \mu}{\sigma} \tag{C4}$$

and use the critical values from the standard normal distribution.

C.2 The Diebold and Mariano test

The Diebold and Mariano [1995] test for equal forecasting accuracy tests the null hypothesis of equal forecast accuracy of two competing forecasts. It uses a forecast error loss differential $d_t = g(e_t^A) - g(e_t^B)$, which is assumed to be a weakly stationary process with short memory. The main rationale underlying this test is that forecast errors are usually serially correlated. In multi-step forecasting (h > 1), forecasts errors are assumed to be at most h-1-dependent. This is a plausible assumption, since two consecutive h-steps-ahead forecasts have h-1

periods with similar information in common. The Diebold and Mariano test is a modified t-test, whereby the modification accounts for the serial correlation of the loss differential. The mean \bar{d} is assumed to be asymptotically normally distributed:

$$\sqrt{T(\bar{d}-\mu)} \xrightarrow{d} N(0, V(\bar{d})), \tag{C5}$$

whereby $V(\bar{d})$ stands for the serially correlated errors corrected variances of the sample mean (\bar{d}) , given by the sum of the variance and the auto-covariances up to lag h-1 assuming that there are no auto-correlations at a lag equal to or greater than h:

$$V(\bar{d}) = \frac{1}{T} \left(\gamma_0 + 2\sum_{\tau=1}^{h-1} \gamma_\tau \right)$$
(C6)

where T denotes the sample size and the autocovariance is ghiven by:

$$\gamma_{\tau} = \frac{2}{T} \sum_{t=\tau+1}^{T} (d_t - \bar{d}) (d_{t-\tau} - \bar{d})$$
(C7)

The asymptotically normally distributed test statistic DM can be obtained by

$$DM = \frac{\bar{d}}{\sqrt{V(\bar{d})}} \tag{C8}$$

In small samples, the t-distributed modified test statistic DM^* should be preferred [Harvey, Leybourne and Newbold 1997]:

$$DM^{*} = \frac{DM}{\sqrt{\frac{T+1-2h+\frac{h(h-1)}{T}}{T}}}$$
(C9)

If the value of the test statistic is greater than the critical value, the null of equal forecasting accuracy should be rejected.

C.3 Harvey, Leybold, and Newborn test for forecast encompassing

Harvey, Leybourne and Newbold (1998) proposed a test for forecast encompassing under the null that forecast A encompasses forecast B, i.e. forecast B adds no predictive power to forecast A. The test uses a linear combination of two competing forecasts y_t^A and y_t^B of variable y_t with a combined forecast error ε_t :

$$y_t = (1 - \lambda)y_t^A + \lambda y_t^b + \varepsilon_t .$$
(C10)

In terms of individual forecast errors $e_t^i = y_t - y_t^i$, for i = A, B, equation (10) can be written as

$$e_t^A = \lambda(e_t^A - e_t^B) + \varepsilon_t \tag{C11}$$

This equation has to be estimated by OLS. If the null of forecast encompassing holds, than λ should equal zero.

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