

Asset liability management and interest rate risk in Solvency II – an empirical study

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1 Introduction

The variation in the net value that arises due to changes in the term structure of the interest rates, used for the calculation of the present values of both sides of the balance sheet of insurance companies, is one of the most important risks that they have to allocate capital (eligible own funds) for. Under the new supervisory rules for insurances in the European Union – usually referred to as Solvency II – the relevant capital requirements for this risk are defined within the market risk module, in the interest rate risk sub-module.

In general, Solvency II allows for two different approaches to determine the relevant capital requirements to compensate for a reduction in the company's net value. Firstly, the determination is based on the worse outcome of two predefined scenarios, one reflecting an upshift of the risk free interest curve and one defined as a downshift, usually referred to as the standard approach. Secondly, the determination can be based on an internal model for the relevant risk. If an insurance company chooses to use an internal model, the model has to be calibrated in such a way that it measures the value-at-risk for a confidence level of 99.5%, based on a one year horizon. Both approaches have to be applied on the total balance sheet, thus capturing interest rate sensitivity of assets and liabilities. Given the typically long term oriented liability profile of e.g. life insurers, proper management of (long term) interest rate risk is the most demanding part of asset liability management in the insurance sector.

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The following sections present an empirical study to the task of measuring interest rate risk. Based on the historic development of German government bonds with various maturities, the changes in the term structure within one year seen historically are calculated and applied to four different cash flows. This allows for the calculation of the changes in the *Net Asset Values (NAV)* and subsequently the construction of an empirical cumulative distribution function of these changes. From these empirical cumulative distribution functions, the regulatory relevant value-at-risk 99.5% can be derived.

These results are of special interest when compared to the capital needs stemming from the use of the standard approach, which uses more or less parallel shift scenarios. The question is, whether this type of stress calibration is suitable for various types of cash flow profiles (balance sheets) when compared to actual changes in the own funds of these balance sheets.

2 The cash flows

This study elaborates on the risks due to changes in the interest rate term structure for four different cash flow profiles (“stylized” balance sheets). These cash flows are named *Demo*, *Balanced*, *Unbalanced* and *Matched* and shall be described in this section. It is worth mentioning that the four cash flows have identical liabilities and identical amount of own funds (defined as the difference in present value of assets minus liabilities) and only differ on the maturity profile of the asset side of the balance sheets.

Chart 1 further down this section shows the asset and the liability side of the four cash flows. It can easily be seen that the maximum maturity of all cash flows is 61 years, as it was defined in the latest quantitative impact study conducted by EIOPA and national supervisors (Financial Market Authority, FMA in the case of Austria) in 2014, based on 2013 year end company data.²

2.1 The “Demo” cash flow

The first cash flow *Demo* represents the cumulative term structure of Austrian insurance companies and therefore can be seen as a representative example of the structure of the balance sheet for an average insurance company as of year-end 2013. The present values of assets and liabilities are used to calibrate the other three cash flows to ensure that the results are easily comparable. The cash flow represents an

² The FMA’s official homepage contains detailed information on conditions and results of the QIS6 assessment: www.fma.gv.at/de/sonderthemen/solvency-ii/informationen-fuer-versicherungsunternehmen/qis-6eiopa-stresstest.html.

insurance company with equity EUR 100,000³, the sum of assets equals EUR 2,915,838 and the sum of liabilities equals EUR 4,168,163. Applying the term structure of the risk free rate r_{base} on this cash flow results in a present value of assets equal to EUR 2,448,004 and a present value of liabilities of EUR 1,800,290. Subsequently the NAV for this cash flow under the risk free term structure equals EUR 647,714.

For the same term structure of the risk free rate the duration of the asset and liability cash flows can be calculated as well. For the Demo cash flow we get 7.2 for the Macaulay duration of the assets and 21.1 for the Macaulay duration of the liabilities. As the euro duration⁴ we get EUR 170,606 for the assets and EUR 368,302 for the liabilities.

This is already a first indication on the inherent interest rate risk of insurance balance sheets, as the difference in euro duration for assets and liabilities is a rough approximation for the net change in company value for a one percent change in the risk free interest rate level. Such a change would wipe out almost EUR 200,000 in value which was only determined at around EUR 650,000.

2.2 The “Balanced” cash flow

The second cash flow named *Balanced* represents the balance sheet of an insurance company which has exactly the same liabilities, but a different structure of assets. The asset side of the cash flow has been calibrated in such a way that the euro duration of the assets equals the euro duration of the liabilities. Therefore, this cash flow represents an insurance company who has tried to immunize its balance sheet against the risk associated with parallel changes in the interest rate term structure via duration matching. Obviously, such a profile would be exposed to non-parallel shifts, such as twists, which are not modelled under the standard approach.

The cash flow represents an insurance company again with equity amounting to EUR 647,714, the sum of assets equals EUR 3,981,784. With the term structure of the risk free rate r_{base} we get a present value of assets equal to EUR 2,448,004, a Macaulay duration of 15.3 and EUR 363,717 for the euro duration. Note that the present value of assets is exactly the same as for the *Demo* cash flow. This was one of the conditions for calibrating the cash flows and holds as well for the *Unbalanced* and the *Matched* cash flow.

³ For this and the following sections the term equity is used to denote the assets of the company with maturity 0 years.

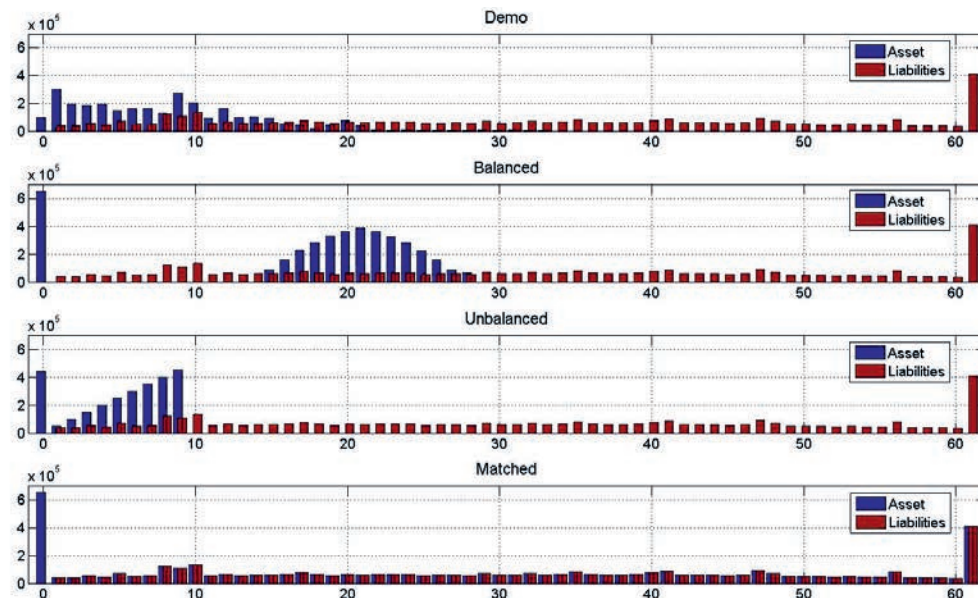
⁴ For the calculation of the euro-duration the following formula has been used:
 $\text{D€} = \text{PV} * \text{modified Duration} / 100.$

The sum of liabilities, the present value of liabilities and the corresponding durations of liabilities clearly equal the values of the *Demo* cash flow as the liabilities are identical.

2.3 The “Unbalanced” cash flow

The third cash flow *Unbalanced* represents the balance sheet of an insurance company who has taken no effort to immunize its balance sheet in respect to changes in the risk free rate term structure. While the liability side of the cash flows is the very long term, which is typical for insurance companies due to their contract portfolio especially of life insurance products, the asset side is rather short term with maximum maturity of 9 years. This reflects the asset allocation of a company that is waiting for interest rates to rise again before taking longer fixed rate securities.

Chart 1: Modelled cash flow profiles of assets and liabilities



Source: FMA, authors' calculations.

The cash flow represents an insurance company with equity EUR 437,047, the sum of assets equals EUR 2,687,047. Applying the term structure of the risk free rate r_{base} results in the same present value of assets equal to EUR 2,448,004, but a drastically lower Macaulay duration of 5.1 and a corresponding EUR duration of EUR 121,113.

The sum of liabilities, the present value of liabilities and the durations of liabilities once again equal the values of the *Demo* cash flow.

2.4 The “Matched” cash flow

The fourth cash flow represents the balance sheet of an insurance company which has fully immunized its balance sheet in respect to changes in the term structure of the risk free rates. Its assets equal the liabilities for all maturities taken into consideration. Therefore, any change in the interest rate does not affect the NAV of the company as the changes in the interest rate affect the assets exactly the same way as the liabilities; both effects abrogate the subsequent effect to the NAV.

The calibration of this cash flow has resulted in an equity of EUR 647,174, which is equal to the equity of the *Balanced* cash flow. The sum of assets equals to EUR 4,815,877. With the term structure of the risk free rate r_{base} we get the same present value of EUR 2,448,004, a Macaulay duration of 15.5 and a euro duration of EUR 368,302. Note that the euro duration of the asset side of this cash flow by design equals the euro duration of the liability sides of all four cash flows.

3 Interest rates

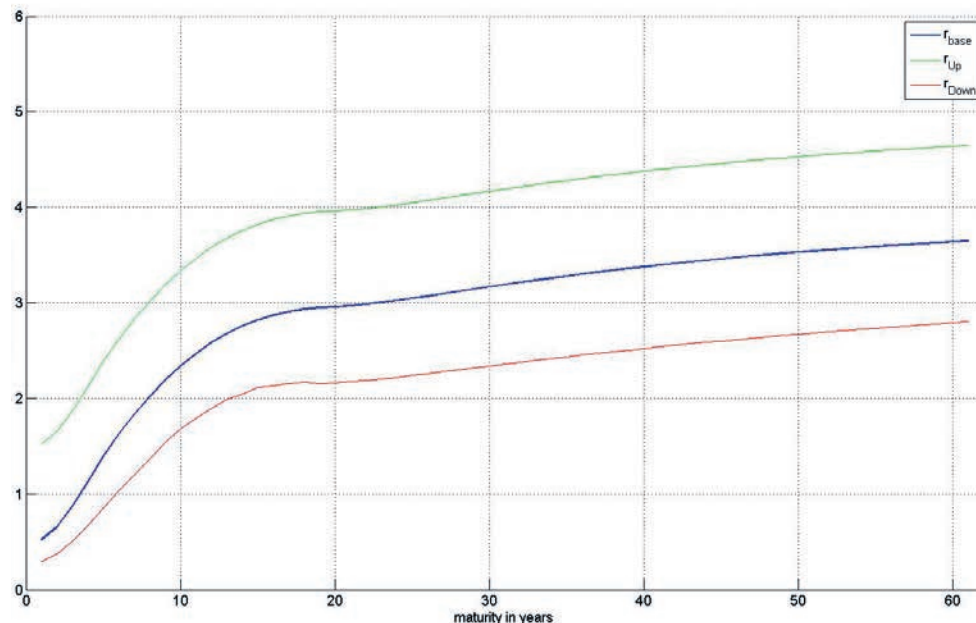
In general this study uses similar inputs for interest rates as used in the Quantitative Impact Study 6 (QIS6). QIS6 refers to a voluntary study conducted by many European insurance companies in order to elaborate on the effects of the new regulatory framework denoted as Solvency II. Within this study several parameters have been set as a default, foremost it proposed a term structure for the risk free rate which has to be used as standard for the present value calculations.

3.1 Risk free rate r_{base}

In this study, the predefined term structure of the risk free rate is usually referred to as r_{base} . It has been used to calibrate the three cash flows *Balanced*, *Unbalanced* and *Matched* and represents the basis scenario. For QIS6 the interest rate term structure has been given for maturities up to 150 years, while for this study only the first 61 years have been taken into account as 61 years represent the maximum maturity of the cash flows taken into consideration.

Chart 2: *EIOPA's risk-free interest rate structure and shocked levels by end of 2013*

interest rates in %



Source: EIOPA.

In chart 2 the term structure of the risk free interest rate r_{base} is shown as bold solid blue line.

3.2 Interest rate shock in the standard approach of Solvency II

One aim of this study is to compare the results of an empirical analysis of historical interest rate changes and their effects on balance sheets of companies to the corresponding measurement of interest rate risk as defined in the standard approach of Solvency II.

In order to do so, the standard scenarios for an interest rate shock as defined in articles 146 and 147 of the Draft Delegated Acts to Solvency II have to be specified. In general within Solvency II shocks are always defined as shocks in both directions, the capital requirement for this risk module is then derived from the worse effect on the company's change in NAV.

For this study the shocks as defined for the QIS6 study have been taken into consideration. In difference to the legal definition of *Up* and *Down* shocks for the interest rates which is only given as a relative increase and decrease of the risk free

rate term structure (e.g. the increase for the 10 year maturity is defined as 42%), the predefined scenarios used for QIS6 are given in absolute values. This results in two additional term structures for the stress scenarios which can also be seen in chart 2. The shock of a sudden increase in interest rates is shown as green line, while the shock of a sudden decrease is shown in red.

3.3 Historic interest rates of German government bonds

The aim of this study is to conduct an empirical analysis of the effects of changes in the term structures of interest rates on the NAV of insurance companies. The idea is to use observed historical shifts in the interest rate term structure to analyze the effects these changes have on the cash flows defined in section 2.

German government bonds are nowadays somewhat the standard yield curve for the whole euro area. Furthermore, German bonds represent financial instruments traded in “deep, liquid and transparent financial markets”. This requirement for the choice of the relevant risk free interest rate term structure is part of the Solvency II regulation.

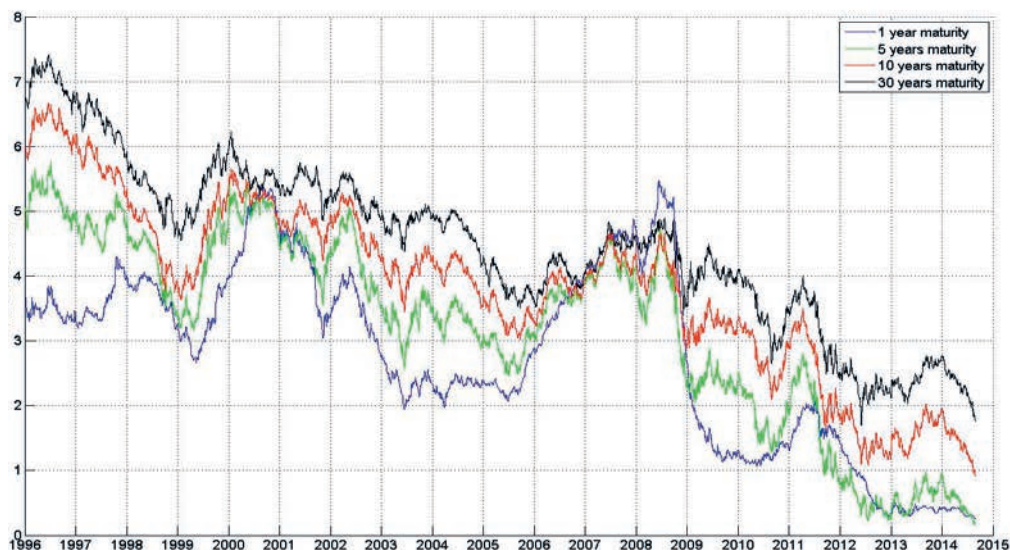
The historical German bond yields⁵ are given for the maturities 1, 2, 3, 5, 7, 10, 20 and 30 years. The data starts January 1st, 1996 and ends August 28th, 2014. The overall data set therefore consists of 8 interest rates, given for 4,868 trading days.

Chart 3 shows the development of the interest rates for 1, 5, 10 and 30 years. It can easily be seen that the historic development of the interest rates does not follow the generalized idea that interest rate term structures should change similar to parallel shifts. Such a parallel shift of the interest rates can be seen in the base scenario of *Up* and *Down* shocks as defined in QIS6, but history indicates that other forms of interest rate changes are possible as well. They are not even rare, but rather the most common development. Phases where the interest rate term structure flattens – or even inverts as seen in 2008 – exist besides phases where the interest rate term structure steepens. This empirical study includes such effects, which can lead to very different assessments of interest rate risk, as discussed in section 5.

⁵ Source: Thomson Reuters, German Bond redemption yields.

Chart 3: Yield of German sovereign bonds by maturity

interest rates in %



Source: Thomson Reuters.

4 Interpolation of interest rates

In order to use the yields of German government bonds, given for different maturities, the term structure has to be completed in order to obtain interest rates for maturities which are not given by the historical data. For this study, two methods of interpolation have been used, a simple linear interpolation and the Smith-Wilson method.

4.1 Linear interpolation

To calculate the missing interest rates with linear interpolation, one has to simply connect the given yields, e.g. for maturities A and B, with a straight line. The yields for all maturities in between the given ones, e.g. all maturities C with $A < C < B$, are then given by the function value of the straight line for this maturity.

This method allows for two different choices of parameters. Firstly, an *Ultimate Forward Rate (UFR)* has to be specified. The UFR represents the interest rate, which can be expected in the very long run. For this study an UFR of 4.2%, in line with the EIOPA specifications, has been chosen.

Secondly, the maturity at which the UFR is reached has to be fixed. In geometric form this equals the fixing of the endmost point on the very right side which can then be used to determine all the interpolated interest rates between the last historically given interest rate (in our case the rate for 30 years maturity) and the maximum maturity of the cash flows. For this study the UFR is reached after 61 years, in other words at the last maturity of the given cash flows.

4.2 Smith-Wilson method

The Smith-Wilson method is a sophisticated mathematical method for fitting yield curves to given spot rates. It was developed by Andrew Smith and Tim Wilson, further information can be found in the original paper *Fitting Yield Curves with Long Term Constraints* (2001), Research Notes, Bacon and Woodrow. The Financial Supervisory Authority of Norway published a more application orientated paper titled *A Technical Note on the Smith-Wilson Method*, which is publicly available on their homepage.

Some of the main advantages of this method include that it is a purely mechanized approach to yield curve fitting which can be easily implemented. It provides a perfect fit of the estimated term structure, while the UFR is reached asymptotically. Furthermore, it is a uniform approach including both interpolation between given spot rates and extrapolation beyond the last given spot rate. In contrast to the linear interpolation, the resulting yield curves have no kinks at maturities for which yields are given.

In order to conduct the yield curve fitting for given data, two parameters have to be chosen. Firstly an *Ultimate Forward Rate (UFR)* has to be fixed; for this study the UFR is 4.2%. Secondly the value for the parameter α has to be set. The parameter determines the weight of the ultimate forward rate within the model. Larger values give greater weight to the ultimate forward rate, while smaller values give more weight to the input data. For this study α equal to 0.1 has been chosen.

5 Change in net asset values

Both interpolation methods described in the previous section can be used to construct a full yield curve out of the given yields of German government bonds. The resulting interpolations can then be used as indication for possible changes in the yield curve within one year. For this empirical study the historic changes in the yields of German government bonds within one year are used as scenarios to model the effects of changes in the risk free term structure.

5.1 Calculation of the change in the interest rate term structure

The basis of risk assessment within Solvency II the value-at-risk 99.5% for a horizon of one year is used. For the submodule interest rate risk this means that the regulatory relevant risk is the change in the Net Asset Value (NAV)⁶ of the insurance due to changes in the risk free rate term structure.

The standard approach defines an upside shock and a downside shock on the interest rates out of which the more adverse effect on the NAV is relevant for the capital requirements. This can be problematic as other changes in the term structure, such as flattening or steepening of the risk free rate term structure, can have more severe effects on the NAV but are not covered by the standard approach.

For the empirical study, we do not try to identify adverse scenarios on the basis of the yield curve but rather conduct simulations based on *all* observed changes in the risk free rate term structure. In order to be in line with the regulatory framework, changes for the horizon of exactly one year have been taken into account. In order to simplify calculations, observed interest rate term structures for February 29th have been excluded from the data.

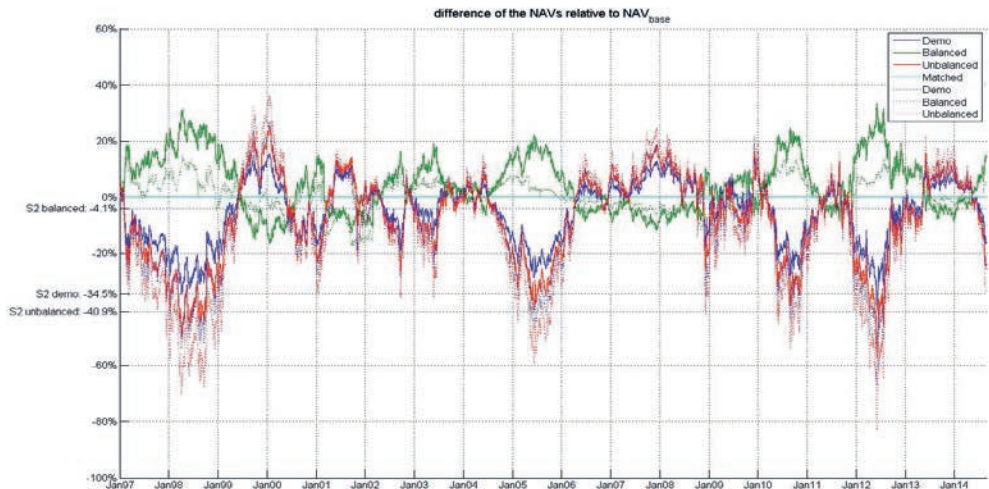
This allows for the calculation of movements in interest rates for different maturities for each day, if the interpolated interest rate term structure of the previous year is known. As our data starts January 1st, 1996 the first change in the interest rate term structure we can observe is the change from January 1st, 1996 to January 1st, 1997. Equivalently, the last change in the interest rate term structure our data contains is the change between August 28th, 2013 and August 28th, 2014.

For this paper the change in interest rates for a given maturity is defined as the absolute change. Other definitions of changes, such as relative changes as given in the legal definition of the standard approach to interest rate risk or absolute/relative changes in the discount factors are possible as well. Further research to study the effects of the various definitions of “change” seems promising.

Overall the empirical method derives a set of two times 6,444 changes in the risk free rate term structure which then can be applied to the four defined cash flows.

⁶ For this study net asset value (NAV) refers to the difference of the present value of assets and the present value of the liabilities.

Chart 4: Relative change of NAV over a rolling 12 month period
%



Source: Authors' calculations.

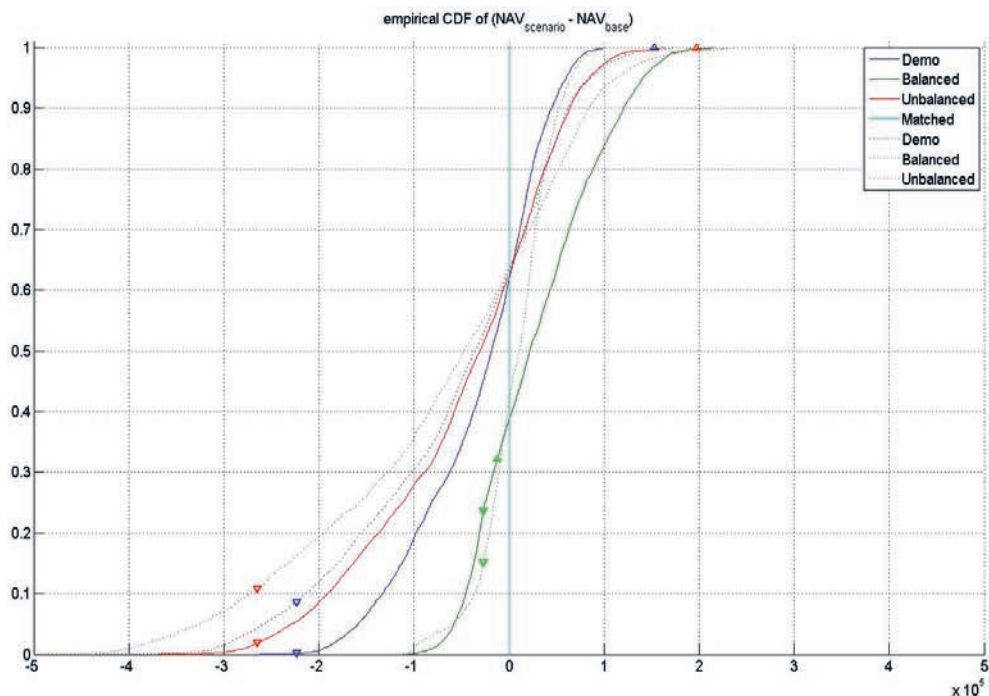
Chart 4 shows the relative difference of the NAV of a certain cash flow calculated with the term structure of the risk free rate (usually denoted as NAV_{base}) and the NAV calculated with the term structure of the risk free rate plus the movement of the interpolated German government bond yields (denoted as $NAV_{scenario}$) in % of NAV_{base} . The results calculated with linear interpolation are shown as solid lines while the results obtained via Smith-Wilson interpolation are shown as dashed lines. Additionally, the results of the standard approach to interest rate risk as defined in Solvency II are shown in the grid of the graph. It can be easily seen that the NAV of the *Balanced* Cash Flow is reduced by 4.1% under the standardized interest rate shock. Similarly the NAV of the *Demo* Cash Flow is reduced by 34.5% and the NAV of the *Unbalanced* Cash Flow is reduced by 40.9%.

5.2 Empirical cumulative distribution functions (empirical CDF) of Delta NAVs

After the historically observed changes in the term structure of interest rates have been calculated, these changes can be used to calculate the effect they have on the NAV of the cash flows defined in section 2.

Depending on the movement of the interest rate term structure and the structure of the cash flows, the effects on the NAV can vary widely. For some cash flows a flattening is much worse than for other cash flows, while for the cash flow *Matched* all changes in the interest rate term structure do not result in a change of the NAV as both sides of the cash flow are perfectly matched.

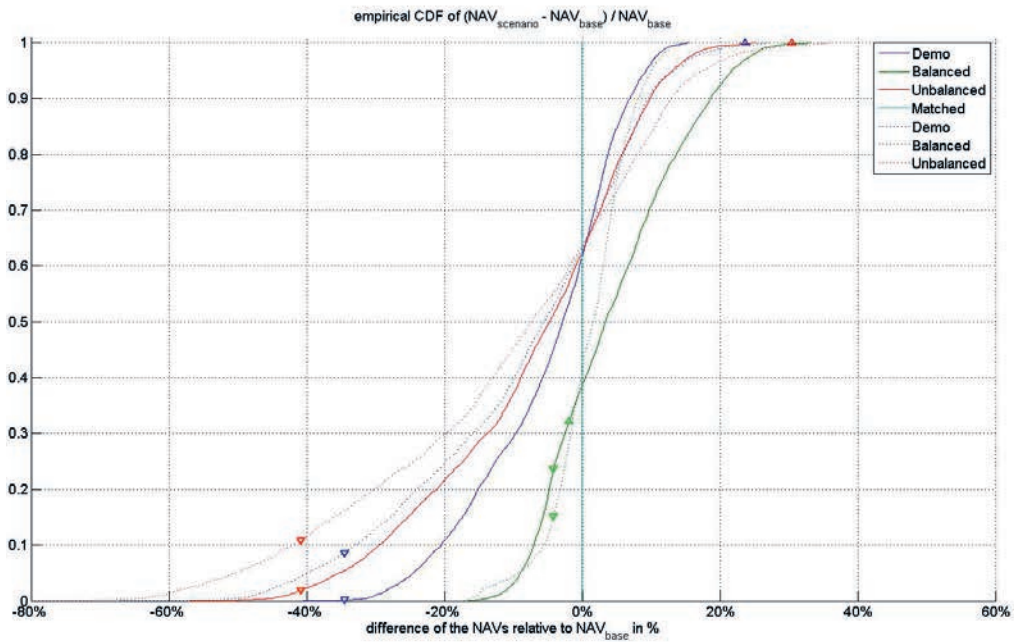
Chart 5: *Distribution of absolute changes in NAV over a rolling 12 month period*



Source: Authors' calculations.

In order to analyze the effects of the changes in the interest rate term structure, an empirical cumulative distribution function of the changes in the NAV can be constructed. Formally an empirical cumulative distribution function is constructed for the values of $NAV_{\text{scenario}} - NAV_{\text{base}}$. Chart 5 shows the empirical cumulative distribution function for absolute values of the change in the NAV. The results for the scenarios where the term structure of the interest rate are obtained with linear interpolation are shown as solid lines, the results for the interpolation with the Smith-Wilson method are shown as dashed lines.

Chart 6: *Distribution of relative changes in NAV over a rolling 12 month period*



Source: Authors' calculations.

Chart 6 shows a similar empirical cumulative distribution function, but for relative values of the change in the NAV, formally the empirical distribution function of the values of $(NAV_{\text{scenario}} - NAV_{\text{base}}) / NAV_{\text{base}}$. The results for the scenarios where the term structure of the interest rate are obtained with linear interpolation are shown as solid lines, the results for the interpolation with the Smith-Wilson method are shown as dashed lines.

In both pictures, the absolute and relative change in the net asset value for the interest rate shock in the standard approach are included as triangles. The triangle pointing upside represents the outcome of the shock of interest rates moving up; the triangle pointing downside represents the outcome of the downside shock.

The construction of the empirical cumulative distribution function allows for the empirical calculation of the regulatory relevant value-at-risk 99.5%. Table 1 shows the empirically calculated value-at-risks 99.5% in EUR, both for the linear interpolation of the interest yield curve and the Smith-Wilson interpolation method. The last column shows the corresponding results for the standard approach as given by the down shocks defined for QIS6.

Table 1: Absolute value-at-risk comparison for various methods of calculation

	VaR 99.5% in EUR linear interpolation	VaR 99.5% in EUR Smith-Wilson	VaR 99.5% in EUR standard approach
Demo	-203.040	-324.881	-223.187
Balanced	-88.588	-104.271	-26.728
Unbalanced	-291.730	-414.443	-264.608
Matched	0	0	0

Source: Authors' calculations.

It can easily be seen that the linear interpolation method roughly results in the same value-at-risk values as the standard approach especially for the *Demo* and *Unbalanced* cash flow. The value-at-risk 99.5% for the cash flow *Balanced* differs much more, but from a much lower level. In contrast to that the empirical results, obtained with Smith-Wilson interpolation, indicate a much higher level of risk. Table 2 shows the same results for relative values.

Table 2: Relative value-at-risk comparison for various methods of calculation

	VaR 99.5% in % of NAV linear interpolation	VaR 99.5% in % of NAV Smith-Wilson	VaR 99.5% in % of NAV standard approach
Demo	-31,3%	-50,2%	-34,5%
Balanced	-13,7%	-16,1%	-4,1%
Unbalanced	-45,0%	-64,0%	-40,9%
Matched	0,0%	0,0%	0,0%

Source: Authors' calculations.

6 Conclusion

The results outlined above give rise to three main conclusions.

Firstly, it indicates that the overall calibration of the standard approach seems to be underestimating the potential effect of interest rate changes, mainly due to its design as a shock relative to the current level of rates. Opposite to this, especially in recent years the movement of rates has been very strong already on an absolute level.

Secondly, it can be concluded that a lot of influence comes from technical aspects such as the interpolation method. Especially for poorly balanced cash flows

the effect of linear versus other interpolation methods can be of the magnitude of 20% of NAV.

Thirdly, and as the main result, we find that not only parallel shifts have to be considered when assessing the risk arising from changes in interest rates. Paradox, the relative risk estimation error between parallel (standard) approach and historic variation is higher for assumedly hedged balance sheets, in our analysis represented by the balanced cash flow profile. Whereas an alleged insensitivity towards rate movement as measured by the duration sensitivity measure would impose very little capital requirements on a company, the variation in NAV and thus own funds can be significantly higher in reality.

These findings suggest that further studies and more detailed data on the cash flow profiles of insurance companies and their sensitivities to interest rates as well as an impact study on technical specifications would be needed in order to properly assess the risk arising from adverse movements of the risk free interest rate curve.