

# New Quantitative Models of Banking Supervision



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## Preface

A reliable and financially sound banking sector is an essential prerequisite for a country's stability and economic growth. For this reason, monitoring and examining the financial situation of banks is of great interest to regulatory authorities throughout the world. Regulators – even within the EU – use a variety of approaches to attain this goal, which is due to the different structures not only of the authorities themselves but even more so of the financial centers (in particular the number of banks). As on-site audits require substantial amounts of time and resources and cannot be carried out very frequently due to the large number of banks in Austria, off-site analyses play a major role in the supervision process. Therefore, the Oesterreichische Nationalbank (OeNB) and the Austrian Financial Market Authority (FMA) place great emphasis on developing and implementing sophisticated, up-to-date off-site analysis models to make full use of the resources of both institutions.

The analysis tools used so far turned out to be successful and will continue to exist in a slightly modified form. However, shifting the focus to risk-based supervision has made it necessary to concentrate on the individual risks per se in certain segments and thus to review and expand off-site analysis with regard to risk-based aspects.

As a result, the OeNB and the FMA decided – with university support – to advance the Austrian analysis landscape in a fundamental manner. This publication contains an overview of the new core models; a detailed description of the entire analysis landscape will be made available at the beginning of next year.

We are especially grateful to the employees of both institutions who were involved in the project in general and this publication in particular and who distinguished themselves with their expertise and dedication.

That said, we hope we have aroused your interest with this publication on "New Quantitative Models of Banking Supervision".

Vienna, July 2004



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# *Introduction*

Off-site analysis can use the following methods in the assessment of banks: (i) a simple observation and analysis of balance-sheet indicators, income statement, and other indicators from which the deterioration of a bank's individual position can be derived by experts in an early-warning system ("supervisory screens"); and (ii) a statistical (econometric) analysis of these indicators (or general exogenous variables) that makes it possible to estimate a bank's probability of default or its rating.

This publication describes approaches of the latter category. Specifically, we have selected statistical methods of off-site bank analysis which use econometric methods as well as structural approaches in an attempt to realize, assess, and generally quantify problematic bank situations more effectively.

The first part of this publication describes the procedures selected from the class of statistical models. Using logit regression, it is possible to estimate the probabilities of occurrence for certain bank problems based on highly aggregated indicators gathered from banking statistics reports. Building on those results, a Cox model can be used to compute the distance to default in order to determine the urgency of potential measures.

The second part of this publication deals with the development of a structural model for all Austrian banks. While statistical models forecast a bank's default potential by observing indicators closely correlated with the event, the structural approach is meant to offer an economic model which can explain causal relations and thus reveal the source of risks in order to enable an evaluation of the reasons underlying such problematic developments. Initial attempts to do so using market-based approaches (stock prices, interest-rate spreads) were rejected due to data restrictions. In the end, the decision to model the most important types of risks (credit, market, and operational risks) and to compute individual value at risk proved to be rewarding.

In this document, we will describe the methods, data input, results and also necessary assumptions and simplifications used in modeling. The potential analyses in the structural model are manifold and range from classic coverage capacity calculation (comparison of reserves and risks per defined default probability) to the calculation of expected and unexpected losses (including the related calculation of economic capital) and the possibility of simulating changes by altering input parameters (e.g., industry defaults, interest rate changes, etc.).

# STATISTICAL MODEL

## METHODS AND CALCULATIONS

# Statistical Model – Methods and Calculations

## Introduction

Statistical models of off-site analysis involve a search for explanatory variables that provide as sound and reliable a forecast of the deterioration of a bank's situation as possible. In this study, the search for such explanatory variables was not merely limited to the balance sheets of banks.<sup>1</sup> The entire supervisory reporting system was included, and macroeconomic indicators were also examined as to their ability to explain bank defaults. In a multi-step procedure, a multitude of potential variables were reduced to those which together showed the highest explanatory value with regard to bank defaults in a multivariate model.

In selecting the statistical model, the choice was made to focus on developing a logit model. Logit models are certainly the current standard among off-site analysis models, both in their practical application by regulators and in the academic literature. The results produced by such models can be interpreted directly as default probabilities, which sets the results apart from the output of CAMEL ratings,<sup>2</sup> for example (in the Austrian CAMEL rating system, banks are ranked only in relation to one other, and it is not possible to make statements concerning the default probability of a single bank).

One potential problem with the logit model (and with regression models using cross-section data in general) is the fact that such approaches do not directly take into account the time at which a bank's default occurs. This disadvantage can be remedied by means of the Cox model, as the hazard rate is used to estimate the time to default explicitly as an additional component in the econometric model. For this reason, a Cox model was developed to accompany the logit model.

The basic procedure used to develop the logit model is outlined below; a detailed description will be given in the sections that follow.

The first step involved collecting, preparing, and examining the data. For this purpose, the entire supervision reporting system, the Major Loans Register, and external data such as time series of macroeconomic indicators were accessed. These data were combined in a database which is managed by suitable statistical software.

Next, the data were aggregated and the indicators were defined. The Major Loans Register was also included in connection with data from the Kreditschutzverband von 1870 (KSV), and other sources. Overall, 291 ratios were computed and then subjected to univariate tests.

This was followed by extensive quality control measures. It was necessary to examine the individual indicators to check, for example, whether the values were within certain logical ranges, etc. Some indicators were transformed if this proved necessary for procedural reasons.

Next, estimation and validation samples were generated for the subsequent univariate and multivariate modeling in order to enable verification of the logit model's predictive power.

<sup>1</sup> For simplicity's sake, the terms "defaults" and "default probabilities" will be used in this document to refer to events in which banks suffer problems which make it doubtful whether they could survive without intervention (e.g. industry subsidies).

<sup>2</sup> CAMEL is short for Capital, Assets, Management, Earnings, and Liquidity. For a detailed description, please refer to Turner, Focus on Austria 1/2000.

The predictive power of one indicator at a time was examined in the univariate tests. Then, only those variables which showed particularly high univariate discriminatory power were used in the subsequent multivariate tests. The test statistics used to measure the predictive power of the various indicators were the Accuracy Ratio (AR), also referred to as the Gini coefficient, as well as the area under the Receiver Operating Characteristic curve (AUROC).

In order to avoid a distortion of the results due to collinearity, the mutual pair wise correlations of all ratios were determined. Of those indicators which showed high correlation to one another, only one indicator could be used for multivariate analysis.

Finally, backward and forward selection procedures were used to determine the indicators for the final model. This was followed by an iterative procedure to eliminate ratios of low significance. The model resulted in the selection of 12 indicators including a dummy variable which enables to generate in-sample and out-of-sample AUROCs of about 82.9% and 80.6%, respectively.

Developing the Cox model required steps analogous to those described above for the logit model. In order to get a first impression of the Cox model's capabilities, a traditional Cox proportional hazard rate model was developed on the basis of the results derived from the logit model. The results, which are explained in detail below, show that even this simple model structure is able to differentiate between the average survival periods of defaulting and non-defaulting banks. Nevertheless, the decision was made to develop a more complex Cox model in order to improve on several problem areas associated with the traditional model. The final model using this structure should be available in 2005.

The rest of this chapter is organized as follows: Section 1 briefly introduces alternative methods of statistical off-site analysis, describes their pros and cons, and explains why such methods were included or excluded in the course of the research project. Section 2 then describes the data pool created, after which Sections 3 and 4 discuss the development and evaluation of the logit model, respectively; Section 5 deals with the Cox model. Finally, Section 6 concludes the discussion of statistical models with a brief summary.

## I Theory

In this document, we define a statistical model as that class of approaches which use econometric methods for the off-site analysis of a bank. Statistical models of off-site analysis primarily involve a search for explanatory variables which provide as sound and reliable a forecast of the deterioration of a bank's situation as possible. In contrast, structural models explain the threats to a bank based on an economic model and thus use clear causal connections instead of the mere correlation of variables.

This section offers an overview of the models taken into consideration for off-site statistical analysis throughout the entire selection process. This includes not only purely statistical or econometric methods (including neural networks), but also computer-assisted classification algorithms. Furthermore, this section discusses the advantages and disadvantages of each approach and explains the reasons why they were included or excluded.

In general, a statistical model may be described as follows: As a starting point, every statistical model uses characteristic bank indicators and macroeconomic variables which were collected historically and are available for defaulting (or troubled) and non-defaulting banks.

Let the bank indicators be defined by a vector of  $n$  separate variables  $X = (X_1, \dots, X_n)$ .

The state of default is defined as  $Z = 1$  and that of continued existence as  $Z = 0$ . The sample of banks now includes  $N$  institutions which defaulted in the past and  $K$  institutions which did not default. Depending on the statistical application of this data, a variety of methods will be used.

The classic methods of discriminant analysis generate a discriminant function which is derived from the indicators for defaulting and non-defaulting banks and which is used to assign a new bank to the class of either “healthy” or troubled banks based on its characteristics (i.e. its indicators). The method of logit (probit) regression derives a probability of default (PD) from the banks’ indicators. The proper interpretation of a PD of 0.5% is that a bank with the characteristics  $(X_1, \dots, X_n)$  has a probability of default of 0.5% within the time horizon indicated. This time horizon results from the time lag between the recording of bank indicators and of bank defaults. Using those default probabilities, the banks can be assigned to different rating classes. In addition to the estimation of default probabilities it is also possible to estimate the expected time to default. With these model types, it is possible to estimate not only the PD but also the distance to default.

## 1.1 Discriminant Analysis

Discriminant analysis is a fundamental classification technique and was applied to corporate bankruptcies by Altman as early as 1968 (see Altman, 1968). Discriminant analysis is based on the estimation of a discriminant function with the task of separating individual groups (in the case of off-site bank analysis, these are defaulting and non-defaulting banks) according to specific characteristics. The estimation of the discriminant function adheres to the following principle: *Maximization of the spread between groups and minimization of the spread within individual groups*.

Although many research papers use discriminant analysis as a comparative model, the following points supported the decision against its application:

- Discriminant analysis is based on the assumption that the characteristics are normally distributed and that the discriminant variable shows multivariate normality. This is, however, not usually the case for the characteristics observed.
- When using a linear discriminant function, the group variances and covariances are assumed to be identical, which is also usually not the case.
- The lack of statistical tests to assess the significance of individual variables increases the difficulty involved in interpreting and evaluating the resulting model.
- Calculating a default probability is possible only to a limited extent and requires considerable extra effort.

## 1.2 Logit and Probit Models

Logit and probit models are econometric techniques for the analysis of 0/1 variables as dependent variables. The results generated by these models can be interpreted directly as default probabilities. A comparison of discriminant analysis and regression model shows the following:

- coefficients are easier to estimate in discriminant analysis;
- regression models yield consistent and sound results even in cases where the independent variables are not distributed normally.

We will now give a summary of the theoretical foundations of logit and probit models based on Maddala (1983).

The starting point for logit and probit models is the following simple, linear regression model for a binary-coded dependent variable:

$$y_i = \beta x_i + u_i$$

In this specification, however, there is no mechanism which guarantees that the values of  $y$  estimated by means of a regression equation are between 0 and 1 and can thus be interpreted as probabilities.

Logit and probit models are based on distributional assumptions and model specifications which ensure that the dependent variable  $y$  remains between 0 and 1. Specifically, the following relationship is assumed for the default probability:

$$y_i^* = \beta x_i + u_i$$

However, the variable  $y^*$  cannot be observed in practice; what can be observed is the specific defaults of banks and the accordingly defined dummy variable

$$y = 1 \text{ if } y_i^* > 0$$

$$y = 0 \text{ otherwise}$$

The resulting probability can be computed as follows:

$$P(y_i = 1) = P(u_i > -\beta x_i) = 1 - F(-\beta x_i)$$

The distribution function  $F(\cdot)$  depends on the distributional assumptions for the residues ( $u$ ). If a normal distribution is assumed, we are faced with a probit model:

$$F(-\beta x_i) = \int_{-\infty}^{-\beta x_i/\sigma} \frac{1}{(2\pi)^{1/2}} \exp\left(\frac{-t^2}{2}\right) dt$$

However, several problems arise in the process of estimating this function, as beta and sigma can only be estimated together, but not individually. As the normal and the logistic distribution are very similar and only differ at the distribution tails, there is also a corresponding means of expressing the residues' distribution function  $F(\cdot)$  based on a logistic function:

$$F(-\beta x_i) = \frac{\exp(-\beta x_i)}{1 + \exp(-\beta x_i)}$$

$$1 - F(-\beta x_i) = \frac{1}{1 + \exp(-\beta x_i)}$$

This functional connection is now considerably easier to handle than that of a probit model. Furthermore, logit models are certainly the current standard among off-site analysis models, both in their practical application by regulators and in the academic literature. For the reasons mentioned above, we decided to develop a logit model for off-site analysis.

### 1.3 Cox Model

One potential problem with regression models using cross-section data (such as the logit model) is the fact that such approaches do not explicitly take into account the survival function and thus the time at which a bank's default occurs. This disadvantage can be remedied by means of the Cox Proportional Hazard model (PHM), as the hazard rate is used to estimate the time to default explicitly as an additional component in the econometric model. On the basis of these characteristics, it is possible to use the survival function to identify all essential information pertaining to a bank default with due attention to additional explanatory variables (i.e. with due consideration of covariates). In general, the following arguments support the decision to use a Cox model:

- In contrast to the logit model not only the default probability for a bank for a certain time period is modeled, but from the time structure of the historical defaults a survival function is estimated for all banks (i.e. the stochastics of the default events are modeled explicitly).
- As covariates are used in estimating the survival function, it is possible to group the individual banks using the relevant variables and to perform a statistical evaluation of the differences between these groups' survival functions. Among other things, this makes it possible to compare the survival functions of different sectors with each other.

The Cox model is based on the assumption that a bank defaults at time  $T$ . This point in time is assumed to be a continuous random variable. Thus, the probability that a bank will default later than time  $t$  can be expressed as follows:

$$\Pr(T > t) = S(t)$$

$S(t)$  is used to denote the survival function. The survival function is directly related to the distribution function of the random variable  $T$ , as

$$\Pr(T \geq t) = F(t) = 1 - S(t)$$

where  $F(t)$  is the distribution function of  $T$ . Thus, the density function at the time of default is expressed as  $f(t) = -S'(t)$ . Based on the distribution and density functions of the time of default  $T$ , we can now define the hazard rate, which is represented by

$$h(t) = \frac{f(t)}{1 - F(t)}$$

By transforming this relation, we arrive at the following interpretation: The hazard rate shows the probability that a bank which has survived until time  $t$  will default momentarily:

$$h(t) = \lim_{\Delta t \rightarrow \infty} \frac{Pr(t < T < t + \Delta t | T > t)}{\Delta t} = \frac{S'(t)}{S(t)} = \frac{f(t)}{(1 - F(t))}$$

Estimating the expected time of a bank's default using the hazard rate offers decisive advantages compared to using the distribution and density functions [ $F(t)$ ] and [ $f(t)$ ], respectively (see, for example, Cox and Oakes (1984) or Lawless (1982)). Once the hazard rate has been estimated statistically, it is easy to derive the distribution function:

$$F_t = 1 - \exp \left( - \int_0^t h(s) ds \right)$$

Thus the density function can also be derived with relative ease.

Cox (1972) then builds on the model of hazard rate  $h(t)$ , but assumes that the default probability of an average bank depends also on explanatory variables. Using the terms above, the hazard rate is now defined as  $h(t|x)$ , with  $x$  being a vector of exogenous explanatory variables measured as deviations from the means. We thus arrive at a PHM of

$$h(t|x) = h_0(t)\rho(x)$$

where  $\rho(x)$  is a function of the explanatory variable  $x$ . If we then assume that the function  $\rho(x)$  takes on a value of 1 for an average bank (i.e.  $x = 0$ ) , that is,  $\rho(0) = 1$ , we can interpret  $h_0(t)$  as the hazard rate of an average bank.  $h_0(t)$  is also referred to as the base hazard rate.

In his specification of the PHM, Cox assumes the following functional form for  $\rho(x)$ :

$$\rho(x) = \exp(\beta_1 x_1 + \beta_2 x_2 + \dots + \beta_n x_n)$$

where  $\beta = (\beta_1, \dots, \beta_n)$  represents the vector of the parameter to be estimated and  $x = (x_1, \dots, x_n)$  represents the vector of n explanatory variables.

For the complete Cox model, this yields the following hazard rate:

$$h(t|x) = h_0(t)\exp(\beta_1 x_1 + \dots + \beta_n x_n)$$

It is now possible to derive the survival function from the function  $h(t)$ . We get:

$$S(t, x) = [S_0(t)]^{\exp(\beta x)}$$

where  $S_0(t)$  is the base survival function derived from the cumulative hazard rate.

In particular, it can be argued that an estimate of the survival function for troubled banks yields important information for regulators. Due to this explicit estimation of the survival function and the resulting fact that the time of default is taken into account, it was decided to develop a Cox model in addition to the logit model for off-site bank analysis.

## 1.4 Neural Networks

In recent years, neural networks have been discussed extensively as an alternative to linear discriminant analysis and regression models as they offer a more flexible design than regression models when it comes to representing the connections between independent and dependent variables. On the other hand, using neural networks also involves a number of disadvantages, such as:

- the lack of a formal procedure to determine the optimum network topology for a specific problem;
- the fact that neural networks are a black box, which makes it difficult to interpret the resulting network; and
- the problem that calculating default probabilities using neural networks is possible only to a limited extent and with considerable extra effort.

While some empirical studies do not find any differences regarding the quality of neural networks and logit models (e.g. Barniv et al. (1997)), others see advantages in neural networks (e.g. Charitou and Charalambous (1996)). However, empirical results have to be used cautiously in choosing a specific model, as the quality of the comparative models always has to be taken into account as well.

Those disadvantages and the resulting project risks led to the decision not to develop neural networks.

## 1.5 Computer-based Classification Methods

A second category of computer-based methods besides neural networks comprises iterative classification algorithms and decision trees. Under these methods, the base sample is subdivided into groups according to various criteria. In the case of binary classification trees, for example, each tree node is assigned (usually univariate) decision rules which describe the sample accordingly and subdivide it into two subgroups each. The training sample is used to determine these decision rules. New observations are processed down the tree in accordance with the decision rules' values until a end node is reached, which then represents the classification of this observation.

As with neural networks, decision trees offer the advantage of not requiring distributional assumptions. However, decision trees only enable calculation of default probabilities for a final node in a tree, but not for individual banks. Furthermore, due to a lack of statistical testing possibilities, the selection process for an “optimum” model is difficult and risky also for these approaches. For the reasons mentioned above it was decided not to use such algorithms for off-site analysis in Austria.

## 2. Database

### 2.1 Data Retrieval and Preparation

A wide variety of data sources was used to generate the database. The following figure provides an overview:

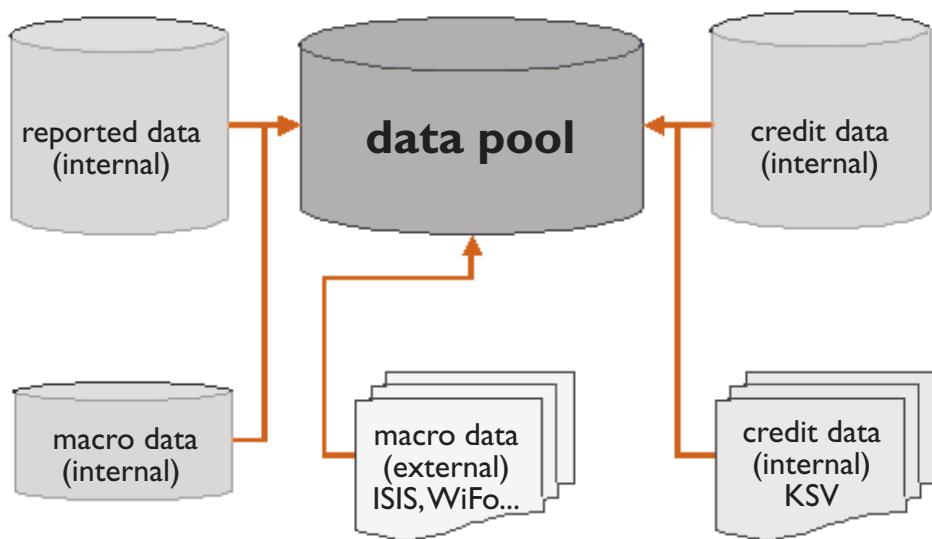


Figure 1: Data sources used

The data from regulatory/supervisory reports were combined with data from the Major Loans Register and external data (such as time series of macroeconomic indicators) and incorporated in a separate database. The data were analyzed on a quarterly basis, resulting in 33,000 observations for the 1,100 banks licensed in the entire period of 30 quarters under review (December 1995 to March 2003).

Data recorded only once per year (e.g. balance sheet items) needed to be aligned with data recorded throughout the year, which made it necessary to carry certain items forward and backward. Items cumulated over a year were converted to net quarters (i.e. increases or decreases compared to the previous quarter), and all macro variables were adjusted for trends by conversion to percentage changes if necessary.

#### 2.1.1 Major Loans Register Data

In essence, the structure of a bank's loan portfolio can only be approximated using the Major Loans Register. Pursuant to §75 of the Austrian Banking Act, credit and financial institutions are obliged to report major loans to the Oesterreichische Nationalbank. This reporting obligation exists if credit lines granted to or utilized by a borrower exceed EUR 350,000. The Major Loans Register thus covers about 80% of the total loan volume of Austrian banks, but its level of individual coverage may be lower, especially in the case of smaller banks.

Data extraction started in December 1995 and was performed every quarter. Between 71,000 and 106,000 observations were evaluated per quarter. Up to 106,000 observations per quarter (with a minimum of 71,000 observations) were analyzed.

The data reported in the past (such as credit lines and utilization) were recently expanded to include ratings, collateral, and specific loan loss provisions. Due to insufficient historical data, however, this new data cannot (yet) be integrated into the statistical model comprehensively.

### **2.1.2 KSV Data**

To analyse the Major Loans Register it was necessary to gather insolvency data for the various industries and provinces and to compare that data with the corresponding exposures for the period under review.

The Kreditschutzverband von 1870 (KSV) was selected as the source of insolvency data with the required level of accumulation for all industries and provinces. The main problem in this context proved to be the comparison of industries and the definition of industry groups in Major Loans Register during the observation period versus the KSV definition. Thus, 20 KSV industry groups had to be mapped to the 28 industry groups defined by the OeNB. The higher level of aggregation applied by the KSV meant that some (industry) data were lost in the computation of certain indicators. Ultimately, however, none of those indicators were used in the final model.

### **2.1.3 Macro Data**

As far as macroeconomic risks were concerned, it was possible to use previously existing papers in the field of stress testing,<sup>3</sup> but despite the availability of this input it was necessary to retrieve the data sets again for the required period and to expand the list of indicators.

Although the time series were disrupted several times as a result of legal and normative changes to the national accounting system, it was possible to use a range of macroeconomic data from various databases. The availability of regional indicators proved to be a particular problem in this context.

In general, the main problem in considering macroeconomic risks consists not only in the availability of the data, but also in the selection of the relevant macroeconomic variables. The factors used in Boss's study,<sup>4</sup> "A Macroeconomic Credit Risk Model for Stress Testing the Austrian Credit Portfolio", formed the basis for our selection of several variables, including indicators on economic activity, price stability, households, and businesses, as well as stock market and interest rate indicators. Other sources were used to include further core data such as other price developments (real estate prices) or regional data (regional industrial production, unemployment rates, etc.). In particular, these data sources were: WISO of the Austrian Institute of Economic Research – Wifo-WSR-DB, VDB of the Oesterreichische Nationalbank and ISIS by Statistics Austria.

<sup>3</sup> See Boss (2002) and Kalirai/Scheicher (2002).

<sup>4</sup> OeNB, *Financial Stability Report 4* (2002).

## **2.2 Data Aggregation and Indicators**

The items defined above were now used to define indicators. The Major Loans Register was integrated in connection with KSV data (among other things) by defining 21 indicators, while the macro variables were usually taken directly as relative change indicators. Overall, 291 indicators were defined and then subjected to initial univariate tests.

Individual subgroups were formed and the indicators were assigned in order to account for various bank risks or characteristics. The total of 291 indicators can be assigned to the subgroups as follows:

<b>field</b>	<b>number</b>
bank characteristics	38
credit risk	56
credit risk based on Major Loans Register	21
capital structure	22
profitability	47
market risk	12
liquidity risk	15
operational risk	11
reputational risk	6
quality of management	14
macroeconomics	49

Extensive quality control measures were taken after the indicators had been defined. First of all, the observation of logical boundaries was tested, after which the distributions (particularly at the tails) were examined and corrected manually as necessary. Furthermore, the indicators were regressed on the empirical default probability and the log odd, and the result was analyzed visually (graphically) (see Section 3.1).

## **3 Development of the Logit Model**

### **3.1 Transformation of Input Variables**

The logit model assumes a linear relation between the log odd, the natural logarithm of the default probability divided by the survival probability (i.e.  $\ln[p/(1-p)]$ ) and the explanatory indicators (see Section 1.2). However, as this relation does not necessarily exist empirically, all of the indicators were examined in this regard. For this purpose, each indicator was subdivided into groups of 800 observations, after which the default or problem probabilities respectively the log odd were computed for each of these groups. These values were then regressed on the original indicators, and the result was depicted in graphic form. In addition to the graphical output, the R<sup>2</sup> from the linear regressions was used as a measure of linearity. Several indicators turned out to show a clear non-linear and non-monotonous empirical relation to the log odd.

As the assumption of linearity was not fulfilled for these indicators, they had to be transformed before their explanatory power in the logit model could be examined. This linearization was carried out using the Hodrick Prescott filter,

which minimizes the squared distance between the actual ( $y_i$ ) and the smoothed observations ( $g_i$ ) under the constraint that the curve should be “smooth”, that is, that changes in the gradient should also be minimized. The value of smoothness now depends on the value of  $\lambda$ , which was set at 100.

$$\text{Min}_{(g_i)} \sum_i (y_i - g_i)^2 + \lambda \sum_i [(g_i - g_{i-1}) - (g_{i-1} - g_{i-2})]^2$$

Once the indicators had been transformed, their actual values were replaced with the empirical log odds obtained in the manner described above for all further analyses.

### 3.2 Determination of Data Set for Model Estimation

The following problem arises in estimating both the univariate and the multivariate models: The underlying data set contains a very low number of defaults, as only 4 defaults were recorded in the observation period.

Therefore, the following procedure was chosen: Basically, a forecasting model can produce 2 types of errors – sound banks can mistakenly be classified as troubled, or troubled banks may be wrongly deemed sound. As the latter type of error has more severe effects on regulation, misclassifications of that type must be minimized.

In this context, one option is to increase the number of defaults in the estimation sample, for example by moving from defaulting banks to troubled banks. This seems especially useful as the essential goal of off-site analysis is the early recognition of troubled banks, not only the prediction of defaults per se. Furthermore, the project team made the realistic assumption that a bank which defaults or is troubled at time  $t$  will already be in trouble at times  $t-2$  and  $t-1$  (2 and 1 quarters prior to default, respectively), and may still be a weak bank at  $t+1$  and  $t+2$  (after receiving an injection of capital or being taken over). This made it possible to construct a data set which featured a considerably higher number of “defaults”: as a result of this approach, the defaults were “over-weighted” asymmetrically compared to the non-defaults. An analogous result could be obtained if an asymmetric target function was used in estimating the logit model. The advantage of this asymmetry is that the resulting estimate reduces the potential error of identifying a troubled bank as sound. However, this approach distorts the expected probability of default. When the estimated model is used, this distortion can then be remedied in a separate step by means of an appropriate calibration.

As shown in the sections to follow, increasing the number of defaults in the sample also makes it possible to conduct out-of-sample tests in addition to estimating the model. However, the distortion resulting from this increase must then be corrected accordingly when estimating the default probabilities.

### 3.3 Definition of Estimation and Validation Samples

In the process of estimating statistical models, one usually tries to explain the dependent variable (here: the default of banks) by means of the independent variables as accurately as possible. However, as the logit model is designed for forecasting defaults, it is important to make sure that the statistical correlations found can be applied as widely as possible and are not too peculiar

to the data sample used for estimation (i.e. to find a model which lends itself to generalization). This is best achieved by validating the predictive power of the resulting models with data which were not used in estimating the model. For this reason, it was necessary to subdivide the entire database into an estimation and a validation sample. The fundamental condition which had to be met at all times was the existence of a sufficient number of defaults in both subsamples. In addition, as both subsamples were to reflect the Austrian banking sector, each of the seven principal sectors was again subdivided into large and small banks. Subsequently, 70% of all defaults and 70% of all non-defaults were randomly drawn from each of the resulting bank groups for the estimation sample, while the remaining 30% were used as validation sample.

### 3.4 Estimation of Univariate Models

The univariate tests mentioned earlier examined the predictive power of one indicator at a time. Then, only those variables which showed particularly high univariate discriminatory power were used in the subsequent multivariate tests.

The Accuracy Ratio (AR) used in finance and/or the Receiver Operating Characteristic curve concept (ROC) developed in the field of medicine could serve as test statistics for the predictive power of the various indicators. As proven in Engelmann, Hayden, and Tasche (2003), the two concepts are equivalent.

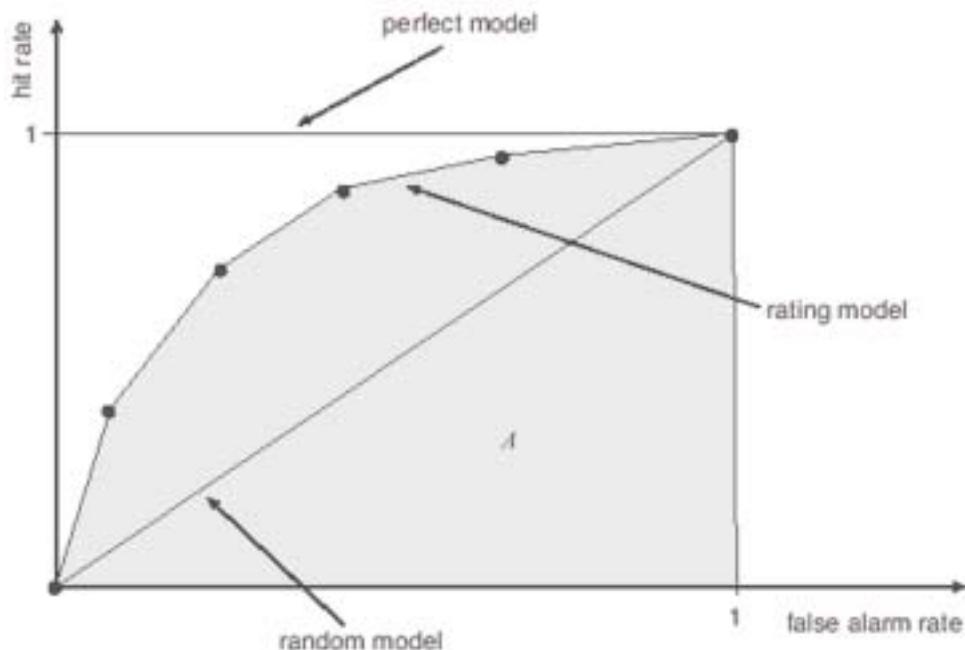


Figure 2: The ROC Model

The ROC curve concept is visualized in Figure 2. A univariate logit model used to assign a default probability to all banks is estimated for the input ratio to be tested. If, based on the probability of default, one now has to predict whether a bank will actually default or not, one possibility is to determine a certain cut-off threshold C and to classify all banks with a default probability higher than C

as defaults and all banks with a default probability lower than C as non-defaults. The hit rate is the percentage of actual defaults which were correctly identified as such, while the false alarm rate is the percentage of sound banks erroneously classified as defaults. The ROC curve is a visual depiction of the hit rate vs. the false alarm rate for all possible cut-off values. The area under the curve represents the goodness-of-fit measure for the tested indicator's discriminatory power. A value of 1 would indicate that the ratio discriminates defaults and non-defaults perfectly, while a value of  $\frac{1}{2}$  signifies a indicator without any discriminatory power whatsoever.

### 3.5 Estimation of Multivariate Models

In order to avoid a distortion of the results due to collinearity, the pair wise correlations of all indicators with each other were determined first. This was followed by an examination of the ratios in the various risk groups (bank characteristics, credit risk, etc.) as to whether subgroups of highly correlated indicators could be formed within those groups. Of those ratios which show high correlation, only one indicator can be used for the multivariate analysis.

In order to estimate the multivariate model, various sets of indicators were defined and used in calculations which each followed a specific procedure. The comparison of the results obtained in this way made it possible to identify a stable core of indicators which were then used to conduct further multivariate analyses. Finally, by integrating a dummy variable which depicts sectors in aggregated form, a multivariate model consisting of 12 input variables in all (including the dummy variable) was generated.

The following three steps were taken to select the variables possible for the multivariate model:

- a. Identification of the indicators with the highest discriminatory power in the univariate case
- b. Consideration of the correlation structure and formation of correlation groups
- c. Consideration of the number of missing values (missings, e.g. due to a reduced time series) per indicator

These three steps were used to generate a shortlist which served as the starting point for the multivariate model. Using the shortlist, a between-group correlation analysis was conducted, as only correlations within a risk group had been examined thus far: the pair wise correlations were analyzed for all indicators on the shortlist. In order to create a stable multivariate model, the following procedure was used to generate four (partly overlapping) starting sets for further calculation.

Those indicators which were highly correlated were grouped together. The indicators combined in this manner were used to decide which of the highly correlated indicators were to be used to the multivariate model. The criteria used in the decision to prefer certain indicators were the following:

- indicators from a sector which was otherwise under-represented;
- ratios generated from a numerator or a denominator not commonly used in the other indicators;
- the univariate AR value;

- the interpretability of the correlation with the defaults (whether the positive or negative correlation could be explained);
- the general interpretability of the indicator;\*
- the number of missings and the number of null reports.

Based on the indicators selected in this way for one of the four starting sets, a run of the multivariate model was performed: using the routines of forward and backward selection implemented for logistic regression in STATA, those indicators which were not significant were eliminated from the starting sets. In a further step, the results were analyzed as to the plausibility of the algebraic signs of the estimated coefficients: economically implausible signs suggested hidden correlation problems.

Ultimately, the four starting sets formed the basis for four multivariate models which could be compared to each other in terms of common indicators, size of the AUROC, highest occurring correlation of the variables with one another, the traditional pseudo-R<sup>2</sup> goodness-of-fit measure, and the number of observations evaluated. It showed that about 20 indicators prevailed in at least half of the trial runs, which means that they were adopted as explanatory indicators in a multivariate model, with each tested indicator being represented in at least two of the four starting sets.

These remaining indicators were then used as a new starting set for further calculations, and the procedure above was repeated.

In the next step, it was necessary to clarify whether the model could be improved further by incorporating dummy variables. Various dummy variables reflecting size, banking sector, and regional structure (i.e., Austrian provinces) were tested. Ultimately, only sector affiliation proved to be relevant. Eventually, aggregating the sectors to obtain two groups turned out to be the key to success (in connection with the 11 indicators selected): Group 1 includes building and loan associations and special-purpose banks, while the second group covers all the other sectors.

Finally, the remaining indicators (including the dummy variable relating to the aggregated sectors) were subjected to further stability tests. The model which ultimately proved most suitable with regard to the various criteria consists of a total of 12 indicators covering the following areas:

field	number
bank characteristics	1
credit risk (incl. Major Loans Register)	4
capital structure	2
profitability	4
macroeconomics	1

This is the final model depicting the multivariate logistic regression in the conceptualization phase and at the same time serves as the basis for the Cox model outlined in Section 6. The explanatory power (measured in terms of AUROC) amounts to 82.9% in-sample and 80.6% out-of-sample, with a pseudo-R<sup>2</sup> of 21.3%. These ranges are compatible with those reported in academic publications and by regulators.

### 3.6 Calibration

Although the output produced by the logit model goes beyond a relative ranking by estimating probabilities, these probabilities need to be calibrated in order to reflect the actual “default probability” of Austria’s banking sector accurately (see the section on designing the estimation and validation samples). This is due to the fact that the “default” event was originally defined as broadly as possible in order to provide a sufficient number of defaults, which is indispensable in developing a discriminatory model and, even more importantly, in minimizing the potential error of classifying a troubled bank as sound. However, the logit model on average reflects those “default probabilities” which are found empirically in calculating the model (this property in turn reflects the unbiasedness of the logit model’s estimators). It is now the goal of calibration to approximate this relatively high “problem probability” to the actual default probability of banks.

As a rule, it is impossible to achieve an exact calculation of default probabilities, as the “default” event in the banking sector cannot be measured precisely: actual defaults occur very rarely, and subsidies, mergers, and declining results and/or equity capital are only indicators of potentially major problems in the banks in question but do not unambiguously show whether the banks are viable or not. As the task at hand is to identify troubled banks and not to forecast bank defaults in the narrow sense of the term, it is not necessary to calibrate to actual default probabilities.

In technical terms, for example, calibration would be possible by adapting the model constant in order to shift the mean of the logit distribution determined (expected value) to the desired value. For the reasons mentioned above, various alternatives may be used as the average one-year default probabilities (or: the probabilities that a bank will encounter severe economic difficulties within one year) to be supplied by the model after the calibration. The figure below shows these probabilities for a selected bank on three different levels referred to as Alternative a), Alternative b), and Alternative c) for the various quarters. Alternative a) shows the estimated model probability, Alternative b) represents the calibration to “severe bank problems”, and Alternative c) shows the actual default probability.

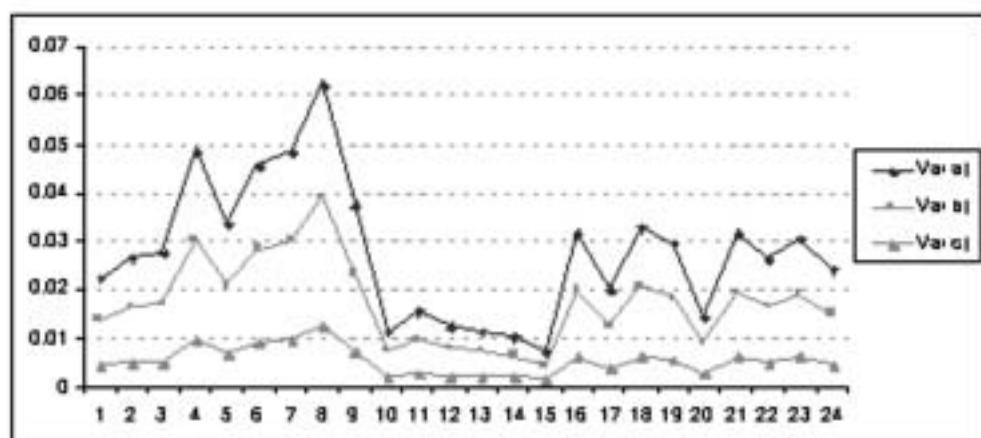


Figure 3: Development of the PDs for different approaches

The figure above illustrates the variability in the probabilities, which raises the question of whether the data should be smoothed with regard to these probabilities. In addition to procedural reasons, there are also economic reasons, such as our input data being based on quarterly observations, that create higher volatility in the probabilities. Additional volatility in the respective probabilities is also caused by changed reporting policies in different quarters, for example concerning the creation of provisions or the reporting of profits. The data could be smoothed, for example, by calculating weighted averages over an extended period of time or by computing averages for certain classes (as is done in standard rating procedures).

Finally, it should be noted that none of the indicators which have only been available for a short period of time could be incorporated in the multivariate model due to the insufficient number of observations, even if univariate tests based on the small number of existing observations did indicate high predictive power. However, these indicators are potential candidates for future recalibrations of the model.

### 3.7 Presentation of Results

If we observe the estimated probabilities of individual banks over time, we see that these probabilities change every quarter. The decisive question that now arises is how one can best distinguish between significant and insignificant changes in these probabilities in the course of analysis. Mapping the model probabilities to rating classes is the easiest way to filter out small and therefore immaterial fluctuations.

Information on the creditworthiness of individual borrowers has been reported to the Major Loans Register since January 2003. The ratings used by the banks are then mapped to a scale – the so-called OeNB master scale – in order to allow comparison of the various rating systems.

This OeNB master scale comprises a coarse and a fine scale; the coarse scale is obtained by aggregating certain rating levels of the fine scale. The rating levels are sorted in ascending order based on their probabilities of default (PD), which means that rating level 1 shows the lowest PD, followed by level 2, etc. Each rating level is assigned a range (upper and lower limit) within which the PD of an institution of the respective category may fluctuate.

In order to ensure that the default probabilities of Austrian banks are represented adequately on a rating scale, an appropriate number of classes is required. This is particularly true of excellent to very good ratings.

The assignment of PDs or pseudo-default probabilities to rating levels reduces volatility in such a way that one only pays particular attention to migrations (i.e. movements from one category to another) over time. Therefore, changes in the form of a migration will occur only in the case of larger fluctuations in the respective banks' probabilities. As long as the default probability of a bank stays within the boundaries of a rating level, this rating will not be changed.

The ideal design of the rating scheme has to be determined in the process of implementing and applying the model. The migrations per bank resulting from the respective rating scheme then have to be examined in cooperation with experts in order to assess how realistic and practicable they are. Furthermore,

it is necessary to evaluate whether and how smoothing processes such as the computation of averages over an extended period of time should be combined with the rating scheme in order to optimize the predictive power of the model.

## 4 Evaluation of the Logit Model

Descriptive analyses as well as statistical tests conducted in order to test the model confirmed the estimated probabilities and the model specification.

### 4.1 Descriptive Analyses

Random tests were used to examine the development of individual banks over time. Next, a cut-off rate was used to classify “good” and “bad” banks. Subsequently, erroneous classifications involving banks considered “good” (viz. the more serious error for the regulator: the default of a bank classified as sound) were examined in general and with regard to whether, for example, sector affiliation and certain time effects played a role in misclassification.

It became clear that neither sector affiliation nor differences in the quarters observed played any significant role in the erroneous classifications. A defaulting bank could be classified as such for up to 5 quarters: at the times  $t-2$ ,  $t-1$ ,  $t$ ,  $t+1$ , and  $t+2$ . The indicators of misclassifications did not show significant discrepancies across those different quarters. Similarly, there were no significant differences in the misclassifications in the individual quarters Q1, Q2, Q3, and Q4 as such.

As far as the development of banks over time is concerned, it must be noted that so far no systematic false estimations have been identified.

### 4.2 Statistical Tests

In this section, we will describe the statistical tests that were conducted in order to verify the model’s robustness and goodness of fit. The tests show that both the model specification and the estimations themselves are confirmed. Moreover, there are no observations which have a systematic or a very strong influence on the estimation.

#### Specification test

First of all, the robustness of the estimation model must be ensured to allow the sensible subsequent use of the goodness-of-fit measures. Most problems concerning the robustness of a logit model are caused by heteroscedacity, which leads to inconsistency in the estimated coefficients (which means that the precision with which the parameter is estimated decreases as sample size increases). The statistical test developed by Davidson and MacKinnon (1993) was used to test the null hypothesis of homoscedacity, with the results showing that the specified model need not be rejected.

#### Goodness-of-fit tests

The goodness-of-fit of our model was assessed in two ways: first, on the basis of test statistics that use various approaches to measure the distance between the estimated probabilities and the actual defaults, and second, by analyzing individual observations which (as described below) can each have a certain strong impact on estimating the coefficients. The advantage of a test statistic is that

it shows a single measure which is easy to interpret and describes the model's goodness of fit.

The *Hosmer Lemeshow goodness-of-fit test* is a measure of how well a logit model represents the actual probability of default for different data fields (e.g. in the field of less troubled banks). Here, the observations are sorted by estimated default probability and then subdivided into equally large groups. We conducted the test for various numbers of groups. In no case was the null hypothesis (the correct prediction of the default probability) rejected, thus the hypothesis was confirmed.

In the simplest case, the *LR test statistic* measures the difference in maximum likelihood values between the estimated model and a model containing only one constant and uses this value to make a statement concerning the significance of the model. A modified form of this test then makes it possible to examine the share of individual indicators in the model's explanatory power. This test was applied to all indicators contained in the final model.

Afterwards, various measures such as the Pearson and the deviance residuals were used to filter out individual observations that each had a stronger impact on the estimation results in a certain way. The 29 observations filtered out in this manner were individually tested for plausibility, and no implausibilities were found. Subsequently, the model was estimated without these observations, and the coefficients estimated in this way did not differ significantly from those of the original model.

In conclusion, we can state that there are no observations that have a systematic or very strong influence on the model estimation, a fact which is also supported by the model's good out-of-sample performance.

## 5 Development of the Cox Model

### 5.1 Cox Proportional Hazard Rate Model

Developing a Cox model required steps analogous to those taken in the logistic regression, but in this case it was possible to make use of the findings produced in the process of developing the logit model. Accordingly, a traditional Cox Proportional Hazard Rate model was calculated first on the basis of the logit model's results. This relatively simple model includes all defaulting and non-defaulting banks, but they are captured only at their respective starting times. June 1997 was chosen as the starting time for the model, as from that time onward all of the required information was available for every bank. In the Cox Proportional Hazard Rate model the assumed connection between the hazard rate and the input ratios is log-linear, and this connection is almost similar to the relationship assumed in the logit model for low default probabilities; therefore, the indicator transformations determined for the logit model were also used for the Cox model. The final model, which – as the logit model – was defined using methods of forward selection and backward elimination, contains six indicators from the following areas:

field	number
credit risk	1
capital structure	1
profitability	4

Model evaluation for Cox models is traditionally performed on the specific basis of the model's residuals and the statistical tests derived from them. These methods were also used in this case to examine the essential properties of the model, viz. (i) its fulfillment of the PHM's assumptions (i.e. the logarithm of the hazard rate is the sum of a time-dependent function and a linear function of the covariate) and (ii) the predictive power of the model, which was evaluated in general on the basis of goodness-of-fit tests. In addition, the concept of the Accuracy Ratio was also applied to examine the predictive power of the Cox model. This could be done as the Cox Proportional Hazard Rate model yields relative hazard rates, which – like the predicted default probabilities of logit models – can be used to classify banks according to their predicted risk and to compute the AR on that basis.

Although the Cox model we developed is very simple, the results show that even this basic version is quite successful in distinguishing between troubled and sound banks. This can be seen in the AUROC attained – about 77% – as well as the following figure, which shows the estimated survival curve of an average defaulting and an average non-defaulting bank. The figure clearly shows that the predicted life span of defaulted banks is considerably shorter than that of non-defaulting banks.

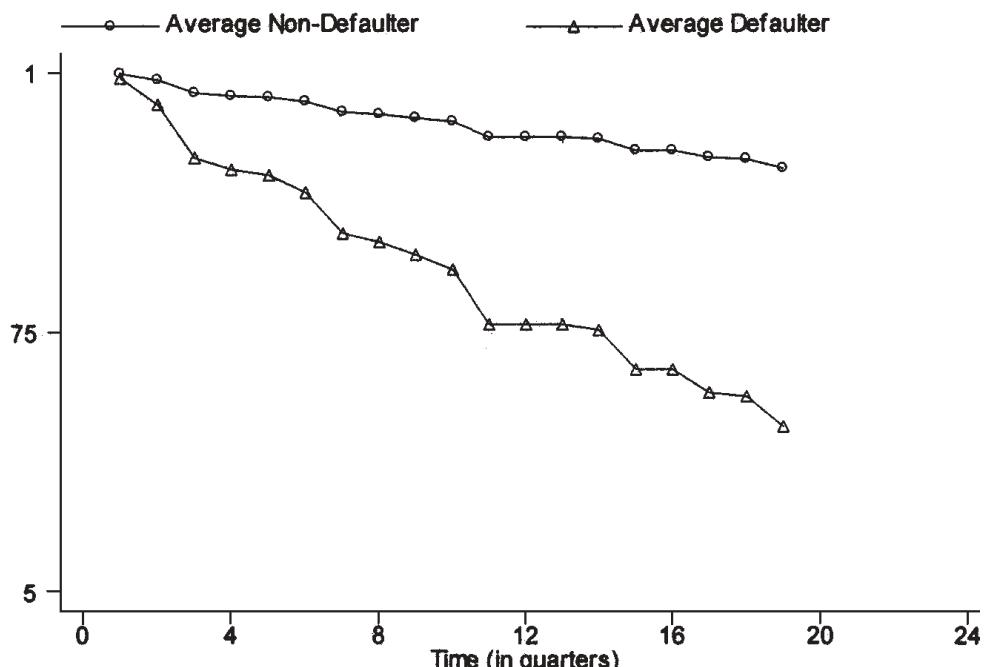


Figure 4: Survival Curves im Cox Model

When interpreting the Accuracy Ratio, it must be taken into account that the logit model, for example, was developed for one-year default probabilities, which means that there was an average gap of one year between the observation of the balance-sheet indicators used and the event of default. In the Cox Proportional Hazard Rate model, however, the explanatory variables are included in the model estimation only at the starting time; thus, in this case, up to five years may pass between the observation of the covariate and the event of default. It is self-evident that closer events can be predicted better than more remote ones.

## 5.2 Further Development of the Cox Model

Although this implemented version of the Cox Proportional Hazard Rate model already demonstrates rather high predictive power, it is still based on simplifying assumptions which as such do not occur in reality; therefore, the following extensions may be added:

- The most obvious extension is the reflection of the fact that the variables are not constant, but can change in every period. By estimating the model with those covariates changing over time, the model would lose its property of time-constant hazard rates, which allow the calculation of an Accuracy Ratio, for example; however, the overall predictive power of the model would increase due to the reduced lag between the most recent indicator and the event of default.
- Furthermore, the traditional Cox Proportional Hazard Rate model assumes that it is possible to observe default events continuously, while in our case the data were collected only on a quarterly basis. This “interval censoring” phenomenon can also be accounted for by applying more complex estimation methods.
- Finally, the assumption – which is common even in the relevant scientific literature<sup>5</sup> – that all banks are at risk at the same time (usually at the starting time of the observation period) is questionable, not least because this would be based on the assumption that actually all banks are at risk of default. Alternatively, one could use the logit model to find out whether a bank’s predicted default probability exceeds a certain threshold and thus decide if or when a bank is at risk. Only once this situation arises would the bank in question be included in the estimation sample for the Cox model. A data set constructed in this way would make it possible to test the hypothesis that certain covariates can predict the occurrence of default better for particularly troubled banks than for the entire spectrum of banks.

At the moment, a more advanced Cox model is being developed which will include the possible improvements mentioned above; the final model in that version should be available in 2005.

## 6 Summary

The Oesterreichische Nationalbank and the Austrian Financial Market Authority place great emphasis on developing and implementing sophisticated, up-to-date off-site analysis models. So far, we have described the new statistical approaches developed with the support of universities.

<sup>5</sup> See e.g. Whalen (1991) or Henebry (1996).

The primary model chosen was a logit model, as logit models are the current standard among off-site analysis models, both in their practical application by regulators and in the academic literature. This version of the model is based on the selection of 12 indicators (including a dummy variable) for the purpose of identifying troubled banks; these indicators made it possible to generate in-sample and out-of-sample AUROC results of some 82.9% and 80.6%, respectively, with a pseudo-R<sup>2</sup> of approximately 21.3%.

In contrast to the logit model, estimating a Cox model makes it possible to quantify the survival function of a bank or a group of banks, which then allows us to derive additional information as to the period during which potential problems may arise. In order to get a first impression of the Cox model's possibilities, a traditional Cox Proportional Hazard Rate model was developed on the basis of the logit model's results. This model is based on six indicators and yielded an AUROC of approximately 77%. In addition, a more complex Cox model is being developed which should improve on several of the traditional model's problem areas. The final model using this structure should be available in 2005.

Finally, we would like to point out that every statistical model has its natural constraints. Even if it is possible to explain and predict bank defaults rather successfully on the basis of available historical data, an element of risk remains in that the models might not be able to identify certain individual problems. Furthermore, it is also possible that structural disruptions in the Austrian banking sector could reduce the predictive power of the statistical models over time, which makes it necessary to perform periodic tests and (new estimations and/or) recalibrations. This would also appear to make sense as currently all those indicators which have only been available for a short time period cannot be incorporated in the multivariate model due to the insufficient number of observations, even if univariate tests based on the small number of existing observations do indicate high predictive power. These indicators, however, could improve the predictive power of the logit and Cox models in the future and are therefore promising candidates for subsequent recalibrations of the models.

# STRUCTURAL MODEL

## METHODS AND CALCULATIONS

# Structural Model

## Methods and Calculations

### Introduction

The objective of the structural model is to capture a bank's risk structure in its entirety and thereby gain insights into the individual risk categories from an economic perspective. At the center of the structural model, we therefore find a detailed analysis of risk drivers and their possible impact on a bank's total risk. The individual risk categories covered by the structural model are as follows:

- market risk
- credit risk
- operational risk

In order to derive the total risk (and thus the respective probability of an event) from those isolated risk positions in an early-warning system for banks, two steps are necessary: First, a concept needs to be developed which makes it possible to capture the risks in the individual categories in a structured manner, and second, these risks need to be combined in a uniform metric. Risk integration and risk aggregation refer to quantitative risk measurement models and methods which enable risks from different categories and/or different business units (banks) to be combined in an integrated manner.

### 7 Aggregation of Risks

This section looks at possible means of aggregation within and across various types of risks, while the ensuing sections deal with calculations for individual types of risks.

#### 7.1 Theoretical Background

In risk management practice, the “building-block” approach is most commonly used to integrate different risks. Within such an approach, the first part of aggregation consists in aggregating the risks within individual risk classes. In our structural model, this means aggregating the risks within the categories of market risks, credit risks, and operational risks. The second part then offers a method of aggregating risks across the individual categories, a process in which it is necessary to make allowances for the dependence structure among individual risk factors.

If we then attempt to aggregate risks from different categories in the process of assessing bank risk, a uniform measurement system is needed in order to measure market, credit, and operational risks. The concept of economic capital has established itself as the uniform metric used in banking and regulatory practice. Economic capital models quantify the capital needed by banks to cover losses from a certain risk category with a predefined probability of occurrence. The value-at-risk (VaR) model has become the most common measure for quantifying economic capital.<sup>6</sup>

If a VaR model is used to integrate risks, the objective of the first block is to find a common distribution of the individual types of risks in a risk class (e.g. market risk: interest rate, foreign exchange and equity position risks), thus making it possible to compute the quantile of the loss distribution for this risk class. It is not until the second step that we have to find a common loss distribution which also covers the dependences between individual risk categories.

<sup>6</sup> See also the document published by the Joint Forum (2003).

Under such a building-block approach, the aggregation of risks within a risk category can thus be described formally as follows: We assume that there are  $m$  risk factors in the category of market risks. Each individual risk factor is represented by a random variable  $x_i$ . Before the market risks can be aggregated, it is necessary to map the risk factors to the individual portfolio positions, which is usually a very complex process characterized by non-linear structures for many products and portfolios.

As the multivariate distribution of all risk factors is usually not known, alternative approaches to quantifying the VaR have to be used.

If we assume, for example, multivariate normal distribution for changes in the economic capital of individual categories, the VaR is completely determined by the variances and correlations between the risk categories, that is, the aggregation is defined completely by the common correlation matrix. If we have no estimates for the correlations, the overall VaR – assuming perfectly positive correlations between the risk factors – can be calculated as the sum of the individual VaRs.

The following sections discuss various approaches to aggregating risks – first within a risk class, then across individual risk classes.

## **7.2 Risk Aggregation within Individual Risk Categories**

The structural model records the risks of banks separately under market, credit, and operational risks, which makes it first necessary to aggregate the individual risks within each class. This aggregation within risk classes is essentially guided by the correlation of risk factors.

### **7.2.1 Aggregation of Market Risks**

A bank's market risk is composed of interest rate risk, exchange risk, equity position risk, and – where applicable – also the non-linearity risk of individual derivative positions. If economic capital or the VaR is used as a uniform metric to quantify market risk with due attention to the dependence structures among individual risk factors, it is possible to capture the entire market risk structure of every single bank. Assuming that the risk factors' distributions of returns belong to the family of elliptical distributions, VaR is a coherent risk measure for the entire market risk portfolio according to Artzner.<sup>7</sup> Elliptical distributions include normal distribution, which unfortunately does not adequately reflect the 'fat tails' property of returns, and t distribution, which – depending on the estimation of the degrees of freedom – is able to capture fat distribution tails in various ways. If we assume  $m$  different market risk factors,  $(X_1, \dots, X_m)$ , the VaR is computed as follows:

$$VaR_{\alpha}^{Market}(X_1, \dots, X_m) = F_{Market}^{-1}(\alpha)$$

<sup>7</sup> Artzner et. al. (1997) define a coherent risk measure as a metric in which a number of axioms are fulfilled; among those is subadditivity, meaning that the risk of a portfolio is smaller than that of the sum of the individual risks. The other properties postulated besides subadditivity are monotony, positive homogeneity, and invariance to transformations.

Assuming normal distribution, it is not necessary to know the risk factors' common distribution function; risk reduction effects can be determined by estimating the covariances.<sup>8</sup>

Under the approach implemented, the total market risk of each bank is calculated using a delta approach, which is based on the assumption of normal distribution. This system captures the dependence structures of risk factors and thus makes it possible to aggregate individual risks. For a closer look at capturing total market risk, please refer to the section describing the market risk model.

### 7.2.2 Aggregation of credit risks

Under the new regulatory capital requirements of Basel II, recognition and quantification of credit risks is a core responsibility of credit institutions. Various statistical models which go beyond the scope of Basel II (the standardized approach and the internal ratings-based approach) have been developed in recent years and already provide an integrated approach to quantifying credit risk with attention to dependences between individual credit risk factors. The probability of default as well as exposure at default and loss given default are fundamental measures used in quantifying credit risks. These measures taken together make it possible to quantify the maximum loss for a given probability providing a uniform metric. This maximum loss to be expected at a certain confidence level is the economic capital associated with credit risk and corresponds to a VaR, meaning that economic capital is also calculated as a quantile of the credit risk factors' distribution.

CreditMetrics and CreditRisk<sup>+</sup> are standard models for consistent measure of credit risk. Both models have different requirements on the market data available for quantifying the credit risk of a bank's entire credit portfolio. Given the data available, only the CreditRisk<sup>+</sup> model can be implemented in practice. A detailed description of the various model properties as well as the methods used for the aggregation of individual credit risks can be found in section 8 describing the credit risk model. Assuming that credit risk is triggered by  $n$  different risk factors ( $Y_1, \dots, Y_n$ ), then the VaR for the credit risk can be calculated in a general statistical model as follows:

$$VaR_{\alpha}^{credit}(Y_1, \dots, Y_n) = F_{credit}^{-1}(\alpha)$$

with  $F_{credit}$  being that distribution function of the credit portfolio value which is formed by mapping the risk factors to the positions. Here, in contrast to market risk,  $F_{credit}$  cannot be assumed to show normal distribution.

### 7.2.3 Aggregation of Operational Risks

Recording operational risks makes great demands on a credit institution's database. Therefore, the regulatory capital requirements under Basel II will be determined in practice using the basic indicator approach or the standardized approach, even if these approaches do not truly quantify operational risk. However, given a loss database which records the frequency and extent of losses

<sup>8</sup> For more detail, see Jorion (1998)

within a period, we can use a statistical model to determine – based on a given confidence level – the loss expected from operational risk as a VaR value in terms of economic capital. The section on the preliminary treatment of operational risks within the structural model presents a simple statistical model in which the frequency of defaults follows a geometric distribution and loss amounts are distributed exponentially, which means that the losses from operational risk within a period are also distributed exponentially. The derived VaR now only depends on two parameters: the significance level and the mean loss. Both values can be determined by the appropriate calibrations; in the first step, operational risk assessment under the method suggested by Basel II (which is not very risk-sensitive) can be used as the basis for calibration. This model records operational risks separately from the specification of individual risk factors, but due the specification of the loss model it still allows the calculation of the VaR, so that – in analogy to market and credit risks – we get the following VaR:

$$VaR_{\alpha}^{Op}(Z_1, \dots, Z_p) = F_{Op}^{-1}(\alpha)$$

Although the individual risk factors for operational risk are not recorded separately, and thus no aggregation of these risks is necessary, the VaR can still generally be interpreted as a quantile of a multivariate distribution.

### **7.3 Aggregation across the Individual Risk Categories**

One major task of the structural model is to aggregate the risks in the individual risk categories into one overall value. In this context, at least three basic principles must be observed:

1. The individual risks must be measured using the same metric. As was already assumed in the analysis of risk aggregation within a risk category, individual risks are quantified as the loss expected with a certain probability (VaR).
2. Individual risks must be quantified for the same time frame. Generally, credit and operational risks are specified for a time horizon of one year, but market risk is specified for a considerably shorter holding period; therefore, it is necessary to use a method which ensures a uniform time frame.
3. The individual risks should be aggregated into a bank's total risk with due attention to the dependences between the individual risk categories. This makes it possible to identify potential diversifications and possibly to reduce economic capital.

In this section, we will take a closer look at Items 2 and 3 above. Using the VaR model ensures the definition of a uniform metric for measuring risks.

#### **7.3.1 Uniform Time Frame to Determine Individual Risks**

In general, both credit and the operational risk are assessed for a time horizon of one year. Especially for credit risk, a shorter period would not make sense, as (for example) changing macroeconomic conditions such as the business cycle do not impact credit defaults immediately but with a certain time lag. Furthermore, the systems commonly used by banks are calibrated to one-year default

probabilities due to the relevant requirements under Basel II. The same is true of operational risk.

As regards market risk, the situation is entirely different. In this case, changes in risk factors have an immediate impact on a portfolio's value. Over an extended period, however, the composition of a portfolio can be changed in order to counteract further increases in market risk. Originally, the VaR model was developed for the treasury field, in which trading positions can usually be modified at very short notice.

For this reason, a shorter reference period appears to be preferable. In order to integrate market, credit, and operational risks into a combined risk indicator, it is also necessary to calculate market risks for a holding period of one year.

In principle, it would be possible to adapt the VaR for a holding period of only one day (i.e. a daily VaR) to 250 days or one year using a scaling factor. However, this procedure usually leads to a considerable overstatement of market risks on a yearly basis. This can happen for two reasons. First, extrapolating daily volatilities for strongly fluctuating values may very well lead to an overstatement of the risks, and second, a bank can easily hedge its market risk position in the course of a year by engaging in active risk management, which means that it has to face less total risk.

In order to meet these requirements, the following procedure seems to be useful for "extrapolating" market risks to a yearly basis. Monthly returns can be used as estimators for the volatilities and correlations between risk factors. This offers the advantage that the distributions of returns are no longer determined by their 'fat tails' property as strongly and that the estimates overall no longer fluctuate as strongly over time. This makes it possible to secure a uniform time basis for all three risk categories.

### 7.3.2 Consideration of Dependences

Basically, two different procedures can be used to aggregate the risks from individual risk categories into total risk:

The first procedure assumes that the distribution of risk factors is known and determines the joint distribution of the VaR on the basis of the individual mappings (i.e. the projections of the risk factors onto the portfolio positions). In this context, it must be taken into account that the mapping function for the risk factors significantly influences the dependence structure of the individual risk factors as well as that among the individual risk categories. The authors are not currently aware of a consistent implementation of this approach which determines a joint multivariate distribution of all risk factors.

Alternatively, it would be possible to compute the distributions and thus the correlations of profits and losses (P&L) in the individual risk categories directly from historical changes in the values of the individual portfolios. Although this approach is often suggested, it has the major disadvantage that one cannot directly derive the composition of the current portfolio from the historic composition of the market risk portfolio (for example) and thus the distribution of the market risk P&Ls. Below we will present a model based on very restrictive assumptions and discuss the aggregation of individual risks in this context.

### Recording by means of a correlation matrix

Traditionally, individual risks are aggregated by assuming a multivariate normal distribution, thus making it possible to show the dependence between the individual risk categories via the correlation matrix. If we make this assumption in the first step, the VaR for the total risk is derived directly from individual risks, with the dependences between the three risk categories being recorded using the correlations. We thus get the following formula:

$$VaR_{total} \sqrt{\begin{pmatrix} VaR_{credit} \\ VaR_{market} \\ VaR_{Op} \end{pmatrix}^T \begin{pmatrix} 1 & \rho_{C,M} & \rho_{C,O} \\ \rho_{C,M} & 1 & \rho_{M,O} \\ \rho_{C,O} & \rho_{M,O} & 1 \end{pmatrix} \begin{pmatrix} VaR_{credit} \\ VaR_{market} \\ VaR_{Op} \end{pmatrix}}$$

$\rho_{ij}$  represents the correlations between market and credit risks, between market and operational risks, and between credit and operational risks.

The following brief example illustrates the consequences of the correlations under this approach. We assume that a bank has measured risks and economic capital separately and has received the following values:

- At a confidence level of 99%, let the economic capital for market risk be EUR 1m on a yearly basis.
- At a confidence level of 99%, let the economic capital for credit risk be EUR 3m.
- At a confidence level of 99%, let the economic capital for operational risk be EUR 2m.

If we assume perfectly positive correlations (i.e. the most conservative assumption under the normal distribution model), we get  $VaR_{total} = 1+3+2 = 6$  (€ 6 million). If the risks are assumed to be uncorrelated, we get  $VaR_{total} = \text{EUR } 3.74$  million. If we now assume, for example, a correlation between market and credit risk of 0.8 and a correlation between market and operational risk as well as between credit and operational risk of 0.4 each, we get  $VaR_{total} = \text{EUR } 5.02$  million. This example shows that – depending on the underlying correlation structure assumed – the aggregated values can differ significantly from the “conservative” approach. However, when using this approach, we must also consider the following objections which subsequently shed an unfavorable light on the correlation approach for the structural model:

The first and most important objection to the correlation model refers to the assumption of a normal distribution of individual risks. While the use of monthly returns might justify the assumption of normal distribution in estimating market risk, it is definitely impossible to make this assumption for credit and operational risks. Consequently, the correlation loses its significance as a measure of the dependence between risk factors. Embrechts et al. (1999) demonstrate quite impressively the pitfalls that can accompany the uncritical use of the linear correlation coefficient in risk management. In particular, Embrechts et al. (1999) raise the following arguments:

1. In theory, the use of correlations in aggregating risks is acceptable if the distributions of the risk factors belong to the class of elliptical distributions. To this effect, VaR is a coherent risk measure. If the distributions do not belong to this class, the correlations do not contain any information that can be used to estimate aggregated risks adequately.
2. The correlation is a scalar measure which shows linear dependence but which cannot contain all information on the dependences of random variables.
3. Perfectly positively correlated variables do not necessarily show a correlation of 1.
4. A correlation of 0 does not mean that the variables are independent of each other. Only for normally distributed variables does uncorrelatedness also mean independence.
5. The correlation values depend on the marginal distribution. The values of the correlation coefficient cannot always range from -1 to 1.
6. Correlation is not a measure which is invariant to transformations. This means that correlation is not an adequate risk measure in risk management applications where the variables are transformed in many contracts.

All these arguments prompted Embrechts et al. (1999) to maintain that – particularly for the aggregation of risks from different risk categories – the correlation approach is not really suitable. Moreover, it is not clear how the correlation coefficients between the individual risk categories can be estimated. In this context, we must consider in particular the objection made above: it is not possible to derive the composition of the current portfolio directly from the historic composition of, for example, the market risk portfolio and thus the distribution of market risk.

### Copulas as a possible alternative

As an alternative, Embrechts et al. (1999) suggest using copulas. Copulas are functions which establish the connection between the multivariate distribution of random variables and the marginal distributions of the individual variables. Let us assume we are looking at the random variables  $X_1, \dots, X_n$  with the marginal distributions  $F_i(x_i)$ . The joint distribution of  $X_i$  is defined by  $F(x_1, \dots, x_n)$ . The copula  $C$  now denotes that function to which the following applies:

$$F(x_1, \dots, x_n) = C(F_1(x_1), \dots, F_n(x_n))$$

This definition clearly shows that copulas offer the possibility of separating the dependence structure from the structure of the marginal distributions based on the joint distribution of random variables. This means that copulas can be used to capture the dependences between random variables, which is very promising in terms of their application in risk management.

- The major *advantages* of copulas are that
- they reflect dependence structures better than linear correlations; and
  - suitable copula classes are able to capture the tail dependence of distributions. Tail dependence is especially relevant with regard to individual risk classes, as it expresses the probability of an unfavorable development in Asset Y given the unfavorable development of Asset X.

With all these advantages, it would still be rash to opt for the use of copulas in order to capture the dependence structure within the implementation of this project. The arguments against their immediate use are based on the various disadvantages:

- There is a great variety of copula classes. Identifying a suitable class for an application in risk management is not a trivial task.
- Only if the multivariate and marginal distributions of the risk factors are known is it possible to identify copulas for measuring dependences.
- Only if the marginal distributions are continuous does a clear copula distribution exist in each case.
- No empirical studies have shown that tail dependence also exists between different risk categories. So far, studies have mainly investigated dependence structures within certain risk groups such as stocks or bonds.
- The use of copulas to precisely reflect dependence structures only appears sensible if the quantification of all individual risks is sufficiently precise and reliable.

#### **7.4 Selected Procedure**

As the arguments raised support neither the unqualified application of copulas nor the use of the correlation model, the risks in the individual risk categories were aggregated using the “conservative approach”. This means that the following was assumed for the VaR:

$$VaR_{\alpha}^{Total} = VaR_{\alpha}^{Market} + VaR_{\alpha}^{Credit} + VaR_{\alpha}^{Op}$$

where  $\alpha$  = defined confidence level

This signifies a perfectly positive correlation as defined for the correlation model. However, as the assumption of normal distribution is not part of the model, this argument cannot really be brought to bear. In particular, we would like to point out that given the existence of co-monotonous random variables, a simple summation of risks would be the appropriate procedure. For a look at the concept of co-monotonous random variables, see Embrechts et al. (2002).

It should also be noted that from the regulator's point of view, it is more appropriate to assess risks more conservatively by overestimating the extent of possible losses.

Furthermore, the composition of total risk and/or the individual risk types' contributions to total bank risk are extremely interesting issues.

In the following sections, we will illustrate our theoretical explanations using an example in which we compare two different banks:

- Bank A: A large Austrian bank with a focus on retail business and a relatively high equity indicator
- Bank B: A medium-sized Austrian bank with strong expansionary tendencies and a relatively low equity indicator
- The composition of the risk clearly shows the difference between the two banks, but credit risk is obviously the dominant risk for both banks. The impact of market risk on the overall result for Bank B can be put down to a larger number of foreign exchange positions (primarily from the credit sector).

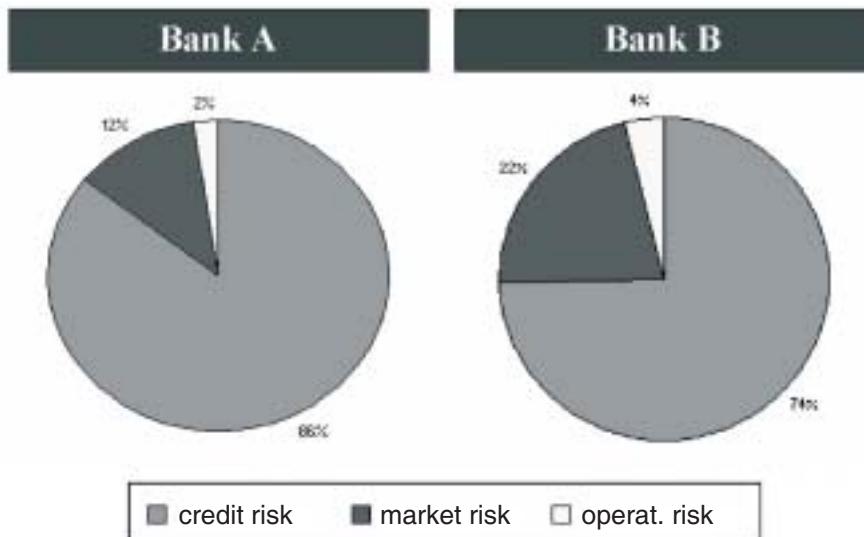


Figure 5: Composition by type of risk

As operational risk depends on the banks' incomes, it is difficult to interpret the results in economic terms.

## 8 Credit Risk

Credit risk is the most important type of risk for Austrian credit institutions. In the last few years, the development of comprehensive quantitative evaluation methods and models has meant that losses attributable to credit risks can be more reliably recognised, which will be even of greater importance in connection with regulatory capital requirements under Basel II.

The increased mobility of capital and faster, more flexible means of transferring credit risks have led to a further increase in the demands on credit risk management and measurement. It has become possible to assess credit risk more precisely particularly as a result of the introduction of credit risk models which do not quantify the risks of individual loans, but measure the credit risk of particular portfolios and for the institution as a whole.

Using a credit risk portfolio model the Austrian banking regulators intend to achieve the following:

1. Calculation of credit risk for individual credit institutions as well as for banks' sub-portfolios or bank groups. This kind of portfolio analysis takes possible diversification effects into account as well as captures concentration risks.
2. Aggregation of credit VaR with market VaR and VaR from operational risk in order to compare the total risk with the institution's available capital cover.

As in the case of market risk portfolio models, a credit risk portfolio model can be used to assess changes in the credit portfolio's value as a result of changes in the credit rating of individual loans. In practice, the main problem in determining the loss distribution in credit risk is the lack of sufficient data of the quality required for precise estimation. If the available time series are too short, for example, cyclical influences cannot be reflected.

The payback profile of the credits is another challenge in modeling credit risk (in contrast to market risk). Credit defaults are rare but they entail larger losses, which means that the loss distribution strongly deviates from a normal distribution as a result of skew and fatter tails (see Figure 6).

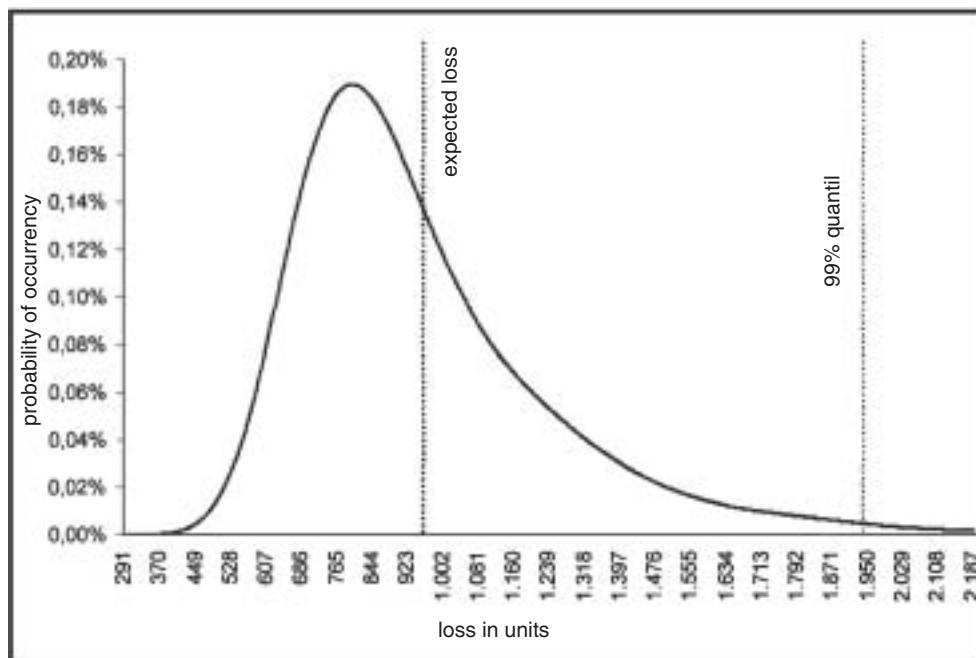


Figure 6: Credit loss distribution of an Austrian credit institution

With the Credit-VaR model currently being developed, it is possible – using the available supervision data in combination with data on companies and defaults compiled by the Kreditschutzverband von 1870 (KSV) – to calculate the following distributions:

- The entire credit loss distribution of each of the roughly 900 Austrian credit institutions;
- the default frequency distribution, which shows the distribution of the total number of defaults in a bank's portfolio for the next period;
- the loss and default frequency distributions of a sub-portfolio or a bank group.

In addition, it is possible to carry out macroeconomic scenario analyses by changing the input parameters, for example by changing the default probabilities of individual sectors.

The method used to calculate credit risk is explained briefly in Section 8.1, after which Section 8.2 describes the raw data available from banking supervision reporting and the data provided by KSV and Statistics Austria.

In Section 8.3, we illustrate the procedure used to calculate the input parameters required for the model on the basis of the raw data available.

Section 8.4 then describes the model in detail, and the findings are discussed in Section 8.5; Section 8.6 concludes with a summary, possible extensions of the model, and final remarks.

## 8.1 Computation of Credit Risk

Currently, the following three models are most commonly used internationally:

Portfolio Manager, CreditMetrics, and CreditRisk<sup>+</sup>

The **Portfolio Manager**<sup>TM</sup> model developed by Moody's | KMV utilizes Merton's idea of treating equity as a call option on the company's assets. The level and volatility of the company's stock price make it possible to draw conclusions about the firm's market value. A company's creditworthiness can then be determined by comparing its value to its debt level. This approach has the key advantage that it does not have to rely on ratings provided by rating agencies, but can use the information efficiency of the stock market as a basis. Unfortunately, this method is of limited use for companies (in this context banks) that are not listed on a stock exchange.

The **CreditMetrics**<sup>TM</sup> model (by J.P. Morgan, RiskMetrics Group) looks not only at losses due to credit defaults, but also integrates the changes in the market value of exposures resulting from rating changes. As a first step, the market value of an exposure is calculated for different ratings, and then the probability distribution of future ratings. The distribution of the exposures' market values is determined using a rating transition matrix. In order to calculate changes in the value of an exposure portfolio, it is necessary to estimate the correlations between the individual exposures. For this purpose, CreditMetrics suggests estimation based either on stock prices or a factor model. The loss distribution can then usually only be generated by means of a processing-intensive Monte Carlo simulation. In any case, the ratings of the individual loans or borrowers as well as a rating transition matrix are required to apply CreditMetrics. Since just a few borrowers on the Austrian market are currently rated, this model can only be applied to a limited extent.

**CreditRisk<sup>+</sup>** was developed by the Credit Suisse Group between 1993 and 1996. The model only calculates explicitly those losses attributable to credit defaults, and it ignores changes in market value resulting from rating changes. In order to reduce the volume of data to be processed, a bank's exposure (with each single exposure assigned a default probability) are combined into exposure bands, so that each credit exposure is approximated by the average credit volume of the respective band. In order to enable the inclusion of economic developments as factors potentially influencing systematic defaults, a volatility level is estimated for the default probabilities; this allows for the fact that default rates can change within an economic cycle.

The PDs modeled as random variables rise or fall depending on independent macroeconomic risk factors. If two borrowers are sensitiv to the same risk factors, their default probabilities move simultaneously. In other words, the risk factors (the economic situation, specific industry or country characteristics etc.) may cause defaults to be correlated even if there is no direct causality between their risk changes and the level of default rates. In this way, dependences are modeled implicitly and very elegantly without requiring default correlations as model inputs. The more precisely the various risk factors can be identified, the more accurately one can determine the correlations and diversification effects.

One of the main strengths of the CreditRisk<sup>+</sup> model is the ability to calculate loss distributions with relatively low numerical effort compared to the other

models. Another important advantage of the CreditRisk<sup>+</sup> approach is that only modest amount of data is required, since it models credit defaults rather than rating downgrades.

In conclusion, it can be stated that the Austrian regulatory authorities' decision to implement a CreditRisk<sup>+</sup> model was based on the relative ease of practical implementation and the availability of the necessary input variables on the one hand, and the fact that the low computational effort allowed a periodical calculation of credit risk for all Austrian credit institutions on the other.

### **8.1.1 Description of the Approach Selected**

#### **Implementation of a one-factor model**

During the development phase, we decided to implement a model with a single risk factor at first. This risk factor can be interpreted as the general economic situation affecting all borrowers to the same extent.

An appropriate probability distribution is assumed for the risk factor, which is modeled as a random variable. The dependences (correlations) between the default probabilities of individual borrowers are accounted for by the volatility of the risk factor. Thus the volatility of the risk factors shows the average correlation of the portfolio.

The decision to use a one-factor model at first was based on the following arguments:

It is difficult to model the economic processes which influence the occurrences of default in the individual sectors in Austria separately. The available database must be expanded and improved continuously in order to model the subtle individual differences in sector developments which influence default probabilities. The various sector default probabilities, which were estimated on the basis of the available data, were also included in the one-factor model in an appropriate manner (as described below).

In a model which uses more factors to calculate risk, the estimated credit VaR is – ceteris paribus – always lower than the corresponding value generated by the one-factor model because diversification is taken into account. Therefore, at this stage the conservative approach appeared to be preferable for the purpose of prudent regulation.

#### **Process Overview**

The following input parameters are required for each exposure in order to implement a CreditRisk<sup>+</sup>-based model

- exposure at default (EAD)
- loss given default (LGD)
- default probability
- volatility of default probability

The available data had to be adapted and processed to obtain the values for the input parameters (see Section 8.3). The process of calculating credit risk involved two steps:

1. calculating the frequency of defaults;
2. calculating the loss given default.

The individual steps necessary and the corresponding model assumptions can be summarized as follows:

- The default probabilities of individual exposures are independent for a specific realization of the risk factor (i.e. a given economic situation).
- An appropriate distribution is assumed for the risk factor (i.e. general economic situation).
- The parameters of this distribution (mean and standard deviation) are calculated on the basis of the expected default probabilities and standard deviations of default probabilities for individual borrowers.
- The distribution of the number of defaults in the entire portfolio for the upcoming period (one year) is generated from the distributions mentioned above.
- The loss distribution is determined by the distribution of credit volumes based on the LGD associated with a borrower, with the current model applying a Basel II-compliant LGD factor of 45% to all borrowers.
- Exposure bands are defined in order to reduce the data volume required as input, with each exposure being approximated by the average exposure of each band. This approximation makes it possible to derive the loss distribution from the distribution of the number of defaults.
- The loss distribution is calculated by combining the distribution of the number of defaults in the entire portfolio for the next period of 1 year with the exposure distribution.

## 8.2 Database

At the moment, the input data comprise supervision data from the Major Loans Register and monthly balance sheet reports from credit institutions, as well as company and default data from the Kreditschutzverband von 1870 (KSV).

### 8.2.1 KSV Data

In our model, the default probabilities of individual exposures are modeled as random variables. A historical time series of defaults is required in order to calculate the expected value and standard deviation of the default probability. Since 2003, rating data for each individual exposure have been reported to the Major Loans Register. Two problems arise in this context: First, the time series are too short to estimate volatilities, and second, data quality is not completely reliable at this time.

KSV industry default data have been available since 1997, with data being reported on a half-yearly basis. This made it possible to estimate industry default probabilities on the basis of the KSV data and then to assign the default probability and volatility of the corresponding borrower's industry as an individual default probability and volatility to each borrower.

The industry default probabilities and volatilities thus serve currently as an approximation to the missing default probabilities and volatilities of the individual borrowers.

As the KSV's industry classification may occasionally differ from that available in the database for company data collected by the OeNB, discrepancies in industry classifications may appear between the two data sources, which in turn may result in inaccuracies in the assignment of bank exposures to indus-

tries. In order to minimize these inaccuracies, groups of industries were defined on the basis of the OENACE divisions which correspond to the OENACE sections (see Table 1). The default probability – expected value and standard deviation – is estimated on the basis of the KSV data for each sector.

On the aggregation level of the OENACE divisions, the data provided by the KSV currently comprise semi-annual data on the bankruptcies of the last 12 months, beginning on June 30, 1997, (broken down into opened bankruptcy proceedings and rejected bankruptcy filings) as well as data on the universe of all Austrian companies.

As there are no KSV data on the private sector, it was temporarily assigned the average default rate of all industry groups.

On the basis of these data, default indicators which reflect the share of defaulting companies in an industry group over the past year are calculated for each cut-off date and industry group.

The expected default probability and standard deviation for each industry group are currently defined as simple mean or standard deviation of the annual industry group default rates (calculated every six months). The most recent data on industry default probabilities are used for the standard quarterly calculations. Figure 7 shows a sample comparison of current industry group default indicators (cut-off date for the calculation: December 31, 2003).

industry group	default probability	standard deviation
agriculture, mining	1.19%	0,21%
manufacturing	2.37%	0,32%
energy supply	0.36%	0.37%
construction	3.94%	0,16%
retailing	2.29%	0.33%
hotel and catering sector	4.68%	0,86%
transport and communications	3.82%	0,28%
finance	1.65%	0,37%
other services	2.08%	0,28%
human health activities	0.85%	0,15%
other	2.33%	0.33%

Figure 7: Default probabilities of the industry groups

### 8.2.2 Data from the Major Loans Register

The quantity and detail of the data contained in the Major Loans Register database makes it a good starting point for further credit risk analyses. At the end of each quarter, credit utilisations, credit limits, rating, and the industries to which the banks' individual borrowers belong are retrieved from the Major Loans Register.

#### Credit utilization and credit limit

The structure of a bank's loan portfolio can basically be approximated using the Major Loans Register. According to the Austrian Banking Act, credit institu-

tions have to report major loans with a credit limit or credit utilization exceeding EUR 350,000 to the OeNB; only the overall exposure (credit limit and utilization) is reported for each borrower, the reports are not based on the individual loans. In the following sections, the terms individual exposure or individual loan are always taken to mean the overall exposure of a borrower.

On average, the Major Loans Register covers about 80% of a bank's credit volume, but its level of coverage may be lower, especially in the case of smaller banks.

### **Data from the Major Loans Register**

Since the beginning of 2003, Austrian credit institutions have also had to report the rating of each borrower who is subject to OeNB reporting requirements. As a rule, the banks also inform the OeNB of their principles and rules for internal credit rating by providing system documentation. This system documentation is intended to describe the procedures and methods used in each institution as well as their integration in credit risk management.

In order to enable a comparison of different rating systems among institutions required to report to the Major Loans Register, the OeNB developed a master scale which allows ratings to be compared across institutions. This master scale consists of a coarse and a fine scale to which ratings are mapped on the basis of the fineness of each institution's grading system. It should be noted that the ratings mapped to the fine scale can also be evaluated in the coarse scale, which means that the coarse scale can cover all ratings. The OeNB's coarse scale contains 6 rating classes, with class 6 denoting default.

Unfortunately, the time series covered by the rating data is too short to estimate a volatility, and classification in the master scale may be inaccurate due to differences in the quality of system descriptions. For this reason, the choice was made to estimate the default probabilities based on the KSV data for now.

The rating data reported to the Major Loans Register was still used to fine-tune the borrower's default probability.

### **Industry classification**

The industry classification used in the Major Loans Register is based on the OeNB's company data. The industry classifications themselves are obtained mainly from the KSV and may be supplemented with internal research or data from other sources. At the moment, no industry information on foreign borrowers is available.

The following aspects are taken into account in assigning the borrowers in the Major Loans Register to industries:

- A borrower is assigned to one industry only, on the basis of its main business activity.
- Loans utilized by several borrowers (joint loans) cannot be assigned to an industry (different industries among the joint loan) Therefore, this sector – in analogy to the private sector – is for the time being assigned the average default rate of all industry groups.

### **8.2.3 Monthly Financial Report Data**

The following data are available from monthly statements: number and value of all receivables from non-banks, in the following bands EUR 0 to EUR 10,000, EUR 10,000 to EUR 50,000, and EUR 50,000 to EUR 500,000.

These data are currently used (as described in Section 8.3.2) to estimate the volume of loans below the Major Loans Register threshold of EUR 350,000.

## **8.3 Preparation of Input Variables**

### **8.3.1 Individual Default Probabilities**

In order to obtain individual default probabilities, the expected value and volatility of the default probability and its volatility (as described in Section 8.2.1) are first calculated for each industry grouping and then assigned to the borrowers.

As the ratings available in the Major Loans Register contain important information concerning the credit risk associated with individual borrowers, it makes sense to use a combination of industry-specific default probabilities and GKE-ratings. The ideal solution would be to combine both rating approaches and to estimate default probabilities for each industry and rating class. At the moment, however, this procedure is not completely feasible because data quality is not reliable and the time series are not sufficiently long.

A simple approximation is now the rating-dependent adjustment of the expected default probability per industry. This adjustment can, for example, be seen in the standard deviations for a given industry's default probabilities. This approach is illustrated on the basis of the  $k$  industry below, with  $\mu_k$  and  $\sigma_k$  being the mean and standard deviation for this industry default probability, and  $\mu_k^{(i)}$  representing the borrower's mean default probability adjusted for the borrower's rating  $i$ :

$$\text{Rating 1 : } \mu_k^{(1)} = \mu_k - \sigma_k$$

$$\text{Rating 2 : } \mu_k^{(2)} = \mu_k - \sigma_k$$

$$\text{Rating 3 : } \mu_k^{(3)} = \mu_k - \sigma_k$$

$$\text{Rating 4 : } \mu_k^{(4)} = \mu_k - \sigma_k$$

$$\text{Rating 5 : } \mu_k^{(5)} = \mu_k - \sigma_k$$

The number of standard deviations by which the values are adjusted in each case is represented by an input parameter which has a current default value of 0.5. If the rating according to the OeNB's coarse scale shows a value of 6 (corresponding to a default), the default probability of the exposure is set to 1. If the default probability after rating adjustment shows a value below a predefined lower limit (the lower limit is also implemented as a changeable parameter and is currently set to 0.03% by default in accordance with Basel II), the expected default probability of the exposure is automatically set to this minimum value.

Each borrower is assigned its industry's standard deviation as its individual standard deviation.

### 8.3.2 Small-scale loans

The aggregate volume of loans below the Major Loans Register reporting limit is approximated as the difference between the total volume of loans under EUR 350,000 from the monthly balance sheet reports and the volume from the Major Loans Register data on exposures between EUR 350,000 and 500,000.

For this purpose, a pseudo number of loans – not the actual number – is used as proxy for the number of exposures below the Major Loans Register reporting limit; this pseudo number is calculated by dividing the total volume of receivables below the reporting limit (or the total volume reduced using the LGD factor, see below) by the bandwidth. As a result, all small loans fall into the first band.

This approach was chosen for two reasons: First, the lack of essential information concerning the distribution of small-scale loans between EUR 0 and EUR 350,000, and second, the fact that the average loan amount in this segment is usually significantly smaller than the corresponding bandwidth. If one were to use the actual number, the discretization error<sup>9</sup> would be too large, as each of the loans is ultimately approximated by the bandwidth.

### 8.3.3 Bandwidth

In order to apply the CreditRisk<sup>+</sup> model, it is necessary to divide the individual loans into bands of equal size based on their loss given default (LGD; see below). As a result, individual exposures are no longer recorded with their respective credit amounts but approximated using the average credit amount of the band. This process is required to reduce the numerical effort and the complexity of the iterative solution algorithm. When credits are assigned to their respective bands, the default probabilities are adjusted in order to keep the expected loss constant.

#### Definition of the bandwidth for individual credit institutions in Austria

The bandwidth is currently defined as the 5% quantile of the LGD distribution for each credit portfolio. This individual definition of the bandwidth makes it possible to limit the approximation error in line with the exposure volume or the size of the bank.

The general rule is that the smaller the bandwidth (i.e. the larger the number of bands) is, the smaller the discretization error becomes. In this context, it has to be added that this discretization error can be quantified and may be ignored if an adequate bandwidth is chosen.

Furthermore, it can be observed that the credit VaR tends to fall when the number of bands is increased. The following figure shows the 95% credit VaR of a sample bank portfolio using various bandwidths as a deviation from the 95% VaR result with 800 bands.

<sup>9</sup> In this context, discretization error refers to the error resulting from classifying exposures into bands; see also Section 8.3.3.

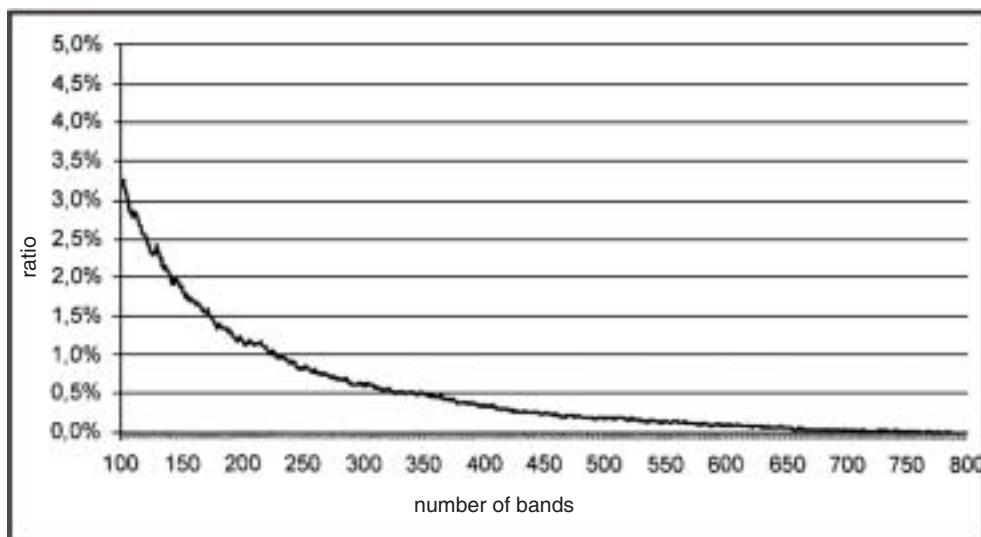


Figure 8: Dependence of the credit Var on the number of bands chosen

#### **Definition of the bandwidth for foreign subsidiaries and groups**

The Major Loans Register does not contain supervision data reported by foreign subsidiaries or data reported at group level. New data sources are currently being considered so that exposure data will include foreign subsidiaries and groups on an aggregated level for each exposure band. The Austrian regulatory authorities are also considering the idea of defining a fixed bandwidth which is identical for all portfolios and would thus make aggregating and collecting data considerably easier.

#### **8.3.4 Loss Given Default (LGD)**

Since the beginning of 2003, Austrian credit institutions have also had to report the value of the collateral provided by each borrower subject to OeNB reporting requirements. The banks also inform the OeNB of their internal principles and rules for assessing collateral values and calculating provisions for bad loans.

As the quality and completeness of the empirical data can not be fully guaranteed at the moment, a fixed factor of 45% was chosen as an approximation for the loss given default as a percentage of the credit exposure in accordance with Basel II. This factor is multiplied by the exposure for each individual credit to calculate the loss given default per credit in euro. The exposure per borrower is currently defined as the higher of the amount utilized (utilization) and the available credit limit.

In the future, improved data in the field of collateral reporting will allow a more accurate (and individual) estimation of the LGD.

#### **8.3.5 Calculation of the Risk Factor's Standard Deviation**

The expected default probabilities and standard deviations for each individual credit are added up (subject to a single risk factor, the defaults are conditionally independent) to calculate the expected number of defaults and the standard deviation per band. Subsequently, the expected number of defaults and the

standard deviation for the total portfolio are calculated by adding up the expected defaults and standard deviations.

The standard deviation  $\sigma_k$  of the risk factor  $k$  is thus estimated by adding up the standard deviations for all borrowers (the model implemented only has one risk factor).

#### 8.4 Detailed Description of the Model

CreditRisk<sup>+</sup> operates on the assumption that defaults occur randomly at a certain point in time, thus making it impossible to predict the exact time or the total number of defaults. Moreover, a credit institution usually has a large number of individual exposures with very low individual default probabilities.

At the end of each period, there are only two possible states for each borrower: default or non-default. If the borrower defaults, the loss realized amounts to the borrower's exposure at that time multiplied by the LGD factor. The time horizon chosen for risk modeling – one year – is not prescribed by CreditRisk<sup>+</sup> but is in line with the commonly used observation of one-year default probabilities.

Stochastic default probabilities are assumed. This assumption is more realistic; fixed default probabilities per loan do not appear appropriate, as empirical studies show that default probabilities are subject to large fluctuations. Various factors such as the general situation of a country's economy influence the condition of borrowers and thus their default probabilities.

In general, the default correlations are driven by a vector  $K$  of independent risk factors  $x = (x_1, \dots, x_K)$ . In order to keep the description of the model as general as possible, the following explanations are based on the assumption of multiple risk factors, although the model applied in Austrian off-site bank analysis uses only one risk factor (i.e.  $K=1$ ).

Contingent on a certain realization of  $x$  (i.e. assuming a specific economic environment), borrower defaults are assumed to be independently binomially distributed. The conditional probability  $p_i(x)$  of a borrower default  $i$  is thus defined by the following function:

$$p_i(x) = \bar{p}_i \left( \sum_{k=1}^K x_k w_{ik} \right),$$

with  $\bar{p}_i$  indicating the borrower's unconditional default probability. The risk factors  $x_1, \dots, x_K$  take positive values and each have an expected value of 1. The intuitive assumption underlying this specification is that the risk factors serve to scale the unconditional default probability. The weights  $w_{ik}$  represent the borrower's sensitivities to the respective risk factors and must add up to 1 for each borrower.

Modeling involves two steps: In the first step, the possible number of portfolio defaults are calculated and used to generate a frequency distribution. In the second step, the distribution of the credit positions is used to generate the distribution of losses.

### 8.4.1 Modeling the Occurrence of Defaults

Instead of modeling the distribution of the number of defaults directly, CreditRisk<sup>+</sup> calculates the generating function for the number of defaults. The generating function  $F_s(z)$  of a discrete random variable  $S$  is a function of a formal variable  $z$ , for which the probability of  $S = n$  is defined by the coefficient of  $z^n$  in the representation of  $F_s(z)$  as a polynomial. The mathematical properties of the generating function therefore make it easier to develop an analytical solution for the desired distribution.

First of all, the conditional generating function  $F(z | x)$  is determined for the number of defaults in the portfolio given realization  $x$  of the risk factors. For borrower  $i$ , this is the generating function for the binomial distribution:

$$F_i(z | x) = (1 + p_i(x)(z - 1)).$$

Using the approximation  $\log(1 + y) \approx y$  for  $y \approx 0$ , the following is true:

$$F_i(z | x) = \exp(\log((1 + p_i(x)(z - 1))) \approx \exp(p_i(x)(z - 1)).$$

As the right side of the expression above is the generating function of a Poisson-distributed random variable, this step is also referred to as “Poisson approximation”. What this means is that as long as  $p_i(x)$  remains small, the condition that a borrower can only default once can be ignored.

The exponential form of the Poisson distribution’s generating function is essential to solving the model numerically.

Contingent on  $x$ , the defaults of the individual borrowers are independent, therefore

$$F(z | x) = \prod_i F_i(z | x) \approx \prod_i \exp(p_i(x)(z - 1)) = \exp(\mu(x)(z - 1)),$$

where  $\mu(x) \equiv \sum_i p_i(x)$ .

In the second step, the unconditional generating function  $F(z)$  is found by integrating over  $x$ . In order to derive an explicit formula, it is necessary to make an assumption concerning the appropriate distribution for each risk factor  $x_k$ . All risk factors  $x_k$  are assumed to be independent  $\Gamma$ -distributed random variables with an expected value of 1 and a variance of  $\sigma_k^2$  as parameters. The distribution is fully defined by expected value and standard deviation and has the advantage of allowing – in connection with the Poisson distribution – analytically comprehensible solution paths for the CreditRisk<sup>+</sup> model.

As described in Section 8.3.5,  $\sigma_k$  is estimated empirically in our specific case.

The following applies:

$$F(z) = \prod_{k=1}^K \left( \frac{1 - \delta_k}{1 - \delta_k z} \right)^{1/\sigma_k^2} \text{ where } \delta_k \equiv \frac{\sigma_k^2 \mu_k}{1 + \sigma_k^2 \mu_k} \text{ and } \mu_k \equiv \sum_i w_{ik} \bar{p}_i.$$

The form of this generating function shows that the total number of defaults in the portfolio at the end of next year is the sum of  $K$  independent, negatively binomially distributed random variables.

### 8.4.2 Modeling the Probabilities of Credit Losses

The second step involves finding the generating function  $G(z)$  for the losses. In this context, we make the preliminary assumption that the loss is a constant component  $\lambda$  of the credit exposure ( $\lambda \equiv LGD - \text{factor}$ ).

Let  $L_i$  be the exposure of borrower  $i$ . In order to take advantage of discrete representation of the exposure and to minimize numerical effort by reducing the input data volume,  $\lambda L_i$  (losses per borrower) needs to be shown as whole multiples of a fixed number (unit), for example EUR 100,000. This number is denoted by  $v_0$ . The loss per borrower  $i$  standardized in this manner is denoted by  $v(i)$  and equals  $\lambda L_i/v_0$  (loss expressed in multiples of  $v_0$ ), with this value being rounded up to the next integer.

Let  $L_i$  be the generating function for borrower  $i$ . The probability of loss  $v(i)$  of units in a portfolio consisting exclusively of credits to borrower  $i$  must equal the probability that this borrower will default, resulting in  $G_i(z | x) = F_i(z^{v(i)} | x)$ .

The conditional independence of defaults is used to determine the conditional generating function for losses in the entire portfolio:

$$G(z | x) = \prod_i G_i(z) = \exp\left(\sum_{k=1}^K x_k \sum_i \bar{p}_i w_{ik} (z^{v(i)} - 1)\right).$$

As above, we now integrate over  $x$  and get the following:

$$G(z) = \prod_{k=1}^K \left( \frac{1 - \delta_k}{1 - \delta_k P_k(z)} \right)^{1/\sigma_k^2}, \text{ where } P_k(z) \equiv \frac{1}{\mu_k} \sum_i w_{ik} \bar{p}_i z^{v(i)}. \quad (1)$$

On the other hand,  $G(z)$  can be developed in a Taylor series with the unconditional probability that the entire portfolio will have  $n$  loss units corresponding to the coefficient of  $z^n$  in the Taylor series of  $G(z)$ . The coefficients of the Taylor series are derived from (1) by means of a recursive connection described in the next section.<sup>10</sup>

### 8.4.3 Iterative Algorithm to Calculate Credit Risk

The iterative algorithm proposed in the CreditRisk<sup>+</sup> model's documentation is based on Panjer recursion, which assumes that the logarithm of the generating function can be defined as a rational function of the form  $A(z)/B(z)$  with the polynomials  $A(z)$  and  $B(z)$ . For certain input constellations, this standard algorithm shows numerical instabilities which arise from the accumulation of rounding errors due to the addition of numbers of similar absolute values but different signs.

The project described here implements an alternative algorithm which is numerically stable, as proven analytically by Haaf et al. (2003). This iterative algorithm's numerical stability<sup>11</sup> results from the fact that only non-negative numbers are added in the key calculations of the iteration. Additionally, the algorithm was improved in order to ensure stability for small variances.

<sup>10</sup> This approach is described in detail by Gordy (1998).

<sup>11</sup> This numerical solution was originally proposed by Giese (2003).

## 8.5 Presentation of Results

The total loss and frequency distributions are calculated periodically for all Austrian credit institutions. In particular, this makes it possible to show the expected and unexpected losses of the credit portfolio, to quantify the risk contributions of individual (large) loans, and to identify concentration risks.

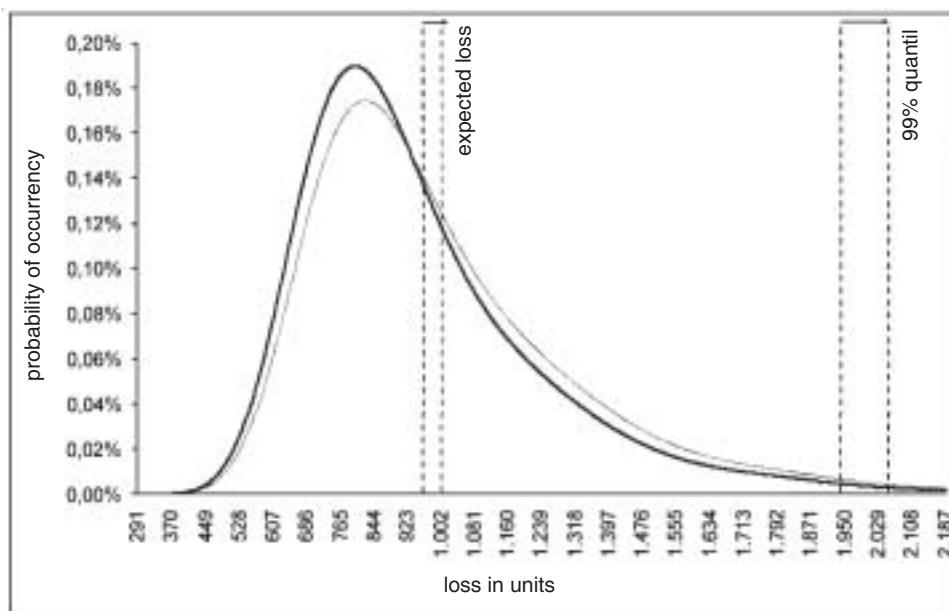


Figure 9: Scenario analysis

Further calculations for individual banks, bank groups, and sub-portfolios or groups of associated customers can also be made as a basis for further analyses. By changing the input parameters, it is possible to define scenarios (e.g. a weak economy can be simulated by increasing the expected default probabilities or the volatilities) and to analyze the effects these changes have on the credit VaR.

Figure 9, for example, shows how the overall loss distribution would shift if the default probability of an industry group in which the credit institution holds 50% of its exposure were to increase by one standard deviation.

When comparing the results of different credit institutions, the absolute VaR is placed in relation to overall credit exposure, total assets, or coverage capital.

## 8.6 Further Development of the Model

More sophisticated algorithms and more detailed data will contribute to the continuing improvement of the credit risk model in line with increasingly rigorous banking supervision requirements.

### 8.6.1 Problems Relating to Estimating Data and Parameters

#### Major Loans Register rating data

In this context, the general objective is to estimate individual default probabilities on the basis of the rating information in the Major Loans Register in the future. The Austrian credit institutions have been reporting rating data since the beginning of 2003. On the one hand, it is necessary to ensure reporting

and mapping quality; on the other, this time series is currently not long enough to allow us to estimate the volatility of the default probabilities. Given improved data, this information is to be integrated more strongly in the modeling process.

### **Collateral and provisions for bad loans in the Major Loans Register**

Since the beginning of 2003, Austrian credit institutions have also had to report the value of collateral and the amount of provisions for bad loans for each borrower subject to OeNB reporting requirements. By submitting system documentation, the banks also inform the OeNB of their internal principles and rules for assessing collateral values, calculating provisions for bad loans, and assigning internal credit ratings. Using these data will allow more precise estimation of LGD in the future.

### **Estimation of industry and borrower default probabilities**

Approximating the expected default probability and standard deviation for an individual borrower with the corresponding industry values requires top-quality industry default data and reliable industry classifications. The current data quality can only be considered moderately satisfactory.

Alternative methods/data sources may be considered for calculating default probabilities and volatilities for each industry and borrower. The primary source from which these alternative data may be gathered is the data reported to Austria's regulatory authorities.

### **Estimation of the risk factor's volatility**

The results of the model are extremely sensitive to the variance of the risk factor. A sound estimate of this variance is very important and constitutes an essential element of the planned improvements in the credit VaR model.

#### **8.6.2 Multi-factor Model**

From a theoretical point of view, a multi-factor model has the advantage of depicting portfolio effects (diversification or concentration) more adequately. The current model assumes that the stochastic, industry-specific default probabilities are determined by a common  $\Gamma$  distribution. This approach ignores the fact that industries may react differently to economic developments, thus it is desirable to implement several stochastic factors in the model.

#### **8.6.3 Validation**

Validating the results generated by the structural credit risk model is problematic for a variety of reasons. First of all, there are no suitable benchmarks, and second, the available data allow only limited back testing at best (the central problems are the lack of time series information and the cross-sectional stochastic dependence of credit risks, see Bühler et al. (2002)).

In principal, validation can be based on a number of methods:

1. Analytical checks: The model can be applied to simulated portfolios for which the result can be calculated analytically, or for which it possible to recalculate certain defaults;
2. Validation of individual input parameter estimates such as the volatility of the LGD factor's default probabilities;

3. Testing of model assumptions (specification tests) or examining the sensitivity of results depending on individual model assumptions;
4. Application of quantitative methods to evaluate the predictive power properties which are independent of the assumptions with regard to the correctness of the underlying model specifications (correctness of calibration), see Bühler et al. (2002);
5. Analysis of the results in comparison to results from other analysis tools;
6. Verification of the results' plausibility in discussions with bank analysts.

## 9 Market Risk

Generally accepted industry standards were established to measure market risk far earlier than in the case of credit risk, not least due to initiatives by companies such as J.P.Morgan (RiskMetrics) and supervisory efforts such as the 1996 Basel Market Risk Amendment and the Capital Adequacy Directive. With regard to positions of the trading book, relevant guidelines which require institutions to measure and meet regulatory capital requirements for market risk have existed for years.

Thus far, market risk has played a less significant role than credit risk, even if that role has been growing due to increasingly volatile markets. Until a few years ago, the equity of some banks was threatened mainly by the poor quality of the credit portfolio. To an increasing extent, however, bank capital is exposed not only to credit risk, but also to interest rate and F/X risks. However, market risk – in contrast to credit risk, which usually builds up over an extended period of time – can take effect almost immediately and destroy equity unless the appropriate countermeasures are taken without delay.

### 9.1 Description of the Methods Used to Determine Market Risk

Therefore, market risk also has to be assessed adequately in the process of recording the total risk of all Austrian credit institutions in a structured manner. For this purpose, Value at Risk is calculated for the following risk categories:

- interest rates
- stocks
- foreign exchange (excluding gold)

Other market risks such as raw material price risks are not covered by the model.

In terms of method, the variance/covariance approach – a parameterized procedure which is sufficiently determined by historical volatilities and correlations – is used for the calculation. Compared to other methods (simulation procedures), the main advantage of this approach is its lower calculation intensity and thus the earlier availability of results. Volatilities and correlations are obtained from the RiskMetrics data service offered by J.P.Morgan, with the time series being applied in their exponentially weighted form and with a decay factor of 0.94.

These calculations yield the following results:

### Absolute VaR (diversified and undiversified):

By default, the VaR is calculated for a confidence level of 95% and a holding period of one year (=250 days). A comparison of diversified and undiversified VaR is performed in order to provide information about the respective portfolio's diversification level. The undiversified VaR is calculated by means of a correlation matrix which assumes the correlations between market risk actors to be perfectly positive (all correlation coefficients equaling +1).

### VaR distribution:

Beyond the default calculation for the 95% confidence level, the entire VaR distribution for confidence levels between >50% and <100% is shown as well; the scaling factors applied correspond to normal distribution:

one-sided confidence interval	scaling factor
99.00%	2.33
98.00%	2.05
97.00%	1.88
96.00%	1.75
95.00%	1.65
...	...
53.00%	0.08
52.00%	0.05
51.00%	0.02

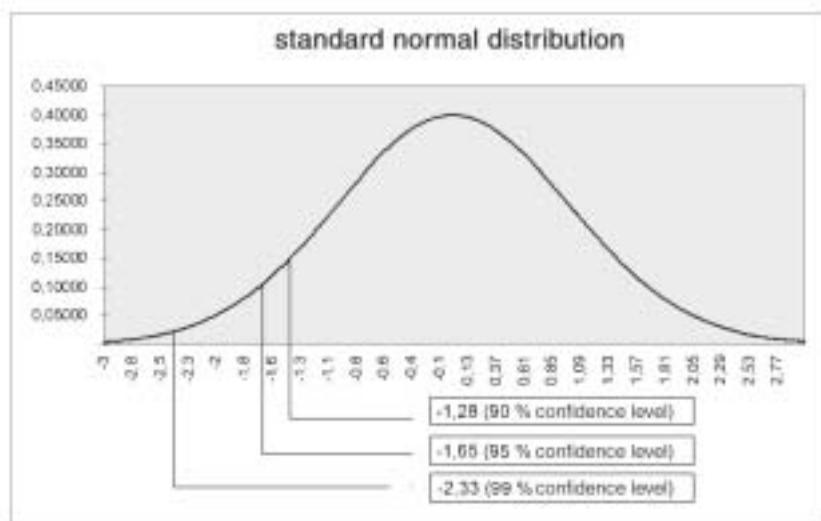


Figure 10: VaR for the market risk

### Relative VaR:

In order to enable the comparison of individual credit institutions, the absolute VaR is placed in relation to the portfolio's market value as well as total assets.

- relative  $VaR_1 = \text{absolute VaR} / \text{portfolio market value}$
- relative  $VaR_1 = \text{absolute VaR} / \text{total assets}$

### Incremental VaR:

Finally, the incremental VaR is used to illustrate the contributions of individual risk positions to the total VaR. The sum of all incremental VaRs equals the total VaR. Above all, this instrument makes it easier to identify those risk positions which (if reduced) can bring down the total VaR most significantly.

The main weaknesses of the parameterized approach, which is based on an assumption of normal distribution, are its disregard for the existence of "fat tails" in the actual distribution of price changes and its inaccuracy for non-linear portfolios. Therefore, it needs to be pointed out that the VaR calculation performed in the structural model only yields linear risk but not the risk of non-linear positions, which is especially relevant in option positions and the calculation of which is usually attempted by means of analytical approximation (Taylor series expansion) or structured Monte Carlo simulation. This simplification is the result of the present data situation, which currently does not allow a more sophisticated approach. Therefore, one has to accept that the VaR calculation presented here tends to underestimate the risk of institutions with very large option positions, as only delta risk is recorded but gamma and vega risks are not taken into account.

Furthermore, it must be noted that the assumed holding period of one year is in no way without its problems. The time scaling of the one-day VaR using the square-root-of-time rule tends to overestimate risk over such a long period for a number of reasons (possibilities to change the structure of the portfolio immediately, mean reversion, autocorrelations, etc.). However, as it is necessary to operate on the basis of a uniform holding period within the overall structural model and this assumption has to be aligned with the credit model in particular, a conscious decision was made to tolerate this simplification as well.

## 9.2 Data Model

The calculations are based exclusively on the data reported in the monthly reporting requirements and the market data provided by J.P. Morgan (interest rates, market prices, volatilities, correlations). After suitable preparation, the data from the monthly statements are transformed into position data (risk loadings).

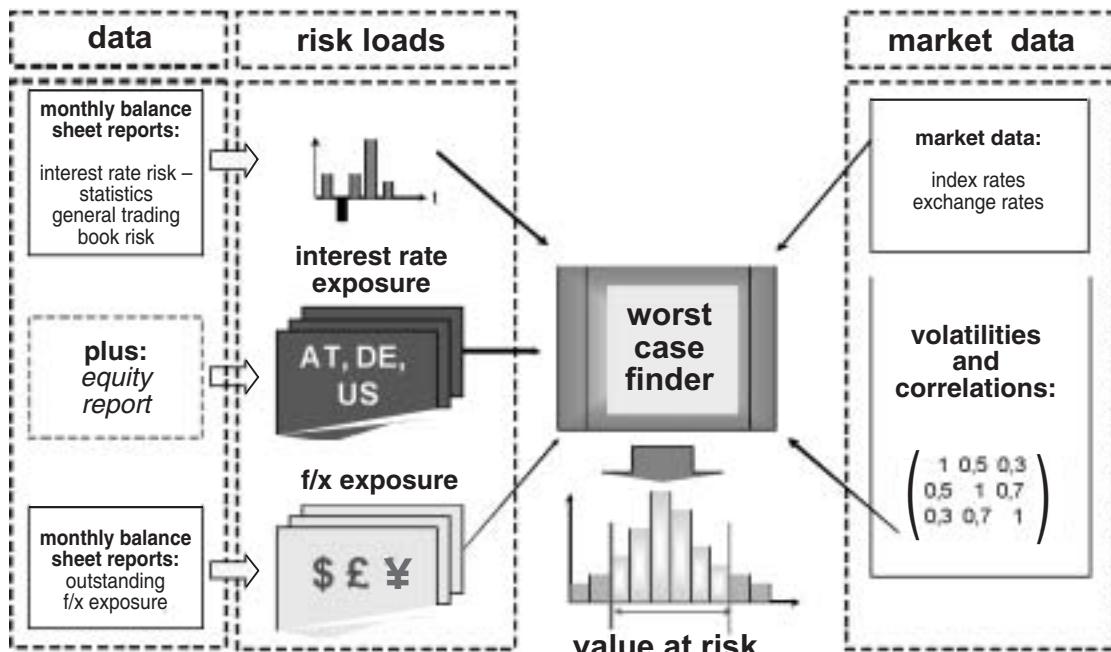


Figure 11: Data preparation for the market risk

### 9.2.1 Description of the Data Reported

#### Interest rate positions

The interest rate risk statistics (monthly balance sheet reports, Part B2) provide data which are basically suitable for banking book calculations (as well as trading book calculations, as long as the limits of §22 para 2 Austrian Banking Act – the so-called “small trading book” – are not exceeded). This report is a simplified table showing exposures sorted by maturity bands in terms of fixed interest periods. All interest-rate sensitive and maturity-driven positions are assigned to the respective maturity bands according to their maturity bands. Products with indefinite interest maturities are estimated by the credit institutions in terms of their maturity and shown as fixed interest rate positions (or as a series of several fixed interest rate positions in cases where such products are shown as replicating portfolios) and assigned accordingly. All interest-rate-sensitive derivatives are shown as synthetic balance sheet items based on their delta equivalent.. Positions with a variable interest rate are classified in the maturity bands for those maturities which correspond to their reference interest rates in the term structure of interest rates. There are a total of 13 maturity bands. The interest rate risk statistics has to be split up by currency, and the currencies EUR, USD, CHF, GBP, JPY, and CAD have to be shown separately. All other currencies have to be aggregated and combined in a residual table. Obviously, the other currencies cannot find their way into the Value at Risk calculation, as it is not possible to assign the corresponding market risk factors. In this way, a comprehensive and consistent representation of all interest-rate business in the banking book and the “small trading book” (with the exception of the other currencies) is ensured.

However, the situation is different for the “comprehensive securities trading book”: Credit institutions whose trading books exceed the limits of §22 para 2 Austrian Banking Act have to subject these positions to a separate process in order to meet regulatory capital requirements and do not assign the interest rate exposures in the trading book to interest rate risk statistics. For institutions without trading books, or with a trading book which is below the limits of §22b para 2 of the Austrian Banking Act, all of a bank’s interest rate risk positions are covered by interest rate risk statistics, while the other institutions only report the interest rate positions in the banking book.

### **F/X Positions**

Austrian credit institutions report their foreign exchange exposures under Part C of the monthly statements, which contains the highest level of foreign exchange exposure for each currency and month. As the total foreign exchange position has to be included in this report, derivative financial instruments are also included in this calculation.

### **Equity positions**

Stock-related information is currently shown in Part A of the monthly statements. However, these data are not classified by markets and therefore cannot be assigned to market risk factors. Furthermore, off-balance sheet items (e.g. stock options) are not included in the report, which – in its current form – is therefore not suitable for calculating VaR.

### **9.3 Transformation of Reported Data into Risk Loads**

The data reported in the interest rate risk statistics have a number of characteristics which render them unsuitable for direct implementation in the VaR model. Therefore it is necessary to make a few simplifying assumptions in order to transform the data into risk loads.

### **Maturities**

As the interest rate positions are reported in maturity bands, the specific maturity per position is not known. Therefore, the structural market risk model assumes that the maturities correspond to the middle of each maturity band:

maturities of the interest rate risk statistics													
short-term (up to 1 year)				medium-term					long-term				
up to 1 m	> 1 m to 3 m	> 3 m to 6 m	> 6 m to 1 y	> 1 y to 2 y	> 2 y to 3 y	> 3 y to 4 y	> 4 y to 5 y	> 5 y to 7 y	> 7 y to 10 y	> 10 y to 15 y	> 15 y to 20 y	over 20 y	
↓	↓	↓	↓	↓	fictitious maturities of the exposures in years								↓
0,04	0,17	0,38	0,75	1,5	2,5	3,5	4,5	6	8,5	12,5	17,5	25	↓

### **Coupon payments**

The interest rate risk statistics only show the principal amounts, not interest payments. This means that the reported cash flows are incomplete and have to be supplemented with additional assumptions concerning interest payment

amounts. Therefore, it is assumed that interest is paid regularly in the following product categories:

- fixed-interest receivables and liabilities
- variable-interest receivables and liabilities
- receivables and liabilities with an indefinite interest rate commitment, and
- swaps.

Current maturity-related market data are used to make assumptions about the amount of interest.

### Cash flow mapping

As the market data supplied by the data provider are available only for predefined default maturities (referred to as grid points) but the fictitious maturities of the positions do not match those grid points, the positions are mapped to the closest standard maturities. The principles of risk-neutral mapping (as detailed in the RiskMetrics Technical Document) are applied in this process.

### 9.4 Detailed Description of the Model

As all non-linear instruments were already accounted for by the banks themselves, calculating the VaR is relatively simple:

- Interest rate positions are mapped to fictitious bonds and zero coupon bonds;
- Stock positions are mapped to the respective indices;
- Foreign exchange positions are included in the calculation as cash positions.

The risk loads are derived for each risk factor by algebraic addition of the individual mapped cash flows. For the delta-normal method implemented, the VaR is calculated using the following formula:

$$VaR = \sqrt{V^T \times K \times V} \times \sqrt{t} \times F$$

where:  
V = vector of the risk loads  
K = covariance matrix  
 $V^T$  = transposed vector of the risk loads  
t = time (in days)  
F = correction factor for the desired confidence level

$V$  is an  $(m \times 1)$ vector  $\begin{vmatrix} 1 \\ \cdot \\ \cdot \\ m \end{vmatrix}$ ;  $V^T$  is a  $(1 \times m)$ vector :  $\overline{1 \dots m}$

$K$  is an  $(m \times m)$ matrix  $\begin{vmatrix} 1 & \cdot & \cdot & \cdot & 1m \\ \cdot & \cdot & & & \\ \cdot & & \cdot & & \\ \cdot & & & \cdot & \\ m1 & \cdot & \cdot & \cdot & mm \end{vmatrix}$ .

Details about these calculations can be found in the RiskMetrics Technical Document.

#### 9.4.1 Calculating the Incremental VaR

The procedure used to calculate VaR shows that Value at Risk is not additive, neither in terms of risk factors nor in terms of assets. This additivity would only exist given a perfectly positive correlation (i.e. with all correlation coefficients equaling one). In any other case, the VaR of a portfolio is smaller than the sum of the VaRs of the individual portfolio assets.

This lack of additivity in the VaR indicator led to the definition of incremental Value at Risk. The incremental VaR of a securities position is that contribution to VaR made by this position in the context of the portfolio. Formally, this is expressed by deriving VaR for the risk weight  $f_i$  (incremental VaR of a risk factor) or the asset weight  $w_k$  (incremental VaR of an asset) multiplied by the specific weight value:

- incremental VaR of the risk factor:  $R_I$

$$incVaR_{R_i} = \frac{\delta VaR_p}{\delta f_i} f_i = VaR_p \frac{\sigma(R_i, P_R)}{\sigma^2(P_R)} f_i$$

- incremental VaR of Asset  $A_k$ :

$$incVaR_{A_k} = \frac{\delta VaR_p}{\delta w_k} w_k$$

## 10 Operational Risk

The next section presents the methods of calculating operational risk under Basel II, the calculation in our structural model, and the weaknesses of the selected approach.

### 10.1 Significance of Operational Risk

Basel II defines operational risk as the “risk of loss resulting from inadequate or failed internal processes, people and systems or from external events. This definition includes legal risk, but excludes strategic and reputational risk.”

This type is a major source of risk for banks, as we have seen in international studies which break the results of various banks' economic capital models down into their various risk components. According to these studies, a significant portion of economic capital is determined by operational risk, an assertion which also appears to be true in Austria. Furthermore, losses from operational risk events have been the cause of bank failures in many cases, both domestically and internationally.<sup>12</sup>

Operational risk is thus a factor which cannot be ignored for the purposes of banking supervision in general and off-site analysis in particular, and it should be measured and evaluated for all banks in the future.

## **10.2 Basel II and the Determination of Operational Risk**

In its new regulatory framework, the Basel II committee has also acknowledged the importance of operational risk and therefore included it in their considerations. On the one hand, Basel II provides for a regulatory capital requirement for operational risk, which can be calculated using at least three different approaches. On the other hand, the second pillar explicitly calls upon banks to set up adequate risk management processes which can be used to measure and reduce material operational risk exposure and to take precautions to ensure continued operations in the event of operational losses.

However, we will demonstrate below that the rather simple approaches in current Basel II regulations – which are likely to be applied by most banks – do not measure actual operational risk adequately.

### **10.2.1 Basic Indicator Approach**

In the simplest of all approaches, the capital requirement is calculated as a fixed percentage (15%) of the bank's gross income. This approach is based on the notion that higher gross incomes are usually linked with higher operational risks and may thus serve as a very rough approximation of operational risk. However, there are serious doubts as to the risk sensitivity of this approach, which renders it rather unsuitable for off-site analyses in the long run.

### **10.2.2 Standardized Approach**

Under the standardized approach, the bank's activities are divided into eight standard business lines, and the average gross income is calculated for each business line over the three previous years. Each of these amounts is multiplied by a different beta factor (12-18%), with the sum of these amounts equaling the capital requirement. The standardized approach assumes different risk situations in the various business lines of a bank. For example, it is assumed that retail banking is subject to a lower level of operational risk than corporate finance; this assumption is reflected in different factors. While the standardized approach certainly reflects a bank's operational risk better than the basic indicator approach does, the basic assumption that it is possible to capture operational risk using gross income remains doubtful.

A number of qualitative criteria have to be met in order for a bank to receive approval for a standardized approach. Among other things, banks are required

<sup>12</sup> e.g. Deutsche Morgan Grenfell (1996); BankersTrust (1994-1998); Barings Bank (1995).

to establish a loss database, a working operational risk management mechanism which is integrated into operations, suitable reporting, documentation, and internal as well as external validation of the systems.

### **10.2.3 Advanced Measurement Approaches**

When it comes to advanced measurement approaches, banks enjoy a large degree of discretion. One of the objectives is, for example, to determine a bank's actual operational risk based on historical experience. According to calculations using the loss database, those losses which will not be exceeded within one year with a probability of 99.9% are to be covered by equity. In addition to this high quantitative threshold, which means that higher losses from operational risk events may be expected only once in 1,000 years (or for one in 1,000 banks), banks are also required to meet comprehensive qualitative criteria. Among other things, banks have to establish an independent, conceptually sound and extensively documented risk management system whose regular reports to business line managers, the supervisory board and management board have an impact on risk management and reduction.

Advanced measurement approaches are certainly best able to determine actual operational risks. However, the high quantitative and qualitative requirements make it doubtful from today's perspective that the investment in developing an advanced equity-based approach will pay off for smaller banks.

Neither the Basel nor the Brussels Consultation Paper states specific requirements as to the way in which such measurement approaches are to be designed. However, the Basel Committee's "Working Paper on the Regulatory Treatment of Operational Risk" outlines three methods which are being applied in practice with increasing frequency:

#### **Internal ratings-based approaches**

These approaches assume a fixed relation between the expected loss (as the expected value of the loss distribution) and the value at the corresponding confidence level. This allows a rather simple calculation of the capital requirement for each combination of risk type and business line, as the relation between expected value and extreme events has to be determined only once and can then be assumed to be constant.

#### **Loss distribution approaches**

Under loss distribution approaches, the distribution of operational losses is estimated for each combination of risk type and business line, then the corresponding confidence level for each of these distributions is assumed in order to determine the capital requirement. As a rule, two separate distributions are assumed in each combination of risk event and business line, for example a Poisson or a geometric distribution for the number of losses, and a log-normal or an exponential distribution for the amount of losses. The VaRs resulting from these combinations can then simply be added up or aggregated using an assumed correlation matrix.

### Scorecard approaches

In scorecard approaches, the capital requirement is determined for operational risk and then modified by a qualitative forecast of operational risks in the individual business lines. However, such scorecard approaches need to be sufficiently substantiated on a quantitative basis and validated by historical data. In general, scorecards can capture operational risks in quite some detail and from an anticipatory perspective, while quantification using a potential amount of loss at a given confidence level proves to be more difficult.

No industry standard has been established to date, but many medium-sized and large banks are setting up loss databases to make it possible to quantify operational risk as well as developing scorecards to capture, manage, and limit operational risk more effectively. What all approaches have in common is that both obtaining the necessary data and applying them in a methodically correct manner often cause problems in practice.

However, advanced measurement approaches generally reflect operational risk appropriately, which does make it possible to use the resulting findings in off-site analyses. Due to the relatively high demands on systems and risk management, however, we cannot expect a large number of Austrian banks to consider developing advanced approaches in the near future.

### 10.3 Selected Procedure

Due to the lack of a complete database, Austria's regulators currently have only limited means of capturing operational risk comprehensively and, in particular, quantifying operational risk through off-site analysis.

Until it is possible to determine operational risk in a theoretically sound manner by qualitative recording and quantification using a loss database, the loss distribution for operational risk is determined as follows:

1. An indicator for K<sub>Basel II</sub> operational risk is defined using the basic indicator approach under Basel II.
2. We assume a geometric distribution with the parameter p for the actual distribution:

$$f(n) = p(1-p)^{n-1} \quad E(N) = p, \quad Std(N) = \frac{1-p}{p^2}$$

3. We assume an exponential distribution with the parameter ...λ for the amount of losses:

$$g(x) = \lambda^{-\lambda x} \quad E(X) = Std(X) = \lambda$$

4. For the loss distribution, this yields  $S_n = \sum_{i=1}^n X_i$

$$P(S \leq s) = \sum_{n=1}^{\infty} P(S_n \leq s | n) f(n)$$

5. However, as the sum of identical independently distributed random variables follows a gamma distribution, the distribution function is defined by

$$P(S_n \leq s | n) = \int_0^s \frac{1}{(n-1)!} \lambda^n u^{n-1} e^{-\lambda u} du$$

6. Or, after solution of the equation:

$$P(S \leq s) = 1 - e^{-\lambda p s}$$

7. This means, however, that the losses from operational risk again follow an exponential distribution with only one parameter  $\mu = \lambda^* p$ :

$$h(s) = (\lambda p) e^{-(\lambda p)s}$$

8. This also means the VaR for operational risk at confidence level  $c$  is clearly defined by:

$$E(S) - s^* = \left(\frac{1}{\mu}\right) [\ln(1 - c) + 1]$$

9. If we then interpret the indicator KBasel II found in Item 1 as a VaR for a certain confidence level, we can estimate the missing parameter  $\mu$  and thus the entire loss distribution for operational risk. This also makes it possible to calculate the VaR for other confidence levels and to perform aggregation with the other types of risks.

The advanced Basel II approaches calculate the capital requirement as the 99.9% confidence level for expected and unexpected losses. If we assume that a calibration to these values was also used for the simpler approaches, they could be used to approximate and estimate the parameter  $\mu$ :

$$K_{Basel\ II} = \left(\frac{1}{\mu}\right) [\ln(1 - 0,999) + 1]$$

However, banks have indicated that the results for the equity requirement are clearly different under the advanced measurement approaches compared to the simpler approaches, as the capital requirement found using advanced approaches is sometimes considerably higher than in the simpler approaches. Thus, there is also the possibility of determining the confidence level implied in the simpler approaches from the calculations of those banks which are already in a position to use simple as well as advanced calculation methods for operational risk.

However, in order to ensure methodical consistency with the supervisory regulations in Basel II, the confidence level applied there is still used, even if this might lead to underestimates of actual operational risk. The actual implementation in calculating the model, however, allows flexibility in modifying this assumption.

The database used for the procedure described above is gross income according to the stated definition. These data are already available in the Austrian reporting system, thus it is possible to calculate the corresponding value as the average over the last three years.

Subsequently, it was possible to calculate the indicator in accordance with the Basel method using the moving values as a basis. Based on the process mentioned above, it was possible to determine the factor of the exponential distribution (formula:  $-(\ln(1-\text{confidence level})+1)/\text{capital requirement}$ ). The loss distribution assumed for operational risk is already defined completely by this factor.

### **Calculation of VaR for operational risk**

It is then easy to compute  $VaR_{\alpha}^{Op}$  by generating the respective confidence levels of the exponential distribution, a procedure which also ensures aggregability with the other risks.

### **II Capacity to Cover Losses**

As a framework for risk management, risk-bearing capacity forms an important basis for management control in banking. It is essential to identify in this respect the extent to which a bank can afford to assume risks.

In general, equity capital serves to cover potential risks; depending on the perspective chosen, one can look at the book value or the net asset value of equity, or use the regulatory definition of capital. The latter definition, however, allows for example hidden reserves only to a limited extent, although they might well be used as potential cover in an internal calculation. As there are other items besides regulatory capital that may be used as coverage capital, it seems appropriate to classify risk coverage capital into different categories.

This gives rise to the following questions concerning risk-bearing capacity for regulatory analysis:

1. What do the capacities to cover expected losses, the changes and level of provisions, and the profitability look like?
2. What do the capital requirements for solvency look like?
3. What does the coverage of expected and unexpected losses from an economic point of view look like?

The next section shows how and to what extent the capacities to cover incurred risks were defined in the structural model.

#### **11.1 Classification of Risk Coverage Capital**

Using a classification of the reserves, i.e. the capacity to cover losses, makes it possible<sup>13</sup> to allow for the fact that risks have different probabilities on the one hand, while the availability of financial resources varies strongly on the other.

##### **Level 1 reserves: internal provisions**

It is not clear whether it is permissible to cover expected losses in the calculation of risk-bearing capacity, as these losses are actually a cost factor which

<sup>13</sup> The definition is based on Schierenbeck (2002).

should be included as standard risk costs. Therefore, they are not a risk in the technical sense, as risks are usually defined as potential negative deviations from the expected result. However, they are listed here for the sake of completeness.

Should losses be realized within expectations, the first measure is to reverse the precautionary measures taken in terms of provisions and risk costs. Next, any profit exceeding expectations can be reduced. The expected minimum profit on paid-in capital, however, is only counted as part of level 3 reserves.

Profits and losses brought forward from previous years also have to be taken into account as additional, easily available risk provisions in the case of profits and as deductible items in the case of losses.

### **Level 2 reserves: accounting surpluses**

Should the incurred losses exceed expectations, it will be necessary to reduce hidden reserves to cover these losses. However, only those parts which do not represent tier 2 capital are treated as level 2 reserves. These hidden reserves arise in connection with securities, investments, and property, as well as excess risk or pension provisions.

The extent of the level 2 reserves to cover risk is certainly a major factor in risk coverage capital and thus needs to be assessed carefully in order to ensure the model's goodness of fit.

Especially level 1 (but also level 2) reserves may be used without attracting publicity, while this is hardly possible in the case of higher level loss coverage.

### **Level 3 reserves: easily available own funds**

Once hidden reserves have been exhausted, it is possible to absorb further losses by reducing or suspending the payout of dividends to the owners.

It is also possible to make use of the fund for general bank risks, which is already part of tier 1 capital. This fund comprises provisions which the credit institution considers necessary to cover special banking risks and thus the access to them must be unrestricted and immediate.

Any excess of own funds above the legally mandated minimum should also have to be considered part of the level 3 reserves, as it could be used in the case of losses without affecting the bank's solvency.

In general, the level 1 to 3 reserves are intended to cover exceptional negative events without threatening the bank's existence. However, as soon as a bank has used up its excess equity, it is likely that regulatory measures will be taken.

### **Level 4 reserves: further coverage by owners (tier 1)**

Should it prove necessary to tap level 4 reserves to cover a loss, this will cut into core balance sheet capital. Disclosed reserves and payed-up capital will be used up first, as these represent the equity which best meets the requirements of offsetting losses and which can be accessed by the credit institutions without restrictions at any time.

50% of the excess equity determined as part of the primary reserves has to be deducted. This split can be explained by the fact that subordinated capital can only be applied up to the level of tier 1 capital, which means that both positions would have to be reduced almost to the same extent if reserves are reversed.

Should realized losses not exceed the level 4 reserves, this would result in default without harming clients or requiring the intervention of a deposit insurance organization.

**Level 5 reserves: Capacity to cover losses:  
coverage by investors (tier 2 + 3)**

Only in extreme emergencies will the banks tap subordinated capital components. In many cases, such capital components are no longer included as risk cover in internal calculations, as – by definition – the non-repayment of subordinated capital already has to be classified as default in the broad sense of the term.

Level 5 risk cover consists of the sum of the tier 2 components (certain pre-defined hidden reserves, supplementary capital, participation capital, revaluation reserves, and the liability sum surcharge, i.e. additional cover by cooperative members) and tier 3 capital. Note that the available components – not the eligible or regulatory components – have to be included in this context. Exhausting quintary reserves ultimately leads to the bank's insolvency, but deposits should not yet be endangered at this stage. Bondholders, on the other hand, would already lose their capital in this case.

Should realized losses also exceed the level 5 reserves, however, this would result in a default, which would harm depositors and require the intervention of a deposit insurance organization.

Furthermore, when looking at the capacities to cover losses it must be noted that sector peculiarities such as "Besserungsgeld" (specific internal short-term liabilities) or intervention on the part of a solidarity fund are not taken into account at this stage.

## 11.2 Capacity to Cover Losses

The table below shows the relative composition of the capacities to cover losses in the two examples.

	bank A	bank B
for expected losses		
– reserves set up	17%	11%
– changes in Specific and general provisions	4%	3%
excess profit/loss for the year	1%	4%
profit/loss carried forward	0%	0%
primary capacity to cover losses	22%	18%
hidden reserves	18%	3%
intangible assets	0%	-1%
secondary capacity to cover losses	41%	20%
minimum profit	1%	4%
fund for general bank risks	0%	0%
excess equity	11%	10%
tertiary capacity to cover losses	53%	33%
disclosed reserves	26%	30%
subscribed capital	4%	6%
deduction item holdings	0%	-1%
deduction of 0.5 of excess equity (tier 1)	-5%	-5%
quartiary capacity to cover losses	76%	63%
supplementary capital	21%	25%
subordinated capital	8%	17%
deduction of 0.5 of excess equity (tier 2)	-5%	-5%
quintary capacity to cover losses	100%	100%

Clear differences also become evident in this case, especially if the tertiary capacity to cover losses is defined as the boundary between default and non-default. While Bank A has more than half of its cover in the tertiary reserves and can access these funds quite easily, the corresponding figure for Bank B is only one third. A substantial part of Bank B's capacity to cover losses results from available equity, which should only be used to cover losses in emergencies.

## 12 Assessment of Total Risk

After calculating the total bank VaRs for different confidence levels and calculating the various reserves, the next and final step is to combine these two components.

Following the explanations above, it is now necessary to determine the full loss distributions of the three types of risk described in order to calculate the bank's overall loss distribution. This is done by adding up the respective VaRs at the same confidence level.

This does not present a problem for credit, market, and operational risk, as the methods stated yield the complete loss distribution anyway. The VaRs can be evaluated for each predefined confidence level, and the expected loss can be calculated for all types of risks. Due to our assumption of normal distribution, the expected loss of market risk is always assumed to be zero.

## 12.1 Theoretical Background

Aggregating the individual risks leads to a total VaR. This value shows that the sum of the losses from the three risk categories (market, credit, and operational risk) will not exceed this level within a year with a probability of  $\alpha$ . From the regulatory perspective, this information concerning the absolute VaR is not particularly important. Instead, the total VaR becomes an indicator if the changes in value over time are observed (i.e. if the extent to which the VaR has changed upward or downward is recorded); alternatively if the economic capital determined using the VaR is related to the coverage capital. Let us assume that a bank's coverage capital is equal to  $D$ . We find the significance level for which

$$VaR^{total}(\alpha) - D = 0$$

One minus that significance level for which the total VaR equals the available coverage capital  $D$  is the bank's probability of default as determined by the model. The graph below illustrates this link. The y-axis of the graph shows the total VaR assigned to a certain significance level, while the x-axis shows the significance level. The curve, which is similar to a section of a hyperbola, shows the relation between the significance level and VaR. If the distributions of the individual loss categories have a support of  $(-\infty, \infty)$ , then the curve must approach the two axes asymptotically. If we now include the bank's coverage capital in the graph, we can immediately read the probability of default.

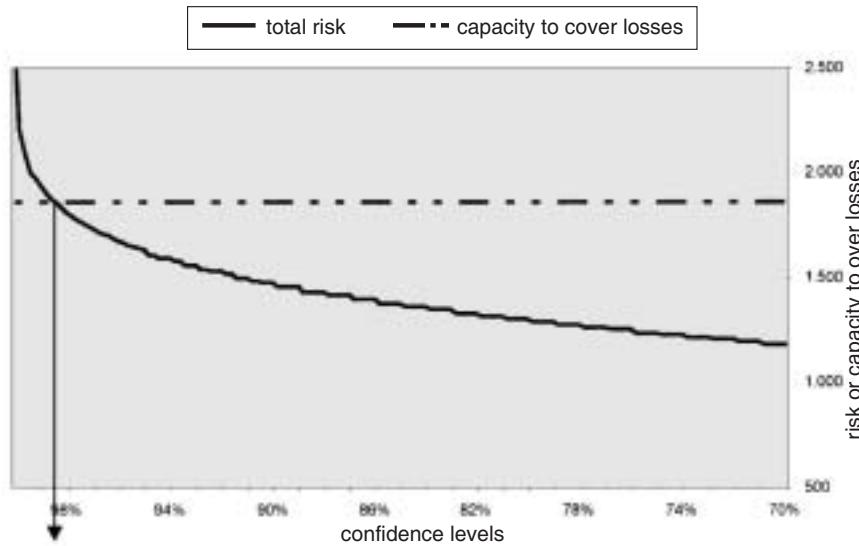


Figure 12: Economic capitals vs. coverage capital

## 12.2 Derivation of Implied Default Probabilities

Using the approaches suggested, it is possible to compare the loss distribution with the various reserves with due attention to the quality of the capital employed to cover losses. The first step usually consists in using the reserves for potential losses or any excess profit. Subsequently, hidden reserves are used,

or the fund for general bank risks may be tapped. Only once these sources have been exhausted will subscribed capital or disclosed reserves be utilized. Subordinated capital components (for more details on the capacity to cover losses see section 11) are touched only in extreme cases. Such hierarchy would allow several intuitively understandable interpretations.

The recommended calculation of the various default probabilities can be implemented directly based on the resulting values by tracing those VaRs in the loss distribution for the bank as a whole which exactly match the respective reserves. By definition, one minus the confidence level implied by the VaR is the corresponding default probability.

This approach can now be illustrated for our two sample banks:

<b>default probabilities</b>	<b>bank A</b>	<b>bank B</b>
probability of losses exceeding that level which was provided for within losses to be expected	>30%	>30%
probability that components of balance sheet equity (excess equity) need to be used to cover the losses, with the bank continuing to exist	1.60%	7.00%
probability that components of balance sheet equity need to be used to cover the losses, with the bank ceasing to exist; i.e. default without harming customers and without deposit insurance taking effect	under 0.01%	0.70%
probability that the bank's total equity (tier 1 & 2) is not sufficient to cover the losses, i.e. default harming customers and requiring deposit insurance	under 0.01%	0.05%

Interestingly, both banks show a high probability that the expected losses will exceed primary reserves. While the default probability is low for both institutions, there is a relatively higher risk for Bank B.

### **12.3 Reviewing Adherence to Coverage Conditions**

The coverage conditions described in section 11 can also be checked directly by means of the resulting total bank loss distribution and the reserves.

For this purpose, the following three loss events are defined and compared to the respective reserves:

1. Normal case: based on expected loss;
2. Negative case: based on a realization of losses at a confidence level of 95%;
3. Worst case: based on a realization of losses at a confidence level of 99.9%.

The results can be illustrated nicely by a traffic light:

	Probability	Result
expected loss < primary reserves	below 70%	exceeded
	below 70%	
potential risk in the negative case < primary to tertiary reserves	95.00%	covered
	98.40%	
potential risk in the worst case < primary to quartiary reserves	99.90%	covered
	>99.99%	

In our case, while the bank appears to have sufficient resources to cover even the worst case losses, the provisions for expected losses are insufficient.

### I 3 Summary

Since the fundamental goal is to develop a model for evaluating all banks operating in Austria, numerous assumptions had to be made in calculating economic capital.

For *credit risk*, we can see that the current reporting system (especially for loans below the Major Loans Register limit of EUR 350,000) allows only limited conclusions as to the actual risk in this area, and that rating information in Major Loans Register is only available from 2003 onward.

Similarly, *market risk* is integrated into the structural model in order to offer a coherent and comprehensive view of risk and to provide information on the extent of market risk and its contribution to total risk. Well-known and established industry standards for the quantitative measurement of *market risk* have existed for some time, so that the concept's implementation in regulatory activities will hardly cause any problems from a theoretical point of view. However, the Austrian regulatory authorities are well aware that given the nature of the current reporting data the VaR calculated in the structural model can only be an approximation. Nevertheless, this approximation appears to be sufficiently realistic for use in supervisory risk assessment and thus represents an appropriate tool within total risk analysis.

The weaknesses resulting from using gross income as an indicator in measuring *operational risk* have already been dealt with in detail. Similarly, the assumption that the capital requirement under Basel II corresponds to a VaR at a certain confidence level seems questionable. It must be noted that the regulators need new data on operational events in order to determine operational risk accurately, as otherwise the required risk sensitivity is not ensured.

In the first step, a pragmatic but reasonable approach to *aggregate* the risks was chosen, and this approach will certainly be refined in the coming years. In this context, it will be necessary to intensify our exchange of knowledge and experience with the banks, which currently face similar problems.

In spite of this less than optimal starting point, we have been able to develop a model which – based on numerous plausibility checks and consultations with experts – can be considered highly effective tool for bank risk analysis. The

FMA and the OeNB are convinced that these models bring Austrian banking supervision to a level comparable with the best European and international standards.



A N N E X

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