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MULTIREG –A Multiregional Integrated Econometric Input–Output Model for Austria*

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1. Introduction

Since Austria is a rather small country and its economy thus very open, attempts to move from the national to a regional level of macroeconomic modeling are not only hampered by severe data restrictions but also by the fact that Austrian regions are characterized by an extremely high degree of openness. This limits the usefulness of single region models since economic impacts from changes in economic policy or public investment projects mostly emerge not within the region where these policies or projects are implemented but in other Austrian regions. In addition, single region models are often top-down-type models where changes in regional economic activity (employment, output, consumption etc.) are derived from changes in the corresponding national variables. In modeling larger regions, e.g. the metropolitan region of Vienna, which accounts for almost 20% of the Austrian population, simultaneity, thus, becomes more and more problematic. Therefore, after having completed two single region models for the federal

* MULTIREG was developed by a the team of researchers including, in alphabetical order and by institutions, Raimund Kurzmann, Gerhard Streicher, Gerold Zakarias (all Joannem Research), Oliver Fritz, Kurt Kratena and Peter Mayerhofer (all WIFO).
We would also like to express our deep gratitude towards numerous members of Statistik Austria for their invaluable support of our work. Above all, Erich Greul and Rudolf Mazanek provided indispensable regional information and Erwin Kolleritsch was always willing to guide us through the shallows of input-output table compilation.
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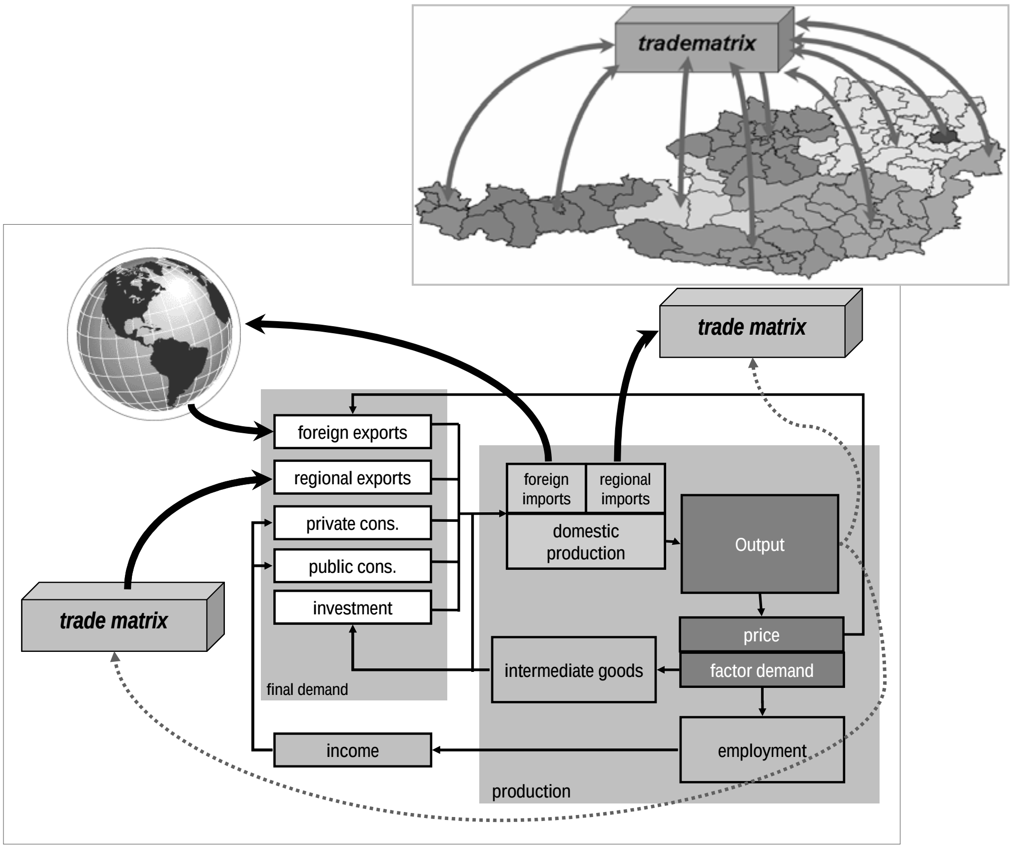
provinces of Styria and Upper Austria (Fritz et al., 2001; Zakarias et al., 2002), an attempt to bring all nine Austrian federal provinces into one multiregional model was undertaken.

MULTIREG integrates two model types, econometric models and input-output models, at the multiregional scale; a first and preliminary version has just been completed and is now undergoing extensive testing. The aim of building an integrated model is to benefit from the advantages of either model type and remedy their respective shortcomings. Integrating econometric and input-output models draws its motivation both from theoretical as well as practical aspects (Rey, 2000): for instance, instead of applying the linear production technology assumption of the standard input-output model, more flexible production functions may be estimated and included in integrated models. Similarly, instead of assuming final demand to be exogenous as is often the case in a pure input-output framework a more theoretically sound treatment of private consumption, investment etc. can be achieved when an econometric modeling approach is applied. A high degree of industrial disaggregation (MULTIREG comprises 32 industries, see also the Appendix), on the other hand, is often put forward as one of the main advantages of input-output models; this becomes especially important when the model is to be applied for impact analysis.

While the single-region models for Styria and Upper Austria were built very much in the tradition of Conway's integrated regional econometric input-output model (Conway, 1990), the modeling approach taken in MULTIREG is closer to the one implemented in MULTIMAC (Kratena, 1994; Kratena and Zakarias, 2001), which in turn was developed along the lines of the INFORUM model family (Almon, 1991) and the European multiregional model E3ME (Barker et al., 1999). This implies that compared to its predecessors MULTIREG not only replaces the single-region framework with a multiregional setting but relies to a much greater extent on functional forms consistent with microeconomic theory instead of pure statistically-driven variable relationships.

MULTIREG's model structure is illustrated in chart 1. A simple description of the model's solution algorithm may start out with total final demand, which is composed of private and public consumption, investment, and regional and foreign exports. This demand can be met either by importing commodities from other regions or abroad or by commodities produced by regional firms. While foreign imports (and exports) are still exogenously determined in the first version of the model but will later be modeled separately, regional imports (and exports) are established in the interregional trade block. Regional production is simulated in the output block, where output prices and factor demand are derived based on cost functions. Factor demand consists of intermediate inputs (which feed back to total regional demand) and labor. By generating income, labour influences final demand. Another feedback channel will operate via output prices, since changing relative prices lead to changes in the demand for foreign exports (and foreign imports). Finally, changing regional production patterns also lead to changes in regional trade patterns.

Chart 1: The Structure of MULTIREG



The paper first discusses conceptual and estimation issues in the construction of the multiregional input-output table for Austria, which is embedded in MULTIREG. The table is based on a multiregional make-use system which was derived from the national make-use system of the year 2000 using an extensive regional data base. Since all regional matrices sum up to the corresponding national matrices the multiregional system is fully consistent with the national system. One of the key conceptual issues to be resolved concerned the distinction in the table between place of production and place of consumption; furthermore, in order to be able to relate consumption to income in the econometric parts of the model, commuters' place of income had to be distinguished from place of work. The paper will also elaborate on the estimation of the interregional trade matrix. Interregional trade flows were first estimated using survey data on regional export activities. These estimations were then used as starting values in a RAS procedure set up to balance the multiregional make-use system.

Following the discussion of the multiregional input-output table construction theoretical and empirical features of the econometric model blocks are presented. Afterwards the paper turns to a more extensive treatment of the way the per se static coefficients of the multiregional input-output tables are transformed into time-variable coefficients; among those are the technical coefficients of intermediate demand as well as the coefficients of the interregional trade matrix. For the latter a gravity model was estimated based on interregional transport data and then used to generate a time series of transport flows between Austrian districts over time. These interregional transport flows were subsequently transformed into interregional trade flow matrices. We finish with a summary and conclusions.

2. The Multiregional Input–Output Block

While most integrated regional econometric input-output models use quadratic input-output tables in MULTIREG the econometric blocks are linked with a complete multiregional make-use table system. As a consequence the model includes industries as well as commodities. In this section the basic identities and definitions of the input-output block are presented before details on the compilation of the multiregional table system are provided.

For the complete vector of commodity output values $\mathbf{g_i}$ in each region *i* the following fundamental identity must hold (commodity balance):

$$\mathbf{g_i} = \mathbf{g_i^d} + \mathbf{m_i^f} + \mathbf{m_i^r} = \mathbf{g_i^{int}} + \mathbf{f_i} \qquad (2.1)$$

where $\mathbf{g_i^{int}}$ is the intermediate demand vector and $\mathbf{f_i}$ is the (total) final demand vector (for both regionally produced as well as imported commodities), $\mathbf{m_i^f}$ are foreign imports, $\mathbf{m_i^r}$ denotes interregional imports and $\mathbf{g_i^d}$ is the vector of output values of regionally produced commodities in region *i*. In MULTIREG the use-matrix provides the basis for the coefficients matrix $\mathbf{A_i}$ (which is hence a commodity-by-industry matrix) in which one element a_{kl}^i is defined as:

$$a_{kl}^i = \frac{u_{kl}^i}{q_l^i}, \qquad (2.2)$$

where u^i_{kl} denotes the value of commodity k used in industry *l* located in region *i* and q^i_l denotes total output of industry *l* in region *i*. Substituting the product of $\mathbf{A_i}$ and $\mathbf{q_i}$ for $\mathbf{g_i^{int}}$ in (2.1) above gives

$$\mathbf{g_i} = \mathbf{A_i} \cdot \mathbf{q_i} + \mathbf{f_i} \; . \qquad (2.3)$$

The final demand vector $\mathbf{f_i}$ is the sum of a vector of private and public consumption, $\mathbf{cp_i}$ and $\mathbf{cg_i}$, a vector of gross capital formation, $\mathbf{i_i}$, as well as a vector of foreign exports $\mathbf{ex_i^f}$ and of interregional exports $\mathbf{ex_i^r}$,

$$\mathbf{f}_{\mathrm{i}} = \mathbf{cp}_{\mathrm{i}} + \mathbf{cg}_{\mathrm{i}} + \mathbf{i}_{\mathrm{i}} + \mathbf{ex}_{\mathrm{i}}^{\mathrm{f}} + \mathbf{ex}_{\mathrm{i}}^{\mathrm{r}} \qquad (2.4)$$

Total output of industries located in region *i*, $\mathbf{q_i}$, follows from multiplying the commodity demand vector with the regional market shares matrix $\mathbf{D_i}$,

$$\mathbf{D_i} = \mathbf{V_i} \cdot \mathbf{\hat{g}_i^{d\,-1}} \; , \qquad (2.5)$$

and

$$\mathbf{q_{i_i}} = \mathbf{D_i} \cdot \mathbf{g_i^d} \; , \qquad (2.6)$$

where $\mathbf{V_i}$ is the make-matrix of dimension industries-by-commodities and $\mathbf{\hat{g}}_{\mathrm{i}}^{\mathrm{d}} = diag(\mathbf{g}_{\mathrm{i}}^{\mathrm{d}})$.

The input-output tables for the nine Austrian federal provinces included in MULTIREG were derived from the national input-output tables compiled for the year 2000 by Statistik Austria. The latter are based on a make-use system comprising 58 sectors (2-digit NACE industries plus imputed financial intermediation services – FISIM) and 57 commodities (corresponding to 2-digit CPA codes). Retaining this level of aggregation, each national table was split up into nine regional sub-tables.

Table compilation proceeded in five steps:

- Estimation of a regional make matrix
- Estimation of regional intermediate and final use matrices independent of the origin of the commodities used
- Preliminary estimation of interregional trade flows by commodities

- Final estimation of interregional trade flows conditional upon balancing the multiregional make-use system with respect to each commodity
- Derivation of regional intermediate and final use matrices for regionally produced commodities

The resulting regional tables may be characterized as hybrid: Table compilation relied on extensive amounts of primary and secondary regional data from official sources and on data from a survey on interregional trade flows. Nevertheless for some sections of the tables, in particular those that depict service industries and commodities, regional data was scarce or did not exist at all. In those cases the structure of the corresponding sections of the national tables had to be retained and the regional information was limited to column sums of the tables (i.e. output levels).

Below the methods applied in the compilation of the matrices of the regional make-use system are described in more detail.

2.1 Regional Make Matrices

Commodity output values by industries included in the regional make matrices were calculated by multiplying total output values by industries and regions with the respective commodity shares. Estimates of regional total output values by industries were based on corresponding value added figures obtained from Statistik Austria's regional accounts. Two problems needed to be resolved: first, regional accounts are published only for 15 1-digit NACE industries (Agriculture, Forestry and Hunting / Fishing are combined, exterritorial units excluded). Statistik Austria provided a custom report for nine groups of the 23 2-digit NACE industries of the manufacturing sector. Further disaggregation into 2-digit industries was accomplished by utilizing indicators from other sources (e.g. employment by industries from the 2001 Austrian census). Secondly, the official regional value added figures were themselves derived from regional total output values, which, however, remained unpublished. Therefore, in order to arrive at consistent regional total output values, Statistik Austria's estimation procedure had to be applied reversely. This was accomplished using information on output to value added ratios by industries and on the development of regional annual revenues by industries.

Finally, the resulting regional total output values for the 57 2-digit NACE industries (FISIM excluded) had to be decomposed into commodity output values. For the different industries of the secondary sector (mining & manufacturing), regional survey information on commodity output levels was available from Statistik Austria. For the remaining industries no such information was at hands; consequently, national commodity output shares by industries as included in the national make matrix had to be applied across all regions. To ensure compliance of the resulting regional make matrices with regional total output levels by industries on the one hand and aggregate, i.e. national commodity output levels by industries as contained in the national make-matrix on the other hand, we used a variant of the familiar RAS method as described in Piispala (2000).

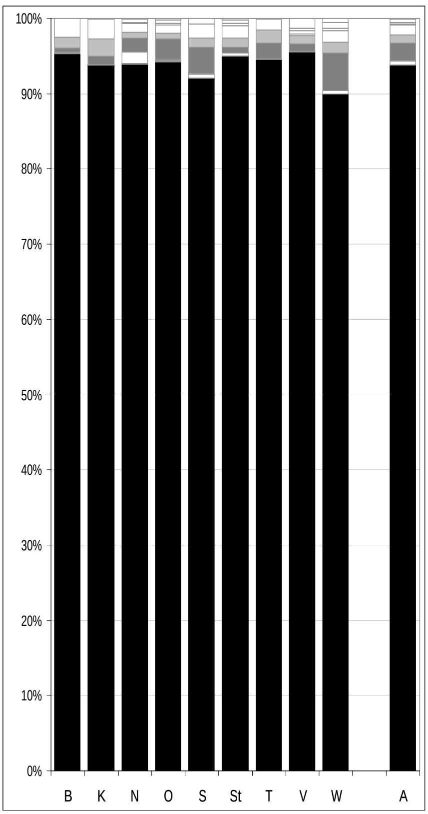
As expected, with respect to the commodity mix regional industrial output is more diverse than national output. However, entries on the main diagonal of the make matrix are strongly dominant at the regional level as well. As an example, chart 2 shows regional and national commodity composition of output for two industries, *Manufacture of Food Products and Beverages* (NACE 15) and *Manufacture of Motor Vehicles, Trailers and Semi-Trailers* (NACE 34). As can be seen, the commodity structure of output is regionally much more diverse for *Motor Vehicles* than in the case of *Food and Beverages*. This is also reflected in the regional commodity structure of intermediate use (see chart 3): the input pattern is much more diverse for *Motor Vehicles* than it is for *Food and Beverages*. In addition, the *Vehicle* industry is much more concentrated: more than 75% of total output is produced in only two regions, Styria and Upper Austria.

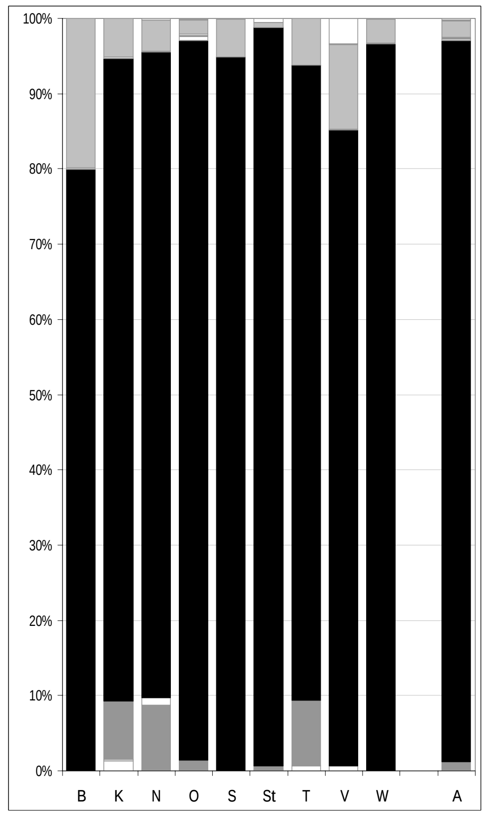
2.2 Regional Intermediate Use Matrices

For the regional intermediate use matrices a very similar approach was taken: here, regional commodity input values by industries resulted from multiplying total intermediate use values by industries and regions with the respective commodity shares. Total intermediate use values were calculated by deducting value added from total output values. Concerning the commodity shares in total intermediate use generic regional information was yet again available only for mining and manufacturing, here both with respect to industries and commodities. Since data on the use of services as well as the use of materials by the service sector is missing, national commodity input shares by industries from the national intermediate use table were used for all service inputs as well as for material inputs in the industries of the service sector. As above in the case of the regional make tables, Piispala's RAS method was applied to ensure consistency with the national intermediate use matrix both with respect to total intermediate use values by industries and total intermediate commodity use.

Hence, for make and intermediate use tables only the mining and manufacturing sections can be truly characterized as survey based, while the sections containing service industries and service commodities (in particular on the intermediate use side) much more reflect the national input-output structure. Both make and intermediate use matrices were finally discussed with experts from Statistik Austria and appropriate adjustments were made where recommended.

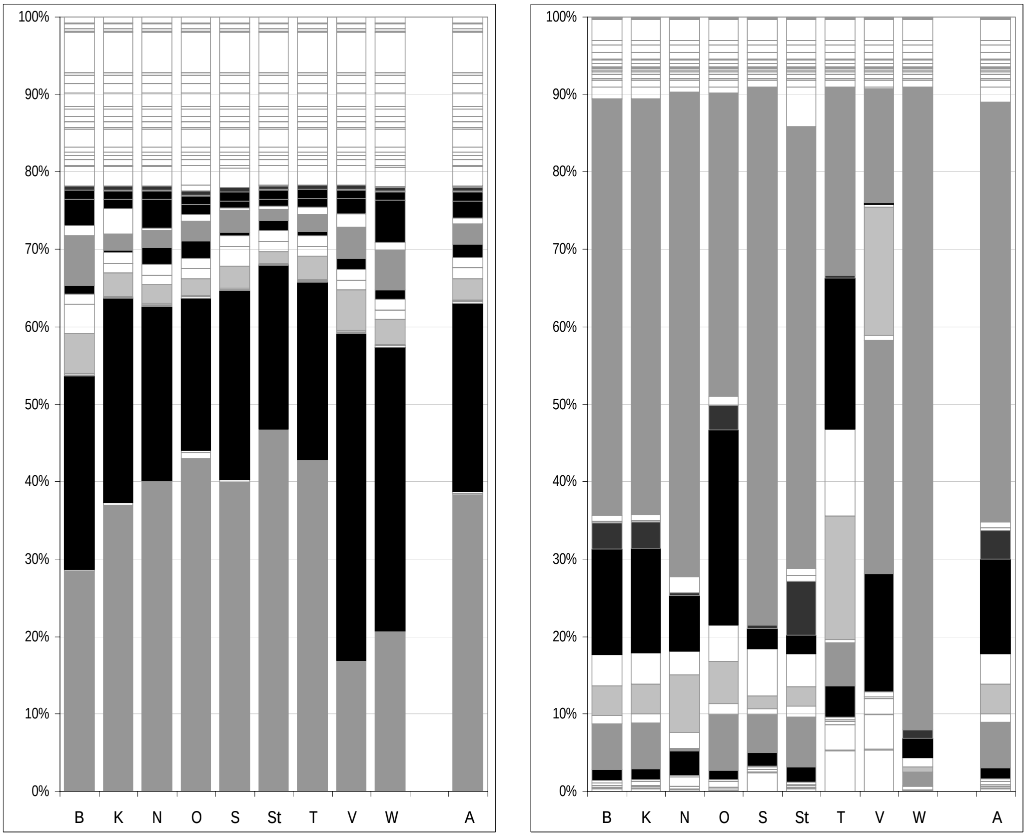
Chart 2: Regional Commodity Shares of Production for Industries NACE 15 and NACE 34 [1]





[1] For a definition of the regional codes see the appendix. Commodities are not designated as it is only the (similiarity in) the structure of regional production which is of interest in the present context.

Chart 3: Structure of Regional Intermediate Use for Industries NACE 15 and NACE 34 [2]



Source: Statistik Austria; authors' calculations.

2.3 Regional Final Use Matrices

Regional final use is separated into private consumption, government consumption, investment and foreign exports.

Regionalization of *private consumption* started by transforming data on household consumption expenditures by regions from expenditure categories into

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[2] For a definition of the regional codes on the abscissa of the diagram see the appendix. . Commodities are not designated as it is only the (difference in) the structure of regional production which is of interest in the present context.

commodities via a bridge matrix. Regional household consumption, however, was measured at the place of residence, while private consumption in the input-output tables was to be based on a place-of-consumption concept. Thus further adjustment for both domestic tourism and shopping was required.

Data on overnight stays by domestic tourists which includes information not only on the location of the accommodation but also on the place of residence of the tourist is available. This allowed calculating net overnight stays for each region, i.e. the number of overnight stays of regional residents outside the region minus the number of overnight stays of non-regional residents within the region. In the case of positive (negative) net overnight stays consumption expenditures measured at the place of residence were increased (decreased) in order to take regional tourism expenditures into account. For this adjustment it was assumed that the expenditures per overnight stay of foreign tourists, both with respect to their level and commodity structure, were equal to those of domestic tourists as well as across regions; multiplying the number of net overnight stays of a region with tourism expenditures per overnight stay provided the amount that was deducted from (in the case of negative net overnight stays) or added to (in the case of positive net overnight stays) the regional consumption expenditures measured at the place of residence.

Further need for adjustment resulted from households shopping outside their region of residence. This mainly concerned the region of Vienna and its surrounding region of Lower Austria, where considerable cross-border shopping takes place in both directions. Results from surveys undertaken in four Austrian regions, including Vienna and Lower Austria, over the last few years shed some light on cross-border shopping and even quantify regional shopping in- and outflows.[3] Even though these surveys are based on very small samples, the data together with a fair amount of assumptions and guesswork was used to further adjust regional consumption expenditures.

With respect to expenditures by foreign tourists, estimations on the regional allocation of these expenditures do not distinguish between consumption categories; such data exists only at the national level. Consequently, the consumption structure of foreign tourists had to be assumed equal across regions.

National *public consumption* expenditures were regionalized with respect to each commodity in part directly by using regional public consumption data provided by Statistik Austria, in part indirectly by applying different regional indicators which were consistent with a place of consumption concept. Specifically, shares of regional population in total Austrian population were used as indicators for commodities that could be classified as public goods like national defense and part of national government services. Education services were

[3] See Österreichische Gesellschaft für Marketing, 1997, Institut für Handelsforschung, 1998, Stadtplanung Wien 1999.

regionalized by the number of students at different levels of education, counted at the location of the educational institution. Public expenditures on health services and pharmaceuticals were first allocated to different (partly regional) health insurance carriers based on the number of insurants and then further regionalized if necessary. Since employees and their dependants are assigned to health insurance carriers based on the location of their employer and furthermore often stay in hospitals outside their home region adjustments for commuting (based on census data) and out-of-province hospitalization (based on data on regional hospital occupancy and the assumption of equal cost per occupied hospital bed across all regions) had to be made in order to comply with the place of consumption concept.

Regional investment was derived from the corresponding column of the Austrian input-output table by assuming, for each industry and each component of investment[4], equal ratios of investment to production across regions. Resulting regional differences in investment commodity expenditures thus merely reflect the different industrial composition of each regional economy.

Regional foreign exports were computed by utilizing information on the location of the exporter contained in the national external trade statistics database. The main drawback of this data is its unit of observation: it is the company level, whereas a meaningful regional input-output table compilation requires the establishment level since especially larger companies not only tend to have several establishments located in different regions but also one single business unit responsible for managing exports and imports for the whole company. Considerable effort and additional data (e.g. sales tax statistics) were used to correct for this problem. The regional foreign exports derived in this way serve only as first estimates and are revised when interregional trade is added to balance the multiregional input-output table system (cf. below).

For the estimation of *regional foreign imports* national external trade statistics are less useful: additional to the company-establishment problem imports are often not declared by the company the imported good is intended for but by the company responsible for its transport. Therefore, in the case of imports for intermediate use, national import ratios were used: for each industry and commodity the ratio of imported to total use was calculated from the Austrian intermediate use table. These ratios were then assumed constant across regions. Imported commodities for the different categories of final consumption were calculated analogously. Again, these results only served as starting values for a balancing mechanism (cf. below).

2.4 Interregional Trade

Since data on interregional trade are usually not collected by statistical offices and short cut methods often proved unreliable, a dedicated survey was conducted

[4] Investment in dwellings; other buildings and structures; machinery, transport equipment; cultivated assets and intangible fixed assets.

among Austrian business establishments, mainly in the manufacturing sector, construction as well as the following service industries: land transport (NACE 60), telecommunications (NACE 64), computer and related activities (NACE 72) and other business activities (NACE 74). Mail questionnaires asked firms about the shares of sales with respect to their regional destinations. A stratified sample of the approximately 90,000 Austrian establishments was used in the survey with the number of employees as stratification criterion: while only a small percentage of small establishments (less than 10 employees) was contacted, with respect to large firms (in most industries this involved establishments with more than 250 employees) a full survey was aimed at. In total about 6,600 establishments received questionnaires.

After conducting a mail follow-up and another follow-up by phone a response rate of 27.7% was achieved. Aggregating over each individual industry the responding establishments account for 19.7% of total employment with industrial response rates ranging between 7 and 65%. Sales to final customers (this included households and other firms except wholesalers, but including retailers) were distinguished from sales to wholesalers, whose location need not be identical to the final customer's location. It turned out that on average some 24% of total sales of responding firms went through a wholesaler. To be able to derive the final regional destination of sales through wholesalers, a separate survey was targeted at the wholesale industry. In this, some 8,000 wholesalers (out of about 20,000) were questioned about the regional distribution of their purchases and sales; however, due to budget restrictions, no follow-ups could be conducted so the response rate did not exceed 10%. In terms of employment the responding firms cover 6.7% of total employment in this industry in Austria.

The results of both surveys were extrapolated using employment weights for different firm size classes in each industry. The final results were then used as starting values in a RAS procedure applied to balancing the whole system of regional input-output tables.

The balancing procedure utilizes the following identity (see also equation 2.1 above): for each region and each commodity the value of total use of a commodity by firms and households within this region plus the value of regional and foreign exports must equal the total value of a commodity available in the region, i.e. the value of production by regional firms and the value of imports from other regions or from abroad. In other terms, whatever is consumed within the region or is exported must be produced somewhere, either in the region itself or in other regions or abroad. Equivalently, for each region and each commodity it must hold that the total value of production is equal to the total value of use of a regionally produced commodity within the region (by firms and households) plus the value of exports of regional production to other regions and abroad. In other terms, whatever is produced within the region must be consumed somewhere.

Chart 4: Balancing of Interregional Trade

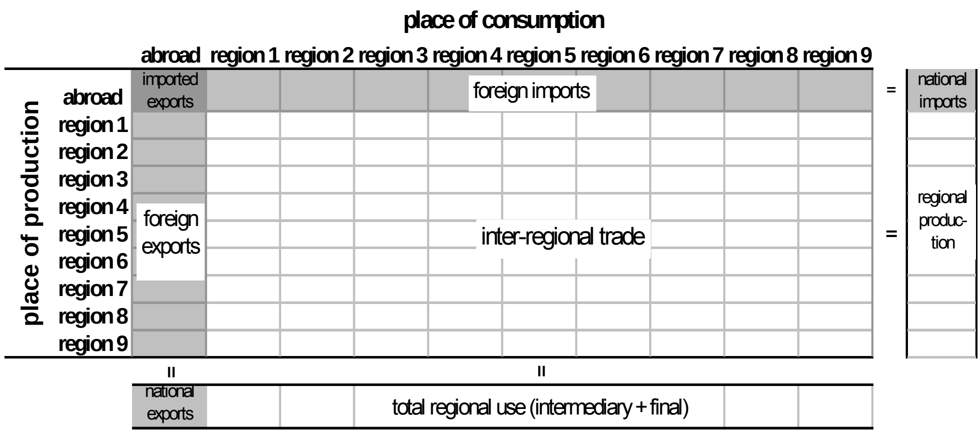


Chart 4 depicts a trade matrix on which these restrictions are imposed and which is set up for each commodity: the column sums contain total use in each region and the sum of foreign exports, the row sums show total regional production and the sum of foreign imports. These column and row sums are known from the regional make and use tables. Moreover, from our preliminary regionalization of national imports and exports we can fill the cells of the first column and the first row. The first column, however, does not contain regional total foreign exports: rather, it contains regional foreign exports net of regional imported exports (i.e. foreign exports of commodities which were previously imported into the region from abroad). The first cell in this column, then, contains total national imported exports, which are known from the Austrian input-output table. As for the interregional trade part, preliminary figures are available from the trade survey.

Assuming row and column sums as fixed, the trade flows can be taken as starting values such that a bi-proportional adaptation method (such as the well-known RAS-method, which was used in the present context) can be applied; the resulting tables represent a balanced multiregional input-output table system.

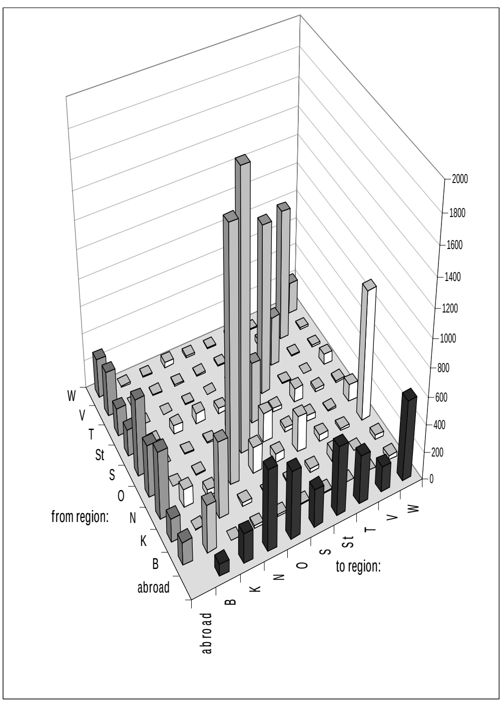
The major advantage of this method is that it allows for "cross-hauling": a commodity can at the same time be bought and sold by each region (instead of assuming that only "surplus production" is exported and only that part of demand is imported which cannot be satisfied out of regional production, respectively). The major drawback is that it disregards the possibility of "trans-shipping": this is the case when a commodity is imported into region 1 from region 2 and sold – unchanged – to region 3. From an input-output point of view this results mainly in a regional miss-allocation of trade (and transport) margins.

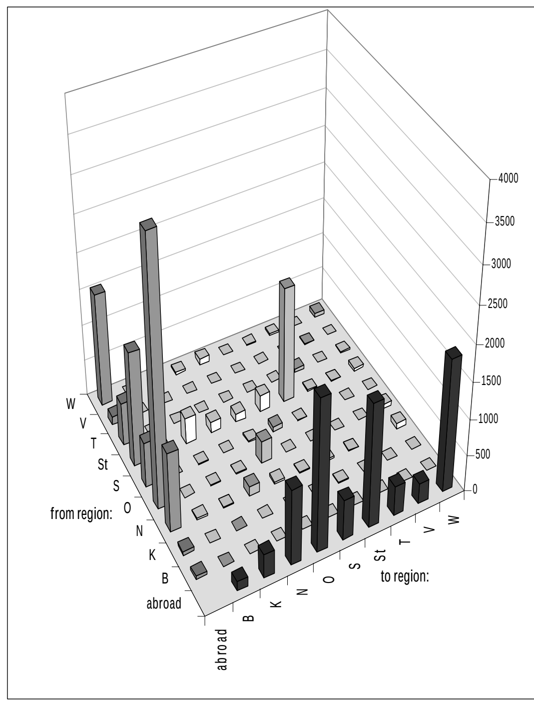
For most commodities the interregional trade flows after completing the balancing procedure do not greatly differ from the flows previously estimated.

Most cases where post-balancing trade flows do deviate significantly from pre-balancing flows concern industries where one or several larger companies did not participate in the survey.

Chart 5 shows interregional and international trade patterns for two commodities: food products (CAP 15, left diagram) are mainly regionally produced, although a significant part is internationally traded. Trade in vehicles (CPA 34, right diagram), on the other hand, is mostly external: although in terms of net exports, Austria is "self sufficient", the gross trade flows reveal that almost all vehicles which are consumed in Austria are imported while practically the whole domestic production of that commodity is exported.

Chart 5: Interregional and International Trade in Food and Vehicles (in Million EUR)





2.5 Regional Use Matrices for Regionally Produced Commodities

The final step of table compilation consisted of computing for each region matrices depicting the intermediate and final use of only those commodities that are produced within the region itself. Given these matrices and the region's make matrix a quadratic regional input-output table can be derived.

One important word of caution is to be issued with respect to these tables: When balancing the multiregional table system, regionally produced commodities were distinguished from commodities produced in other Austrian regions and those imported from abroad only with respect to total use. This implies that uniform import shares across all consuming industries and final consumption categories were assumed. Relaxing this assumption by collecting additional information on commodity use, possibly at a more disaggregated commodity level, is left to future efforts.

3. The Blocks of Econometric Equations

The econometric blocks of equations and their theoretical underpinnings are reported only very briefly in what follows. For a more elaborate description the reader is referred to Kratena and Zakarias (2001).

3.1 Factor Demand and Output Prices

This chapter depicts the determination of factor demand and output prices. The production factors modeled within MULTIREG comprise two variable factors, labor and a compound of intermediate goods, as well as a quasi-fixed, input capital. Following the approach usually adopted in the industrial organization literature, the price setting behavior of firms is treated within an overall model of goods and factor markets. The seminal paper for this approach is Appelbaum (1982), important examples which served as a basis for the approach adopted in MULTIREG include Berndt and Hesse (1986), Morrison (1989, 1990), Flaig and Steiner (1990), Conrad and Seitz (1994) and Meade (1998).

A Generalized Leontief cost function for each sector in each region was estimated and implemented in MULTIREG. Omitting indices denoting regions and commodities, this cost function GL (which is due to Diewert, 1971, and therefore sometimes also called the Diewert cost function) including a trend to capture technological progress (see Morrison, 1989) can generally be stated as:

$$\mathrm{GL}(q,\mathbf{p}^{\mathbf{int}},\mathbf{x},t)=q\left[\sum_{i=1}^{k}\sum_{j=1}^{k}\beta_{ij}\left(p_i^{\mathrm{int}}p_j^{\mathrm{int}}\right)^{1/2}+t^{1/2}\sum_{\mathrm{i}=1}^{\mathrm{k}}\beta_{it}p_i^{\mathrm{int}}+t\sum_{\mathrm{i}=1}^{\mathrm{k}}\beta_{tt}p_i^{\mathrm{int}}\right]+$$
$$+q^{1/2}\left[\sum_{\mathrm{i}=1}^{\mathrm{k}}\sum_{l=1}^{m}\beta_{il}p_i^{\mathrm{int}}x_l^{1/2}+2\sum_{\mathrm{i}=1}^{\mathrm{k}}\beta_{tl}p_i^{\mathrm{int}}t^{1/2}x_l^{1/2}\right]+\sum_{\mathrm{i}=1}^{\mathrm{k}}p_i^{\mathrm{int}}\sum_{l=1}^{m}\sum_{k=1}^{m}\beta_{lk}\left(x_lx_k\right)^{1/2} \quad (3.1)$$

where q is (regional) output, p_i^{int} and p_j^{int} are input prices of the *k* variable inputs summarized in the price vector $\mathbf{p_i^{int}}$, x_l denotes the m fixed factors, t the trend component and the β are the parameters to be estimated. The cost function (3.1) is homogenous of degree one, exhibits constant returns to scale and will be concave in factor prices, if

$$\beta_{ij}=\beta_{ji},\ \forall i\neq j\,,$$

that is, the cross substitution elasticities β_{ij} and β_{ji} will be equal if the symmetry condition is satisfied; this restriction was imposed in the estimation process.

In the present application two variable inputs labor w with price wr and intermediate demand s with price ps as well as a quasi fixed factor, capital k, are used. By Shepard's Lemma the factor demand equations for intermediate inputs and labor can be obtained by taking the first derivatives of the cost function with respect to factor prices ps and wr. Imposing the symmetry condition and dividing the right hand side of the equation by total output q yields the shares of the variable input factors as a share of total output:

$$\frac{\partial\mathrm{GL}}{\partial ps}\cdot\frac{1}{q}=\frac{s}{q}=\beta_{ss}+\beta_{sw}\left(\frac{wr}{ps}\right)^{1/2}+\beta_{st}t^{1/2}+\beta_{tt}t+\beta_{sk}\left(\frac{k}{q}\right)^{1/2}+2\beta_{tk}\left(\frac{k}{q}\right)^{1/2}t^{1/2}+\beta_{kk}\left(\frac{k}{q}\right) \quad (3.2)$$

$$\frac{\partial\mathrm{GL}}{\partial wr}\cdot\frac{1}{q}=\frac{w}{q}=\beta_{ww}+\beta_{sw}\left(\frac{ps}{wr}\right)^{1/2}+\beta_{wt}t^{1/2}+\beta_{tt}t+\beta_{wk}\left(\frac{k}{q}\right)^{1/2}+2\beta_{tk}\left(\frac{k}{q}\right)^{1/2}t^{1/2}+\beta_{kk}\left(\frac{k}{q}\right) \quad (3.3)$$

In order to determine the price vector of regional output ($\mathbf{pd}$) endogenously the system is further expanded by a price equation. To allow for monopolistic competition, output prices must equal marginal costs multiplied by fixed mark-up μ which is determined during estimation:

$$pq = (1+\mu)\frac{\partial \mathrm{GL}}{\partial \mathrm{q}} = (1+\mu)\left[\begin{array}{l}\beta_{ss}ps + \beta_{sw}(ps \cdot wr)^{1/2} + \beta_{ws}(ps \cdot wr)^{1/2} + \beta_{ww}wr + \beta_{st}ps \cdot t^{1/2} + \\ + \beta_{wt}wr \cdot t^{1/2} + \beta_{tt}t(ps + wr) + \frac{1}{2}\left(\frac{\mathrm{k}}{\mathrm{q}}\right)^{1/2}\left(\beta_{sk}ps + \beta_{wk}wr + 2t^{1/2}\beta_{tk}(ps + wr)\right)\end{array}\right] \tag{3.4}$$

The final system of equations estimated within MULTIREG to determine factor demands and output prices for each sector in each region hence consists of equations (3.2), (3.3) as well as (3.4).

3.2 Components of Final Demand

3.2.1 Investment Demand

A common way to model investment demand, which is applied here as well, utilizes the deviation of the actual capital stock from its optimal value, which can be derived from the cost function estimated above. In a first step the shadow price of capital can be obtained by taking the derivative of the cost function with respect to capital k multiplied by (–1):

$$pk^* = -\frac{\partial GL}{\partial k} = -\frac{1}{2}\left(\frac{q}{k}\right)^{1/2}\left[\beta_{sk}ps + \beta_{wk}wr + 2t^{1/2}\beta_{tk}(ps + wr)\right] - \beta_{kk}(ps + wr) \tag{3.5}$$

In equilibrium, the shadow price of capital will equal the price of capital pk (which can be determined only approximately via the composition of capital stock in each sector). Introducing this equality, equation (3.5) is reformulated and the market price of capital substitutes the shadow price. This yields the equilibrium or optimal value of capital, k^*, at each point in time:

$$k^* = q\left[-\frac{\beta_{sk}ps + \beta_{wk}wr + 2t^{1/2}\beta_{tk}(ps + wr)}{2(pk + \beta_{kk}(ps + wr))}\right]^2 \tag{3.6}$$

Given the optimal amount of capital at each point in time, investment demand is made dependent on a stock adjustment process of the current capital stock k to its desired level k^* (Czerny et al., 1997, Appendix A), which – after taking logarithms – is implemented as:

$$\Delta \log\left(k_{i,t}\right) = \tau_1 \left[\log\left(k_{i,t}^*\right) - \log\left(k_{i,t-1}\right)\right] + \tau_2 \Delta \log\left(k_{i,t-1}\right). \tag{3.7}$$

A necessary condition for (3.7) to converge to an equilibrium is for τ_1 to be positive, while the second parameter is subject to no constraint; if τ_2 lies between 0 and 1, adjustment to initial shocks will be slow (the smaller the value, the slower the adjustment) while for estimated values larger than one initial shocks might lead to an overshooting of the desired capital stock k^*. Negative values of τ_2 on the other hand will lead to cyclical fluctuations in the adjustment process.

The model can finally be closed by explaining the desired capital stock k^*. A natural way of doing this would be to utilize capital as explained by the Generalized Leontief cost functions described above, whenever user costs of capital are given. The adjustment process then would depend on the difference between user costs and the shadow price of capital estimated in the cost functions. In Czerny et al. (1997) k^* is furthermore depending on disposable income, interest rates, active population, rents as well as inflation. The absence of user costs of capital in the database, however, prohibits the application of this approach in MULTIREG and hence it has to be assumed that k^* equals some (positive) function of the current level of output only. That is:

$$\log\left(k_{i,t}^*\right) = \mathrm{F}\left[\log\left(q_{i,t}\right)\right]. \tag{3.8}$$

Inserting the optimal capital stock into (3.7) above yields the stock adjustment equation, which is the final equation estimated to determine investment demand in each sector:

$$\Delta \log\left(k_{i,t}\right) = \alpha_k + \beta_k \log\left(q_{i,t}\right) - \tau_1 \log\left(k_{i,t-1}\right) + \tau_2 \Delta \log\left(k_{i,t-1}\right). \tag{3.9}$$

The parameter β_k comprises the (necessarily positive) value of τ_1 and some positive value for the relationship between regional output and the optimal capital stock k^*. Hence, β_k must be positive, while the estimated parameter value on $\log(k_{i,t-1})$ has to be negative (the negative of a necessarily positive parameter).

Finally, adding the depreciation of capital stock in the current period to (3.9) by applying the same rates assumed throughout the set up of the time series for capital stock by industries yields investment:

$$I_i = \Delta \log\left(k_{i,t}\right) + \delta_i \cdot k_{i,t-1}. \tag{3.10}$$

3.2.2 Private Consumption

Private consumption was estimated applying the linear approximation of the well known Almost Ideal Demand System (AIDS; see Deaton and Muellbauer, 1980). For private consumption data at the regional level are not readily available. Their compilation would involve considerable effort which is left to future revisions of the model. Instead, the demand system estimated at the national level was applied to each region. The choice of consumption groups puts emphasis on transport categories and hence the following categories were modeled:

Table 1: Classification of Consumption Categories

1	Food, Drink and Tobacco
2	Clothing and Footwear
3	Medical Care
4	Purchases of Vehicles
5	Operation of Personal Transport Equipment
6	Transport Services
7	Communications and Entertainment
8	Restaurants, Hotels
9	Other Goods and Services

In selecting these groups emphasis was also placed on an approximately equal-sized distribution of the groups. Furthermore, groups consisting mainly of non-durable goods other than demand for vehicles did not enter the demand system; moreover, due to imputed components in the series for rents and housing expenditure, this group – along with durable consumption goods – is treated outside the demand system in single equations.
The budget shares equations for the AIDS can be written as:

$$\mathrm{w_i} = \alpha_i + \sum_j \gamma_{ij} \log p_j + \beta_i \log\left(\frac{\mathrm{x}}{P_1}\right). \tag{3.11}$$

In (3.11) w_i denotes the budget share of commodity *i*, x are the total nominal outlays on the commodities treated within the AIDS-model and P_1 is an aggregated price index, which is set up according to Stone (1954):

$$\log P_1 = \sum_k \mathrm{w}_\mathrm{k} \log p_k \quad (3.12)$$

When the system of budget share equations (3.11) above shall satisfy the standard properties of demand functions, three sets of restrictions have to be implied on the estimated parameters. First, for (3.11) to satisfy the Adding-Up condition it must hold true that:

$$\sum_{i=1}^{n} \alpha_i = 1; \ \sum_{i=1}^{n} \gamma_{ij} = 0; \ \sum_{i=1}^{n} \beta_i = 0 \quad (3.13)$$

It can easily be checked that if restrictions (3.13) are inserted into the budget shares equation (3.11), the sum of budget shares w_i over i will equal one, which is what Adding-Up requires. In terms of interpreting the budget shares equation note that this also means that the shares remain constant if prices and real total expenditure remain unchanged. The Adding-Up property will automatically be satisfied in empirical analysis whenever the data used in estimation add up perfectly. Homogeneity in prices and total expenditure is assured if:

$$\sum_{j=1}^{n} \gamma_{ij} = 0 \quad (3.14)$$

Finally, symmetry of the Slutsky – equation is attained by:

$$\gamma_{ij} = \gamma_{ji} \quad (3.15)$$

Both the homogeneity as well as the symmetry restriction was imposed during the estimation process in MULTIREG.

Disposable income was obtained via an error-correction type equation using total value added in the respective region as regressor. Total outlays on the commodities treated within the AIDS were obtained via a two stage budgeting process, first determining the outlays on the durable consumption goods.

The necessary transition of consumption demand categories estimated in the AIDS-model into demand for commodities in the input-output part of the model was accomplished via a bridge matrix which was available for the 1995-input-output table and whose coefficients were extrapolated to the year 2000.

3.2.3 Foreign Exports and Government Consumption

Both foreign exports as well as public consumption are treated as exogenous in the prevailing version of MULTIREG. Future revisions of the model will include the determination of foreign demand by a simple model of world production.

3.2.4 Foreign Imports

The foreign import shares in nominal terms for each commodity k, $mn_i^{f,k} / qn_i^k$ **,** are modeled by an equation derived from an linear approximate AIDS-model (Deaton and Muellbauer, 1980) to split up between domestic and imported commodities:

$$\frac{mn_i^{f,k}}{qn_i^k} = \alpha_m + \gamma_{md} \log pq_i^k + \gamma_{mm} \log pm^k + \beta_m \log\left(\frac{qn_i^k}{P_i}\right), \qquad (3.16)$$

where P_i denotes the Stone-price-index formed with the output and import price of commodity k; mn, qn are the nominal values of imports and output, with pm and pq the respective prices.

3.2.5 Regional Exports and Imports

Regional exports and imports are linked by the trade shares matrix $\mathbf{T_{ij}^k}$ for *i* and *j* as the regional indices and for k commodities:

$$ex_i^{r,k} = \sum_{j=1}^{n} \left(\mathbf{T_{ij}^k} * m_j^{r,k}\right) \qquad (3.17)$$

As this trade share matrix includes the elements $i = j$, i.e. the deliveries to the own region, the sum of regional imports of commodity k, $m_i^{r,k}$, is given by the commodity balance:

$$m_i^{r,k} = q_i^k - q_i^{d,k} - m_i^{f,k}. \qquad (3.18)$$

The trade shares matrix is directly linked to the road transport flow matrix $\mathbf{TR}^l$ (in volumes) where the commodity index l represents an aggregation of the commodity index k according to different classifications in economic and transport statistics.

The development of each element of this matrix at time *t*, $tr_{ij,t}^l$, is a result of the application of a gravity model at the subregional level. For those commodities for which road transport plays an important role and for which commodity classification l corresponds to classification *k* the estimated development path is applied to the elements of the trade shares matrix $t_{ij,t}^{l}$. This procedure, which is described in more detail in section 0 below, yields an adapted trade shares matrix, which is then re-inserted into the model to derive a new solution for output and all other endogenous variables, especially road transport flows. The loop between output, road transport flows, and trade shares is applied in an iterative mechanism until convergence of the model solution is achieved. This method guarantees a fully consistent modeling of road transport flows, regional production and interregional trade.

Other stochastic equations in MULTIREG, not shown here, comprise wage equations as well as error correction-type equations determining disposable income and total consumption. Furthermore, several price feedbacks are modeled: one is from the domestic output price estimated in (3.4) above to the compound price index of intermediate demand for each industry, another concerns the commodity prices (and hence the inflation rate which is also part of the wage equations) which also respond to changes in the domestic output prices.

4. Updating the Technical Input–Output Coefficients

The issue of updating input-output coefficients has had a long tradition in economics, since there are a number of reasons why those coefficients do not remain constant over time (some of those comprise technological change, variations in the product mix, price changes, input substitutions or shifts in trade patterns). Especially when an input-output model is applied to long-term projections, the per se static nature of its input-output coefficients must therefore be overcome. The updating mechanism of each matrix $\mathbf{A_i}$ incorporated in MULTIREG further expands the approach proposed by Kratena and Zakarias (2004). The updating process "along the rows" (see e.g. Conway, 1990; Israilevich et al., 1996) is thereby supplemented by an adjustment "along the columns". Kratena and Zakarias (2004) demonstrate empirically that their more comprehensive updating procedure results in a better estimation of the true coefficients of the underlying matrix. However, as their method involves only a one time adjustment of columns followed by a one time adjustment of rows, they also find that a full RAS performs even better. As a consequence, in MULTIREG a full bi-proportional adjustment was implemented.

Such an adjustment requires that the row and column sums of matrix $\mathbf{A_i}$, i.e. the constraints necessary to make the RAS approach applicable, are determined endogenously within the model.

To begin with, the endogenous determination of the rows sums of matrix $\mathbf{A_i}$ involves the traditional estimation of the deviations of total intermediate demand from its so-called "hypothetical" value. The latter series is obtained by multiplying total demand in each year by matrix $\mathbf{A_i}$ for the base year.

More formally, the starting point to determine the constraint for the row sum of matrix $\mathbf{A_i}$ is the following relationship already stated above:

$$\mathbf{q_i} = \mathbf{A_i} \cdot \mathbf{q_i^d} + \mathbf{f_i} \,. \tag{4.1}$$

Subtracting the vector of final demand from total goods demand in region *i* yields intermediate demand:

$$\mathbf{q_i^{int}} \,:$$

$$\mathbf{q_i} - \mathbf{f_i} = \mathbf{q_i^{int}} = \mathbf{A_i} \cdot \mathbf{q_i^d} \,. \tag{4.2}$$

Introducing time subscripts to the equation on the left hand side above yields:

$$\mathbf{q}_{\mathbf{i},t} - \mathbf{f}_{\mathbf{i},t} = \mathbf{q}_{\mathbf{i},t}^{\mathbf{int}} \,, \tag{4.3}$$

that is, the time series of intermediate demand. However, if one takes the right hand side of equation (4.2) above and adds time subscripts, the result is a so called "hypothetical" series of intermediate demand, $\tilde{\mathbf{q}}_{\mathbf{i},t}^{\mathbf{int}}$:

$$\tilde{\mathbf{q}}_{\mathbf{i},t}^{\mathbf{int}} = \mathbf{A_i} \cdot \mathbf{q_i^d} \,. \tag{4.4}$$

The difference between the two series is, of course, that matrix $\mathbf{A_i}$ is held constant over time as it is available only for the base year. The deviations of actual intermediate demand from its hypothetical counterpart can be attributed to the changes in coefficients within matrix $\mathbf{A_i}$. Now, the relationship between $\tilde{\mathbf{q}}_{\mathbf{i}}^{\mathbf{int}}$ and $\mathbf{q_i^{int}}$ over time can be stated as:

$$\hat{\mathbf{r}}_{\mathbf{t}} \tilde{\mathbf{q}}_{\mathbf{i},t}^{\mathbf{int}} = \mathbf{q}_{\mathbf{i},t}^{\mathbf{int}} \,. \tag{4.5}$$

where $\mathbf{r_t}$ is an estimable vector. Inserting (4.5) into (4.4) and rearranging terms yields the following relationship:

$$\hat{\mathbf{r}}_{\mathbf{t}} \cdot \mathbf{A_i} \cdot \mathbf{q}_{\mathbf{i},t}^{\mathbf{int}} = \mathbf{q}_{\mathbf{i},t}^{\mathbf{int}} \,. \tag{4.6}$$

Hence, each row of matrix $\mathbf{A_i}$ is updated each year with the respective constant factor from vector $\mathbf{r_t}$. Equation (4.6) is at the same time the relationship that allows for the endogenous determination of intermediate demand within the model, once the elements of $\mathbf{r_t}$ are known. Kratena and Zakarias (2004) suggest estimating an error correction model on (4.5) which is also applied here. As the final demand components are also known – either modeled endogenously (investment, private consumption) or given exogenously (foreign exports and government consumption) – the share of intermediate goods demand in total demand can be computed. As a result, this yields the row sum of matrix $\mathbf{A_i}$ and hence the first restriction needed to make the RAS approach operable.

The derivation of the second restriction necessary to implement a RAS on matrix $\mathbf{A_i}$ is straightforward. Recalling that the factor demand equation (3.2) above derived the share s/q (in the notation introduced above, omitting indices for the region as well as the commodity) it becomes immediately obvious that this share is also equal to the column sum of matrix $\mathbf{A_i}$. Once both restrictions required for the implementation of a RAS are given the adjustment procedure can be implemented, even though this implementation comes at a high programming cost.

5. Updating the Coefficients of the Trade Matrix

Chapter 2 above described the derivation of the trade matrix for the year 2000. This chapter deals with aspect of the dynamization of this static trade matrix.

Conceptually, two components of trade dynamics are distinguished. The first one is the regional structure of production: if production of some commodities expands in a region, regional exports from this region to others should expand as well (probably disproportionately so). But even assuming constant regional production levels, trade between regions is likely to increase due to deepening regional specialisation. These two factors are dealt with separately: the trade impact of regional production levels is modeled by a cross-sectional gravity model, the impact of deepening regional specialisation by a time-series analysis of total transport volumes.

5.1 A Gravity Model

The gravity model is based on transport survey data from which transport is inferred at a sub regional level. Data are disaggregated into 14 commodity groups which are composites of the NSTR-24 classification of commodities (see Table 2). The geographical unit is the district and only transport flows among Austrian districts are considered in the analysis. All in all, transport flows of 14 commodities between 99 districts[5] can be distinguished in the base year. The formal structure of the transport model is rather ad hoc. This is due to the framework for which it had been developed in the first place, viz. a project which aimed at forecasting inter-district transport volumes.

Table 2: Commodity Classification of the Gravity Model

Multireg group	NSTR 24	definition	Multireg group	NSTR 24	definition
A	1,2,3,7	agricultural products	H	13,21	metal products
B	4	timber	I	14,22	construction materials
C	5,23	textile and leather	J	15	crude and processed minerals
D	6	food and animal feed	K	16,17,18	fertilizer, chemical products
E	8	solid mineral fuel	L	19	pulp and paper
F	9,10	crude and refined petroleum	M	20	vehicles, machines, machine parts
G	11,12	iron and metal ores; scrap metal	N	24	special transport goods

The model follows a two-step approach: in the first stage the total volume of shipments which would enter or leave a district given the size of the district's economy was estimated, while in the second stage, total in- and outbound transport was allocated to sending and receiving districts by a gravity model. In this way plausible shipments between districts could be computed that match up with local production possibilities[6]. A RAS was used to ensure that the total inbound transport volume equalled total outbound transport volume. The step 1 model was quite simple: total in- and outbound transport was modeled on various indicators of economic activity (similar to the indicators of "economic mass" used in the gravity model; see below) and/or population.

The second step follows a modified gravity approach: flows between districts are (positively) influenced by "economic mass" and (negatively) by the distance between them. In our case, distance is represented by average travel time between each pair of districts. The indicator of "economic mass" depends on the commodity being modeled: it is approximated by a district's output value of industries which either produce or consume the respective good in significant quantities. For instance, transport of building materials is assumed to be influenced by the size of a district's cement-producing industry on the one hand and the size of its construction industry on the other. Each commodity, therefore, is modeled by a gravity model which is specific with respect to the indicator of "economic mass". The measure of distance, though, is identical for all commodities.

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[5] Austria is divided into nine federal provinces and 121 districts. The 23 districts making up Vienna, though, are lumped together, leaving 99 different (groups of) districts.

[6] The "pure gravity approach", i.e. omitting step 1, resulted in what might be termed the "small neighbour problem": for small districts which are located close to large ones the model sometimes produced implausibly high transport volumes.

The main modification as compared with usual applications of the gravity model concerns its functional form: whereas in most cases the model is estimated in logarithms here levels are used. The reason for this is that at the district level quite a few industries which might influence transport volumes are present only in a minority of districts. This is a problem as some transport commodities are regressed on the output levels of as much as five or six different industries. Using a logarithmic function, districts where only one of those industries has zero production would drop out of the estimation, thus appreciably reducing the sample size. Moreover, the heteroscedasticity-correcting quality inherent in the logarithmic approach has some drawbacks in the present case. The basic form of the equations, thus, is

$$TR_{ij}^{c} = \alpha_l + \sum_n \beta_n \left(q_i^n / dist_{ij} \right) + \sum_m \beta_m \left(q_j^m / dist_{ji} \right), \qquad (5.1)$$

with TR_{ij}^{c} being the transport volume of commodity c from district i to district j, and $dist_{ij}$ being the average travelling time between the respective districts. q_i^n are output levels of the n industries thought to influence transport volumes on the part of the source district; typically the industries used here are those that produce the commodity for which the model is set up. Analogously, q_{ji}^{m} are m industry output levels which determine the transport flows in the target district; here mostly industries are included which use the respective commodity.

A second modification was implemented as well: for all NSTR commodities, not all of the 99x99 transport relations among districts exhibit positive values (for less important commodities, e.g. group E – solid mineral fuel – positive transport values for less than 3% of all 9801 district relations are observed). This is not really surprising given the very detailed level of geographical disaggregation (after all, on average, a district accounts for as little as about 30 000 employees). The gravity model implemented in levels, though, allots a value to each and every district relation. To solve this dilemma, a two-stage gravity model was estimated: in the first stage, a binary (probit) model is used to determine whether some relation should be attributed any transport flow at all; only if the model results do suggest that such flows exist, a concrete value for the transport volume is estimated by the proper gravity model. Technically, the regressors used in the probit model are essentially the same as those used in the respective gravity equation (thus mimicking a "threshold model").

For their implementation in MULTIREG the transport flows estimated for the 99 districts were aggregated to the level of the nine regions modeled in MULTIREG. At the regional level the transport flows were translated into trade flows via a 9x9 bridge matrix. The link between the different classifications of the gravity model on the one hand and the trade model on the other hand was solved econometrically: for each of the 55 NACE commodities, a linear combination of the 14 NSTR transport matrices was estimated to ensure maximum correlation.

The reason for modeling trade and transport flows at different levels of regional aggregation is due to data availability issues: as transport data were available for one year only, a pure cross-section approach had to be applied. An aggregation to the level of the nine regions at this pre-modeling stage, however, had to be ruled out, as it would render the distance variable meaningless (the nine regions feature highly diverse areas: the largest region, Lower Austria, covers about a quarter of Austria and thus more than the four smallest regions combined). For this reason, gravity modeling proceeded at the district level. Consequently in simulations with MULTIREG results are broken down from the regional to the district level (distinguishing between different types of districts in the process: rural, urban, peripheral) to be fed into the gravity model; results from the gravity model are then re-aggregated to the regional level and fed back into the other blocks of MULTIREG.

5.2 Simulations with the Transport Model

One problem of the gravity model concerned its restriction to cross section data; any time series information was lacking. This drawback was overcome by introducing a block of transport equations that links regional economic output to an overall amount of transport volume shipped as inland traffic. Apart from changing output levels, these equations also consider time-varying transport intensities in each industry.

The simulation of interregional trade thus proceeds as follows: in the first step total transport volume $\mathbf{TR}_{\mathbf{tot}}$ is calculated based on the transport equations. Then, for given output levels for all districts, this total transport volume is broken down to the 99 districts, separately for inbound and outbound transport volumes ($\mathbf{TR}_{\mathbf{j}}$ and $\mathbf{TR}_{i}$, respectively). In the third step, for each cell of $\mathbf{TR}_{i\mathbf{j}}$, the probit-model determines whether this particular relation from district *i* to district *j* carries any transport at all. If so, the proper gravity model is used in step four to estimate the transport volume between districts *i* and *j*. A RAS is employed to ensure the compliance of the gravity model's results with the previously determined in- and outbound transport volumes.

Once the model is solved at the district level, a 9x9 regional transport matrix is compiled by simply by aggregating over all districts within a region (see chart 6). The last step involves using the bridge matrix linking transport and trade at the regional level to finally arrive at the new trade matrix.

Chart 6: The Transport Matrix

TR_{tot} / to district: / from district:	1	2	3	4	5	6	7	8	9	.	.	98	99	total outbound transport
1	$TR_{1\,1}$	$TR_{1\,2}$	$TR_{1\,3}$	$TR_{1\,4}$	$TR_{1\,5}$	$TR_{1\,6}$	$TR_{1\,7}$	$TR_{1\,8}$	$TR_{1\,9}$					$\Sigma = TR_{1.}$
2	$TR_{2\,1}$	$TR_{2\,2}$	$TR_{2\,3}$	$TR_{2\,4}$	$TR_{2\,5}$									$\Sigma = TR_{2.}$
3	$TR_{3\,1}$	$TR_{3\,2}$	$TR_{3\,3}$	$TR_{3\,4}$										$\Sigma = TR_{3.}$
4	$TR_{4\,1}$	$TR_{4\,2}$	$TR_{4\,3}$											$\Sigma = TR_{4.}$
5	$TR_{5\,1}$	$TR_{5\,2}$												$\Sigma = TR_{5.}$
6	$TR_{6\,1}$													$\Sigma = TR_{6.}$
7	$TR_{7\,1}$													$\Sigma = TR_{7.}$
8	$TR_{8\,1}$													$\Sigma = TR_{8.}$
9	$TR_{9\,1}$													$\Sigma = TR_{9.}$
.														
.														
98													$TR_{98\,99}$	$\Sigma = TR_{98.}$
99												$TR_{99\,98}$	$TR_{99\,99}$	$\Sigma = TR_{99.}$
total inbound transport	$\Sigma = TR_{.1}$	$\Sigma = TR_{.2}$	$\Sigma = TR_{.3}$	$\Sigma = TR_{.4}$	$\Sigma = TR_{.5}$	$\Sigma = TR_{.6}$	$\Sigma = TR_{.7}$	$\Sigma = TR_{.8}$	$\Sigma = TR_{.9}$			$\Sigma = TR_{.98}$	$\Sigma = TR_{.99}$	$\Sigma = TR_{tot}$

6. Summary and Conclusions

While regional models integrating econometric and input-output approaches are fairly widespread, only a few of them truly operate on a spatially disaggregate level.[7] Modeling the nine Austrian federal provinces, MULTIREG links a multiregional make-use system with region-specific econometric equations and thus qualifies to be a member of the exclusive club of spatially disaggregate integrated models.

Three strong points of MULTIREG are to be stressed: In developing the model, an extensive regional database was set up and used both for the compilation of input-output block of the model as well as for estimating its econometric equations. Furthermore, most of these econometric equations are derived from microeconomic theory; hence MULTIREG departs from the much more empirically oriented econometric approach of its single-region predecessors.

Rather innovative ways of modeling are followed with respect to time-adjusting both the technical coefficients of intermediate input use as well as the interregional

7 For a discussion of multiregional linkages in integrated models see Rey (2000). He identifies only three models, by Dewhurst and West (1990), Kort and Cartwright (1981) and Rey and Dev (1997), that are spatially disaggregate.

trade flows. The static nature input-output coefficients has been at the core of input-output model criticism; at the interregional level constant trade flows were strongly brought forward against such models. Therefore, even though there are numerous shortcomings the MULTIREG approach may suffer from, allowing for time-dependent changes in input-output coefficients and trade flows tackles important modeling problems.

Any discussion of MULTIREG's merits and flaws, however, is strongly hampered by the fact that a first version of the model has just been completed but is still being extensively tested by its developers. Therefore, no model simulations are available at this time. Further evaluation of the model will have to await these simulations as well as the models performance "in practice", i.e. when applied to answer regional economic policy questions.

Appendix

Chart A1: Map of Austria and the Nine Federal Provinces Included in MULTIREG

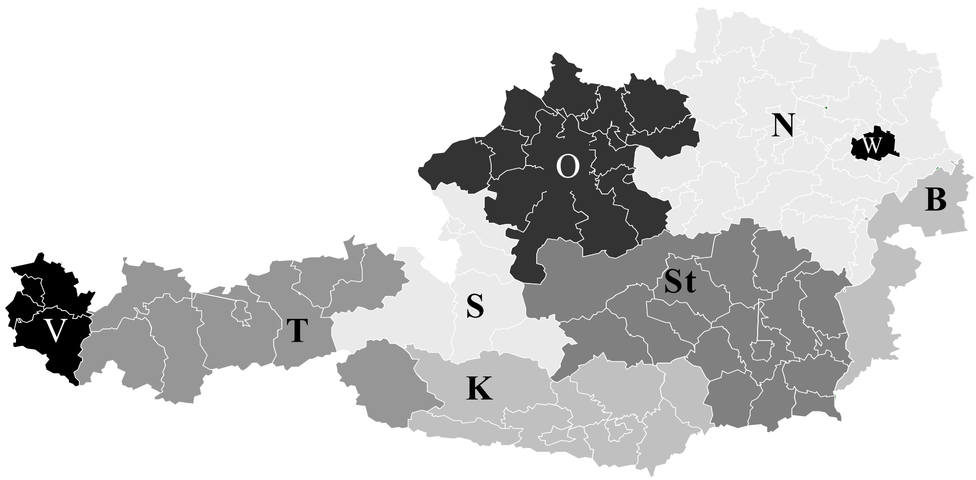

Table A1: Summary Statistics on the Nine Regions Included in MULTIREG

Code	Region		population 2000	GRP 2000 [Mio €]	GRP/pop [1000 €]
B	Burgenland		277,962	4,467	16.1
K	Kärnten	*Carinthia*	563,207	11,549	20.5
N	Niederösterreich	*Lower Austria*	1,542,393	30,901	20.0
O	Oberösterreich	*Upper Austria*	1,379,524	31,605	22.9
S	Salzburg		517,096	13,785	26.7
St	Steiermark	*Styria*	1,202,275	24,418	20.3
T	Tirol	*Tyrol*	669,710	16,189	24.2
V	Vorarlberg		349,421	8,658	24.8
W	Wien	*Vienna*	1,608,656	52,840	32.8
A	Österreich	*Austria*	8,110,244	194,413	24.0

Source: Statistik Austria.

Table A2: Industries Included in MULTIREG

NACE	MultiREG Sector	Definition
1	1	Agriculture, hunting and related service activities
2	1	Forestry, logging and related service activities
5	1	Fishing, operation of fish hatcheries and fish farms; service activities incidental to fishing
10	2	Mining of coal and lignite; extraction of peat
11	2	Extration of crude petroleum and natural gas; service activities incidental to oil and gas extraction excluding surveying
12	2	Mining of uranium and thorium ores
13	2	Mining of metal ores
14	2	Other mining and quarrying
15	3	Food products and beverages
16	3	Manufacture of tobacco products
17	4	Manufacture of textiles
18	4	Manufacture of wearing apparel; dressing and dyeing of fur
19	4	Tanning and dressing of leather; manufacture of luggage, handbages, saddlery, harness and footwear
20	5	Manufacture of wood and of products of wood and cork, except furniture; Manufacture of articles of straw and plainting material
21	6	Manufacture of pulp, paper and paper products
22	7	Publishing, printing and reproduction of recorded media
23	8	Manufacture of coke, refined petroleum products and nuclear fuel
24	8	Manufacture of chemical and chemical products
25	9	Manufacture of rubber and plastic products
26	10	Manufacture of other non-metallic mineral products
27	11	Manufacture of basic metals
28	11	Manufacture of fabricated metal products, except machinery and equipment
29	12	Manufacture of machinery and equipment n. e. c.
30	13	Manufacture of office machinery and computers
31	13	Manufacture of electrical machinery and apparatus n. e. c.
32	13	Manufacture of radio, television and communication equipment and apparatus
33	13	Manufacture of medical, precision and optical instruments, watches and clocks
34	14	Manufacture of motor vehicles, trailers and semi-trailers
35	14	Manufacture of other transport equipment
36	15	Manufacture of furniture; manufacturing n. e. c.
37	15	Recycling
40	16	Electricity, gas, steam and hot water supply
41	16	Collection, purification and distribution of water
45	17	Construction
50	18	Sale, maintenance and repair of motor vehicles and motorcycles; retail sale of automotive fuel
51	18	Wholesale trade and commission trade, except of motor vehicles and motorcycles
52	18	Retail trade, except of motor vehicles and motor cycles; repair of personal and household goods
55	19	Hotels and Restaurants
60	20	Land transport; transport via pipelines
61	21	Water transport
62	21	Air transport
63	22	Supporting and auxiliary transport activities; activities of travel agencies
64	23	Post and telecommunications
65	24	Financial intermediation, except insurance and pension funding
66	24	Insurance and pension funding, except compulsory social security
67	24	Acivities auxiliary to financial intermediation
70	25	Real estate activities
71	25	Renting of machinery and equipment without operator and of personal and household goods
72	26	Computer and related activities
73	27	Research and development
74	27	Other business activities
75	28	Public administration and defence; compulsory social security
80	29	Education
85	30	Health and social work
90	31	Sewage and refuse disposal, sanitation and similar activities
91	31	Activities of membership organizations n. e. c.
92	32	Recreational, cultural and sporting activities
93	32	Other service activities
95	32	Private households with employed persons

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