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Evaluating Euro Exchange Rate Predictions from a Battery of Multivariate Models

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Abstract

We compare the accuracy of vector autoregressive and vector error correction models in forecasting the exchange rates of the euro (EUR) against the U.S. dollar (USD) and the Japanese yen (JPY) when using monetary and capital flows related variables. For the EUR/USD exchange rate monetary and capital flows models tend to outperform the random walk model for long-term predictions (more than six months), but fail to reject the test of equality of forecasting accuracy against the random walk model for all forecasting horizons but one. On the other hand, the best monetary model for the EUR/JPY exchange rate outperforms the random walk model on all horizons, and does so significantly for more than six months ahead. Models based on capital flow variables fail to beat the predictions of the random walk model for all forecasting horizons.
Keywords: Vector Autoregression; Cointegration; Forecasting; Exchange Rates.

1. Introduction

Exchange rate prediction is a subject of major interest for researchers and economic policy actors. The surprising result presented in Meese and Rogoff (1983), namely that exchange rate forecasts based on the random walk model outperform the predictions of theory-based and (both univariate and multivariate) time series models, gave rise to an ever-growing branch of literature aimed at finding econometric models which deliver good out-of-sample exchange rate forecasts. Hoque and Latif (1993), Liu *et al.* (1994), Finn (1986), MacDonald and

Taylor (1993), Boothe and Glassman (1987) or van Aarle *et al.* (2000) are, among many others, examples of this direction of research.

This paper presents the results of a systematic comparison of multivariate time series models in terms of the accuracy of out-of-sample point forecasts for the euro (EUR) against the U.S. dollar (USD) and Japanese yen (JPY) when using monetary and capital flows related variables. We use a collection of linear multivariate models which comprises the most important models used in the literature: unrestricted and restricted vector autoregressions (henceforth, VAR and RVAR, respectively) and vector error correction models (henceforth, VEC). Features of such an exercise are quite appealing, both as a source of knowledge about the insights of exchange rate determination and as a consulting instrument for portfolio choices in the increasingly globalized world economy.[1]

The structure of the paper is the following. Section two presents a brief exposition of the various multivariate specifications used throughout the paper. The results of the forecasting exercise for the euro against the U.S. dollar and the Japanese yen are presented and commented in section three, and section four concludes.

2. Forecasting Euro Exchange Rates

2.1 Analytical Framework

The variables used in the analysis are those suggested by the theoretical framework of the monetary model of exchange rate formation (for the original formulations, see Frenkel, 1976, Dornbusch, 1976 or Hooper and Morton, 1982). In our case, (logged) exchange rates (E_t) are put in relation with their own lagged values, lagged values of domestic and foreign (logged) money supply (M_t^d and M_t^f), domestic and foreign (logged) output − the data actually used is industrial production − (Y_t^d and Y_t^f), domestic and foreign short-term interest rates (R_t^d and R_t^f), and domestic and foreign long-term interest rates (π_t^d and π_t^f) in the form of a VAR model.[2]

Depending on whether relative or country-specific variables are used, we will differentiate between structural or unstructural models. An unstructural VAR (u-VAR) model is specified as

[1] Crespo Cuaresma and Hlouskova (2004) perform a similar exercise involving also Bayesian vector autoregressions for five Central and Eastern European currencies against the U.S. dollar and the euro.

[2] See Appendix for data characteristics and sources.

$$X_t = \psi(0) + \sum_{s=1}^{p} \psi(s) X_{t-s} + \varepsilon_t, \tag{1}$$

$$X_t = \left[E_t, M_t^d, M_t^f, Y_t^d, Y_t^f, R_t^d, R_t^f, \pi_t^d, \pi_t^f \right]',$$

$$\varepsilon_t = \left[\varepsilon_t^1, \varepsilon_t^2, ..., \varepsilon_t^9 \right]' \sim NID(\vec{0}, \Sigma)$$

where $\psi(l)$ ($l = 1,..., p$) are (9×9) matrices of coefficients.

The structural VAR model (s-VAR) arises when imposing the restriction that the parameters corresponding to the domestic variables be equal in absolute value and contrary in sign to those of the corresponding foreign variable. The model specified in (1) can then be written as

$$Z_t = \omega(0) + \sum_{s=1}^{p} \omega(s) Z_{t-s} + \mu_t, \tag{2}$$

$$Z_t = \left[E_t, m_t, y_t, r_t, \pi_t \right]',$$

$$\mu_t = \left[\mu_t^1, ..., \mu_t^5 \right]' \sim NID(\vec{0}, \Sigma_\mu),$$

where $m_t = M_t^d - M_t^f$, $y_t = Y_t^d - Y_t^f$, $r_t = R_t^d - R_t^f$, $\pi_t = \pi_t^d - \pi_t^f$, and $\omega(l)$ ($l = 1,..., p$) are (5×5) matrices of coefficients. Both the u-VAR and s-VAR models will be specified in levels or in differences (the latter will be denoted u-DVAR and s-DVAR), and the models in differences can be augmented by including one or more error correction terms among variables of the system, giving rise to the u-VEC and s-VEC models.

The monetary model rests on two important simplifying assumptions: (i) domestic and foreign assets are perfect substitutes (implying perfect capital mobility) and (ii) current account effects (surplus or deficit) are negligible. These assumptions could be relaxed if the possible role of capital flows in explaining exchange rate movements is taken into account (see Bailey et al., 2001 or Aliber, 2000). Thus, it might be possible to tie together movements in the exchange rates, the real interest rate, equity prices and current account balance. Rather than explicitly incorporating current account data in the model, we may choose to do so *implicitly* by using productivity figures. Current account dynamics are the result of changes in productivity. For instance, a positive productivity shock raises expected future output in the home country. This will tend to induce capital inflows for at least two reasons. On the one hand, if consumers in the home country expect to be richer in the future, they will want to borrow from abroad to increase their consumption today (in case that consumers are sufficiently forward-looking to smooth their consumption over present and future time periods). On the other hand, the expected increase in future productivity raises expected future profits, increasing equity prices, thereby stimulating investment demand; insofar the additional demand for funds to finance such investment is not available domestically, which causes inflows of capital (FDI and portfolio investment).

The VAR and VEC models with capital flow variables that will be evaluated in terms of forecasting ability include short and long-term interest rates, leading indicators, stock market indices and earnings. The vectors X_t and Z_t above are thus modified accordingly. For the sake of brevity we use the term *capital flows model* to denote this model class.

Unrestricted VAR models are known to forecast poorly due to their overfitting of parameters (see, e.g., Fair, 1979), therefore restricting linear combinations of the parameters in the VAR model to be equal to certain constants may result in improved forecasting features of the VAR model. Such restrictions may be imposed under the light of the available economic theory, as in the case of the structural models of exchange rate described above, or based on empirical grounds (in a similar way as, e.g., Kunst and Neusser, 1986). That is, an unrestricted VAR is estimated and insignificant lags of the endogenous variables are removed from the model specification. The class of estimated models where insignificant parameters have been removed will henceforth be denoted restricted VAR (RVAR).

2.2 Estimation and Forecasting Comparison

The forecasting exercise is carried out following a systematic procedure for all models and countries (see Appendix for the range and characteristics of the datasets).[3] Models in first differences and in levels are estimated for each class. The model selection concerning the number of lags to be included in the VAR specification is done by evaluating the AIC criterion for each lag length $l = 1,\ldots,8$ in the original estimation period and choosing the lag length with a minimum value of the information criterion. For the VEC models, the number of lags and error correction terms to be included is done by choosing the specification with a minimum AIC among all VEC models with lag length l ($l = 1,\ldots,8$) and all possible combinations of cointegration relationships in the original estimation period.

For the case of restricted models, the restrictions are imposed by setting to zero those parameters whose t-test statistic for parameter insignificance falls within the central 80% region of the t-distribution[4] in the estimated VAR specification for the original in-sample period.

The parameters of the model of interest are estimated for the available data up to 2000:12 (the periodicity of the data is monthly, and seasonally unadjusted series are adjusted using additive seasonality filters) and forecasts up to twelve months ahead are drawn from the estimated model. A new observation (the one corresponding to 2001:1) is added to the sample, the model is re-estimated, new forecasts are drawn from it and compared to realized values. This procedure is repeated until no usable observation is left. At this stage two statistics evaluating the forecast accuracy of the point forecasts of the model being studied (Root Mean Squared Error, RMSE, and Mean Absolute Error, MAE) are computed by comparing the forecasts with the actually realized values,

$$\mathrm{RMSE}(k) = \sqrt{\sum_{j=0}^{N_k-1} \frac{[F_{t+j+k} - A_{t+j+k}]^2}{N_k}},$$

[3] For each currency, the following models are estimated: u-VAR, u-RVAR, u-DVAR, u-RDVAR, u-VEC, s-VAR, s-RVAR, s-DVAR, s-RDVAR, s-VEC.

[4] This (unusual) level of significance was chosen after several experiments with lower significance levels lead to equations with too few regressors.

$$\text{MAE}(k) = \sum_{j=0}^{N_k - 1} \frac{| F_{t+j+k} - A_{t+j+k} |}{N_k},$$

where $k = 1,...,12$ denotes the forecast step, N_k is the total number of k-steps ahead forecasts in the projection period for which the realized value of the exchange rate A_t is known, and F_t is the forecast value for the exchange rate.

The Diebold-Mariano test (Diebold and Mariano, 1995) will be used to compare the accuracy of forecasts against random walk predictions. When comparing two forecasts, the question arises of whether the predictions of a given model, A, are significantly more accurate, in terms of a loss function $g(\cdot)$, than those of the competing model, B. The Diebold-Mariano test aims to test the null hypothesis of equality of expected forecast accuracy against the alternative of different forecasting ability across models. The null hypothesis of the test can be, thus, written as

$$d_t = E\left[g(e_t^A) - g(e_t^B)\right] = 0 \qquad (3)$$

where e_t^i refers to the forecasting error of model i when performing h-steps ahead forecasts. The Diebold-Mariano test uses the autocorrelation-corrected sample mean of d_t in order to test for (3). If n observations and forecasts are available, the test statistic is, therefore,

$$S = [\hat{V}(\overline{d})]^{-1/2} \overline{d},$$

where

$$\hat{V}(\overline{d}) = \frac{1}{n} \left[\hat{\gamma}_0 + 2 \sum_{k=1}^{h-1} \hat{\gamma}_k \right],$$

and

$$\hat{\gamma}_k = \frac{1}{n}\sum_{t=k+1}^{n}(d_t - \overline{d})(d_{t-k} - \overline{d}).$$

Under the null hypothesis of equal forecast accuracy, S is asymptotically normally distributed.

3. Results of the Forecasting Exercise

Tables 1 and 2 show the ratios of RMSE and MAE statistics of the best monetary and capital flows models (in terms of smallest average RMSE and MAE for the out-of-sample period considered) and the benchmark model (the random walk model) for the EUR/USD and EUR/JPY exchange rates. The results of the test of equal forecasting accuracy against the random walk model are included as well. The tables present the ratios of forecasting error for one to twelve months ahead, together with the ratio of the average prediction error for the period ranging from one to twelve months ahead. The column RMSE/RMSE(RW) [MAE/MAE(RW)] refers to the ratio between the root mean squared error (mean absolute error) of the model considered and that of a simple random walk model for the exchange rate. In all cases the best model chosen by minimizing average root mean squared error is the same, namely the restricted structured VAR model on differences (s-RDVAR).

The performance results of the EUR/USD exchange rate (see Table 1) using the monetary variables are not too convincing. The model fails to reject the null of equal forecast accuracy to the benchmark random walk model over all horizons when using both RMSE and MAE as loss functions. The random walk model actually outperforms the best monetary model over the two to six months horizon when using the RMSE as the loss function and over the three to six months horizon when using the MAE as the loss function. The forecasting performance when using the capital flows related variables is marginally better. When the RMSE is used as the loss function, the random walk model outperforms (albeit marginally) the best capital flows model only for five and six months ahead but for the rest the performance of the best model is not significantly better. On the other hand, when the MAE is taken as the loss function, the Diebold-Mariano test for forecasting horizons nine and ten months ahead is rejected at 10%. For the three to five months horizon the random walk model seems to outperform the best capital flows model and for the rest of horizons the best model outperforms the random walk model but not significantly.

The forecast performance of the best monetary model of the EUR/JPY exchange rate is more satisfactory. Taking the RMSE as the loss function, with exception of the one and two months horizon, the best monetary model outperforms the benchmark model significantly. More specifically, for the three and five months horizon the Diebold-Mariano test is rejected at 10%, for the four months horizon and from the six to ten months horizon the test for equal forecast accuracy is rejected at 5%, and for eleven and twelve months ahead the Diebold-Mariano test is rejected at 1%. In contrast, the forecast performance of the best capital flows JPY/EUR model is very poor. The random walk outperforms the best capital flows model on all horizons.

To summarize, the forecasting exercise delivers mixed results concerning the predictability of euro exchange rates. For the EUR/USD exchange rate monetary models tend to outperform the random walk model for long-term predictions (more than six months), but fail to reject the test of equality of forecasting accuracy against the random walk model for all forecasting horizons. On the other hand, the best monetary model for the EUR/JPY exchange rate outperforms the random walk model on all horizons and significantly for more than six months ahead. Models based on capital flow variables, on the other hand, tend to have worse predictive power than simple monetary models.

4. Summary and Conclusions

A battery of multivariate time series models has been compared to the naive random walk model in terms of forecasting accuracy in the prediction of the euro exchange rate against the U.S. dollar and the Japanese yen. The results partly confirm the conclusions in Meese and Rogoff (1983), namely that the random walk model performs as well as more sophisticated models of exchange rate determination for short-term predictions, including in this case VAR, VEC and restricted VAR models in different (structural and unstructural) specifications. For long-term predictions, however, multivariate time series models present clearly better forecasting accuracy than the simple random walk in the case of the monetary model for the EUR/JPY exchange rate, and marginally better forecasting accuracy in the case of the monetary and capital flows models for the EUR/USD exchange rate.

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Appendix: Data Sources and Characteristics

All time series have monthly periodicity (January 1980 to June 2004), and have been extracted from Thomson Financial Datastream. The variables used for EU-11, U.S.A. and Japan are:

- Money supply: M1 aggregate, indexed 1990:1=100. Seasonally unadjusted
- Output: Industrial production index 1990:1=100
- Short-term interest rate: 3-month interbank offered rate
- Long-term interest rate: 10-year rate interest rate on government bonds

- Leading indicator for Germany as a proxy for Europe: IFO Index
- Leading indicator for U.S.A.: ISM Index
- Earnings
- Stock market indices covering at least 80% of market capitalization in the respective country

*Table 1: Out-of-Sample Forecast Performance for USD/EUR: Best Monetary Model (Best Capital Flows Model) – RMSE and MAE. *(**)[***] Indicates Rejection of the Null Hypothesis of Equal Forecasting Accuracy at 10% (5%) [1%]*

U.S. dollar				
	Monetary model, s-RDVAR		Capital flows model, s-RDVAR	
Horizon	RMSE/RMSE(RW)	MAE/MAE(RW)	RMSE/RMSE(RW)	MAE/MAE(RW)
1 month	0.9382	0.9819	0.9642	0.9334
2 months	1.0055	0.9989	0.9856	0.9598
3 months	1.0413	1.0665	0.9967	1.0243
4 months	1.0505	1.0605	0.9935	1.0331
5 months	1.0468	1.0297	1.0018	1.0143
6 months	1.0270	1.0091	1.0015	0.9960
7 months	0.9872	0.9729	0.9748	0.9461
8 months	0.9608	0.9705	0.9467	0.9183
9 months	0.9556	0.9622	0.9363	0.9007*
10 months	0.9572	0.9601	0.9297	0.9082*
11 months	0.9538	0.9726	0.9250	0.9207
12 months	0.9583	0.9733	0.9362	0.9234
Average	0.9902	0.9965	0.9660	0.9565

Source: Authors' calculations.

*Table 2: Out-of-Sample Forecast Performance for JPY/EUR: Best Monetary Model (Best Capital Flows Model) – RMSE and MAE. *(**)[***] Indicates Rejection of the Null Hypothesis of Equal Forecasting Accuracy at 10% (5%) [1%]*

Japanese yen				
	Monetary model, s-RDVAR		Capital flows model, s-RDVAR	
Horizon	RMSE/RMSE(RW)	MAE/MAE(RW)	RMSE/RMSE(RW)	MAE/MAE(RW)
1 month	0.9865	0.9977	1.0114	1.0062
2 months	0.9540	0.9365	1.0388	1.0410
3 months	0.8763*	0.9086	1.0325	1.0535
4 months	0.8402**	0.8588	1.0699	1.0572
5 months	0.8218*	0.8453	1.1151	1.1215
6 months	0.7646**	0.8006	1.1401	1.1661
7 months	0.7198**	0.7378*	1.1486	1.1952
8 months	0.7112**	0.6888**	1.1629	1.1914
9 months	0.6632**	0.6176***	1.1675	1.1860
10 months	0.6346**	0.5794***	1.1794	1.1932
11 months	0.5930***	0.5179***	1.1984	1.1653
12 months	0.5522***	0.4789***	1.2077	1.1957
Average	0.7598	0.7473	1.1227	1.1310

Source: Authors' calculations.