Are the Exchange Rates of EMU Candidate Countries Anchored by their Expected Euro Locking Rates?

This paper examines whether the exchange rates of the Czech koruna, the Hungarian forint and the Polish złoty were anchored by the market expectations for their euro locking rates in the period from December 15, 2004, to August 3, 2006. First, I derive the process of the exchange rate as a function of the processes of the following three factors: latent exchange rate, market expectations for the euro locking rate and locking date. Then the expected final conversion rates are filtered. The time-varying volatilities of the state variables are estimated from cross-sectional data on option prices.

1 Introduction

This paper investigates the stabilizing feature of market expectations for the euro locking rate. The analysis is applied to three Economic and Monetary Union (EMU) candidate countries: the Czech Republic, Hungary and Poland. First, an economic model is constructed where the exchange rate is a function of three factors, namely the latent exchange rate, the market expectations for the final conversion rates and the locking dates. Then, in the empirical part of the paper, the historical exchange rate changes are decomposed into changes of each of the factors. In order to filter the factors, some parameters need to be estimated or calibrated. To estimate the time-varying volatilities of the filtered factors, a theoretical option pricing model is derived in the paper and cross-sectional data on option prices with different maturities are used. The identification of the time-varying volatilities is based on the following: Options with longer maturities depend more on the volatility of one of the filtered factors than options with shorter maturities. By investigating the filtered market expectation of the euro locking rate, I make inferences about the exchange rate-stabilizing effect of locking.

Rather than filtering market expectations for the locking rate, it would have been possible to derive the market expectations from an alternative source (Reuters polls) or with an alternative method (estimating equilibrium exchange rates). However, for the reasons outlined below, filtering has some advantages over its alternatives.

Reuters regularly surveys the expectations of analysts regarding the entry dates to EMU and the exchange rate mechanism II (ERM II) and regarding the central parities in ERM II. The central parity expected by respondents may be considered as the market expectations for the final conversion rate. Yet extracting market expectations for the final conversion rate from daily historical exchange rate data may yield more accurate and more up-to-date information than the monthly or quarterly Reuters polls. Moreover, the higher

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2 The latent exchange rate is defined as the exchange rate that would prevail if the currencies were not going to be locked against the euro.
frequency of the filtered expectations enables us to investigate the stabilizing
effect the prospect of locking has on the exchange rate.

As EMU candidate countries aim at having their final irrevocable conversion
rates set equal to their equilibrium exchange rates, reliable estimates on the
latter also reflect market expectations for the final conversion rate. Yet in the
context of our research this concept poses at least three kinds of problems.\footnote{Égert, Halpern and MacDonald (2006) survey a number of issues related to the equilibrium exchange rates of transition economies. They conclude that “…deriving a precise figure for the equilibrium real exchange rates in general and also for the transition economies is close to mission impossible as there is a great deal of model uncertainty related to the theoretical background and to the set of fundamentals chosen.”}
First, economists use a number of concepts to define, and a number of methods\footnote{Williamson (1994) gives an overview of the widely used methods: Fundamental Equilibrium Exchange Rate (FEER), Behavioral Equilibrium Exchange Rate (BEER), NATural Real Exchange (NATREX).} to estimate, the equilibrium real exchange rate. Second, these estimates refer
to the real rather than the nominal exchange rate. Third, market expectations
might deviate from the estimated nominal equilibrium exchange rate, especially
if the choice of the final conversion rate is based not only on economic, but also
on political considerations.

The novelty of this paper is that it filters the subjective market expectations
for the final conversion rate, which not only mirrors economic but also possible
political considerations. By comparing the time series of the filtered subjective
market expectations with the time series of the historical exchange rate we
can make inferences about the stabilizing effect of locking on the exchange
rate.

With regard to market expectations for the time of locking, the Reuters
polls appear to yield reliable results, which are therefore also used in this paper.
Csajbok and Rezessy (2005), for instance, estimate the expected euro area
entry date of Hungary from the forint and euro yield curves and find the
estimates to be relatively close to analysts’ expectations provided by the
Reuters polls.

The paper is structured as follows: Section 2 presents the exchange rate
model. Section 3 derives an option pricing formula used to estimate parameters
in the empirical part of the paper. In section 4 I start by defining the filtering
problem, then I show how the parameters are set and finally I present the
results of the Kalman filtering. Section 5 concludes.

2 Exchange Rate Model
The exchange rate model is similar to Krugman’s target-zone model (1991).
First, in Krugman (1991) the target-zone exchange rate is derived from the
fundamental. Similar to Krugman’s approach our starting point is that the
exchange rate subject to future locking is a function of the fundamental $v$.
Second, in Krugman’s model the logarithm of the target-zone exchange rate is
equal to the fundamental plus a term proportionate to the conditional expected
change of the logarithm of the exchange rate. Moreover, the exchange rate
would be equal to the fundamental if there were no target zone. In our model
the log exchange rate subject to future locking is equal to the fundamental plus
a term proportionate to the conditional expected instantaneous change of the
log exchange rate. In the absence of future locking, the term proportionate to

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the conditional expected instantaneous change of the exchange rate would be zero, so that the exchange rate would be equal to the fundamental. Given this relationship, the fundamental $v$ is referred to as the log latent exchange rate, i.e. the exchange rate that would prevail if the currency was not going to be locked against the euro at some point.

Third, the implicit relationship between the target-zone exchange rate and the fundamental in Krugman’s model is the same as the relationship between the exchange rate subject to future locking and the latent exchange rate in this model. This relationship can be expressed as follows in a reduced form:

$$s_t = v_t + c \frac{E_t(d_s)}{dt}.$$  \hspace{1cm} (1)

Here, $s_t$ is the log exchange rate and $v_t$ is the log latent exchange rate. The constant $c$ is the time scale. The term $\frac{E_t(d_s)}{dt}$ is the expected instantaneous change of the exchange rate.

While Krugman investigates the stabilizing feature of the target zone with a floating regime as a benchmark, this contribution explores the stabilizing effect of future locking on the exchange rate as compared with a “no locking” benchmark regime.

In the following, the latent exchange rate $v_t$ is defined as a function of some macro variables, as explained in (4f) of footnote 5:

$$v_t = -\alpha y_t + q_t + c \psi_t - p^*_t + m_t + c i_t^r.$$  \hspace{1cm} (2)

In this respect, $y$ denotes domestic real output, $q$ is the real log exchange rate, $\psi$ is the risk premium, $p^*$ is the foreign log price, $m$ denotes the domestic nominal money supply and $i^r$ denotes the foreign interest rate. For the sake of simplicity, $p^*$, $m$ and $i^r$ are assumed to be constant and normed to 0.

As mentioned above, the EMU candidates will aim to have their exchange rates fixed at their equilibrium levels, for which the concept of the behavior equilibrium exchange rate (BEER) is chosen. Given that the strong law of purchasing power parity (PPP) should hold for the locking rate under this equilibrium concept, the log nominal exchange rate at the time of locking is equal to the difference between the domestic and foreign log prices:

$$Svensson (1991)$$ presents one possible structural model for the reduced form (1):

$$m_t - p_t = \alpha y_t - ci_t^r \quad \alpha > 0 \quad c > 0 \quad \text{money market equilibrium}$$

$$q_t = s_t + p^*_t - p_t \quad \text{real exchange rate}$$

$$\psi_t = i_t - i^r - \frac{E_t(d_{as})}{dt} \quad \text{risk premium}$$

$$v_t = -\alpha y_t + q_t + c \psi_t - p^*_t + m_t + c i_t^r \quad \text{fundamental/latent exchange rate}$$

In this model the parameter $c$ can be interpreted as the interest rate elasticity of the money demand.

Two different types of expectations are considered in this paper. One is the subjective market expectation and the other is the mathematical expected value of a random variable. Here, reference is made to the latter. In order to distinguish between the two, the first type of expectation is referred to as market expectation. However, under rational expectations the two are the same.
$s_T = p_T - p^*_T$. Under rational expectations the market expects the final conversion rate at time $T$ to be $x_T = E_T(s_T)$, which gives

$$x_t = p_t + \int_t^T E_t(\pi_\tau)d\tau$$

(3)

where $\pi$ denotes the inflation rate.

Neither the definitions (2) and (3) nor the corresponding macrodata are used directly in the empirical part of the paper – mainly because of the low frequency of these data, but also because of a possible misspecification of the underlying macro models. For instance, the examples of the current EMU countries show that the locking rates caused deviations from the strong law of purchasing power parity. Still, it is the equilibrium real exchange rate that should be the key determinant of the locking rate chosen on the basis of economic considerations. Therefore, while not used directly in the following, these definitions motivate the interpretation of our results and the choice of the processes assumed for the underlying factors of the model.

2.1 Dynamics

In this subsection the processes of the factors are specified. These processes will be used to derive the process of the exchange rate.

We start by assuming that all three factors, i.e. $T_t$, $v_t$ and $x_t$, follow Brownian motions. First, the process of market expectations regarding the log euro locking rate, $x_t$, can be derived from equation (3) and expressed in a discrete time framework as $\Delta x_t = [\pi_{t+1} - E_t(\pi_{t+1})] + \sum_{i=t+1}^{T_t} [E_i(\pi_i) - E_i(\pi_i)] d\tau$. Under the assumptions that both expectation errors and changes in expectations are independent and normally distributed with zero mean, the process of $x_t$ can be rewritten in a continuous time framework as

$$dx_t = \begin{cases} \sigma_{x,t} dz_{x,t}, & \text{if } t < T_t \\ 0, & \text{otherwise} \end{cases}$$

(4)

where $dz_{x,t}$ is a Wiener process.

Second, the discrete time process of the log latent exchange rate, $v_t$, can be derived from (2), (4) and from two additional equations of the model. By defining $\chi_t$ by its discrete corresponding process as $\Delta \chi_t = [\alpha + \gamma] (\beta \sum_{i=t+1}^{T_t} [E_i(\pi_i) - E_i(\pi_i)] + c \Delta \psi_t)$, the process of the latent exchange rate is $dv_t = -(\alpha + \gamma) \sigma_{x,t} dz_{x,t} + d\chi_t$. If $\chi_t$ is assumed to follow Brownian motion then the continuous time process of the log latent exchange rate is

$$dv_t = \sigma_{x,t} dz_{x,t}$$

(5)

where $dz_{x,t}$ is a Wiener process. By assuming the expectation error $(\pi_{t+1} - E_t(\pi_{t+1}))$ to be orthogonal to the sum of changes of expectations $(\sum_{i=t+1}^{T_t} [E_i(\pi_i) - E_i(\pi_i)])$, and by assuming the risk premium $\psi_t$ to be

7 The model is extended with a supply curve and an equation capturing the Balassa-Samuelson effect:

(5f) $y_{t+1} = \beta y_{t+1} - \beta(\pi_{t+1} - \pi_{t+1}) \beta > 0$ supply curve

(6f) $dq_t = -\gamma dy_t \gamma > 0$ Balassa-Samuelson effect (real appreciation).
orthogonal to both the expectation error and the sum of changes of expectations, we find the correlation between $dz_{v,t}$ and $dz_{x,t}$ to be

$$\rho(dz_{v,t}, dz_{x,t}) = - (\alpha + \gamma) \beta \frac{\sigma_{s,t}}{\sigma_{v,t}}. \quad (6)$$

Third, the assumed process of market expectations regarding the time of locking, $T_t$, is the following martingale:

$$dT_t = \begin{cases} (T_t - t) \sigma_{T,t} dz_{T,t} & \text{if } t < T_t \\ T & \text{otherwise} \end{cases} \quad (7)$$

where $dz_{T,t}$ is a Wiener process.

Given that EMU candidates need to fulfill the Maastricht criteria, the market expectations regarding the time of locking $T_t$ depend on both inflationary and fiscal shocks. The market expectation for the log final conversion rate $x_t$ reacts mainly to the inflationary shocks, whereas the log latent exchange rate $v_t$ is more strongly related to real output and hence to fiscal shocks. $T_t$, in turn, depends on $x_t$ (the log final conversion rate) and on $v_t$ (the log latent exchange rate). To be more precise, higher uncertainty relating to $x_t$ and $v_t$ makes $T_t$ more volatile. Moreover, the higher the interest rate elasticity of money demand $c$, the more efficient monetary policy can be by influencing inflation and output – in other words, the less dependent on $x_t$ and $v_t$, the expected time of locking will be.

Along these lines it is possible to make restrictions on the process of the expected time of locking. However, the restrictions to be posed are not uniquely determined by the above intuitive requirements. In this paper, the restrictions (8) and (9) below were chosen for technical reasons, motivated by the demand for an analytical solution to the functional relationship $s_t = f(t, v_t, x_t, T_t)$.

$$\rho(dz_{T,t}, dz_{x,t}) \frac{\sigma_{s,t}}{x_t} = \frac{1}{c} (T_t - t) \sigma_{x,t} \quad (8)$$

$$\rho(dz_{T,t}, dz_{v,t}) \frac{\sigma_{s,t}}{v_t} = \frac{1}{c} (T_t - t) \sigma_{v,t} \quad (9)$$

### 2.2 Functional Relationship between the Exchange Rate and Underlying Factors

To obtain the functional relationship $s_t = f(t, v_t, x_t, T_t)$ between the log exchange rate on the one hand and the log latent exchange rate $v_t$ as well as market expectations for the log final conversion rate $x_t$ and the time of locking $T_t$ on the other hand, Itô’s stochastic change-of-variable formula is used.\(^8\)

$$s_t = f(t, v_t, x_t, T_t) = \left(1 - e^{-\frac{x_t}{v_t}}\right) v_t + e^{-\frac{x_t}{v_t}} x_t. \quad (10)$$

\(^8\) Proof is available from the author upon request.
This function satisfies (1), (8) and (9), Ito's formula given by (11) and the terminal condition (12).

\[
\begin{align*}
    df &= \left[ \frac{\partial f}{\partial t} + \frac{\partial f}{\partial v} \mu_v + \frac{\partial f}{\partial x} \mu_x + \frac{\partial^2 f}{\partial T^2} \sigma^2_v + \frac{\partial^2 f}{\partial x^2} \sigma^2_x + \frac{\partial^2 f}{\partial T \partial v} \rho \sigma_v \sigma_x + \frac{\partial^2 f}{\partial T \partial x} \rho \sigma_v \sigma_x \right] dt + \\
    &+ \frac{1}{2} \frac{\partial^2 f}{\partial v^2} \sigma^2_v (T_t - t)^2 + \frac{1}{2} \frac{\partial^2 f}{\partial x^2} \sigma^2_x (T_t - t)^2 \\
    &+ \frac{1}{2} \frac{\partial^2 f}{\partial T \partial v} \rho (dz_v, dz_x) (T_t - t) \sigma_v \sigma_x + \\
    &+ \frac{1}{2} \frac{\partial^2 f}{\partial T \partial x} \rho (dz_v, dz_x) (T_t - t) \sigma_v \sigma_x \\
    &+ \frac{\partial f}{\partial v} \sigma_v dz_v + \frac{\partial f}{\partial x} \sigma_x dz_x + \frac{\partial f}{\partial T} (T_t - t) \sigma_T dz_T.
\end{align*}
\]

This function satisfies (11), (8) and (9), Ito's formula given by (11) and the terminal condition (12).

\[
(11)
\]

Equation (10) shows that the log exchange rate is the weighted average of the log latent exchange rate and the expected log final conversion rate. The weights change over time; if the time until locking is infinite or, in other words, if the currency will not be locked, then the weight of the latent exchange rate is 1 and the weight of the expected final conversion rate is 0. As the time until the locking decreases, the weight of the expected final conversion rate increases. Finally, as the time until locking approaches 0, the weight of the expected final conversion rate approaches 1.

In order to examine the dynamics of the exchange rate, equation (11) is rewritten as follows: By substituting (8), (9) and (10) and \( v_t = \frac{1}{1 - e^{-\frac{5T_t}{c}}} s_t - \frac{e^{-\frac{5T_t}{c}}}{1 - e^{-\frac{5T_t}{c}}} x_t \) into equation (11) we obtain

\[
\begin{align*}
    ds_t &= \frac{1}{c} e^{-\frac{5T_t}{c}} (x_t - s_t) dt + \left( 1 - e^{-\frac{5T_t}{c}} \right) \sigma_v dz_v + \\
    &+ e^{-\frac{5T_t}{c}} \sigma_x dz_x + \frac{1}{c} e^{-\frac{5T_t}{c}} (x_t - s_t) (T_t - t) \sigma_T dz_T.
\end{align*}
\]

Equation (13) shows the dynamics of the exchange rate to be such that it converges to the actual market expectation for the final conversion rate. Moreover, the closer the time of locking, the faster the convergence is.

Equations (4), (5), (7) and (10) define a three-factor model. One factor is the market expectation for the final conversion rate; another factor is the market expectation for the time of locking; the third factor is the latent exchange rate. This model is linear in two of the factors, but not in \( T_t \).

### 3 Option Pricing

This section presents a pricing formula for European-type options that fits our model. This option pricing formula is used to estimate the time-varying volatilities of the filtered factors. The historical option prices are given in terms
of implied volatility; consequently, the option prices are derived in terms of volatility as well.

In the theoretical model the uncertainty is present due to the stochastic innovations \( (dz_{\tau t}, dz_{x,t}, dz_{y,t}) \) of the factors; consequently, the price of an option is a function of the variances and covariances of these normally distributed innovations. From equation (13), we can derive that the instantaneous variance of the log changes of the exchange rate at time \( t \) is

\[
\sigma^2_{s,t} = \sigma^2_{s,t} + \left( \frac{1}{c} e^{-\frac{T - T}{\tau}} \right) \left( x_t - s_t \right)^2 (T - t)^2 \sigma^2_{y,t} + \\
-2 \frac{1}{c} e^{-\frac{T - T}{\tau}} (x_t - s_t) (T - t) \sigma_{y,t} \left( \frac{1}{c} e^{-\frac{T - T}{\tau}} \right) \sigma_{x,t} \rho(dz_{x,t}, dz_{y,t}) + \\
-2 \frac{1}{c} e^{-\frac{T - T}{\tau}} (x_t - s_t) (T - t) \sigma_{x,t} e^{-\frac{T - T}{\tau}} \sigma_{z,t} \rho(dz_{z,t}, dz_{x,t})
\]

where \( \sigma^2_{s,t} \) is

\[
\sigma^2_{s,t} = \left( 1 - e^{-\frac{T - T}{\tau}} \right) \sigma^2_{v,t} + \left( e^{-\frac{T - T}{\tau}} \right) \sigma^2_{y,t} + 2 \left( 1 - e^{-\frac{T - T}{\tau}} \right) \left( e^{-\frac{T - T}{\tau}} \right) \sigma_{v,t} \sigma_{x,t} \rho(dz_{x,t}, dz_{y,t}).
\]

The magnitude of the terms of (14) other than \( \sigma^2_{s,t} \) are negligible compared with the magnitude of \( \sigma^2_{s,t} \). Consequently, these terms will be disregarded in the theoretical option pricing formula and \( \sigma^2_{s,t} \) will be approximated by \( \sigma^2_{s,t} \). Moreover, the following simplification is made. Until now, \( \sigma_{v,t}, \sigma_{x,t}, \sigma_{y,t} \) \( \sigma_{v,t} \) were allowed to change over time. While we do not rule out this possibility, the influence of volatility changes on option prices would appear to be limited. The pricing formula for the stochastically changing volatility case is different from the one derived here, however, the one derived is a good approximation for the theoretical option pricing formula and

\[
g(t, m, \sigma_{s,t}, \sigma_{v,t}, \sigma_{x,t}, \rho(dz_{v,t}, dz_{x,t})) = \left[ \int_t^{t+m} \sigma^2_{s,t} d\tau \right]^{1/2} = \\
\left[ \int_t^{t+m} \left( 1 - e^{-\frac{T - T}{\tau}} \right) \sigma^2_{v,t} + \left( e^{-\frac{T - T}{\tau}} \right) \sigma^2_{y,t} + \\
+2 \left( 1 - e^{-\frac{T - T}{\tau}} \right) \left( e^{-\frac{T - T}{\tau}} \right) \sigma_{v,t} \sigma_{x,t} \rho(dz_{x,t}, dz_{y,t}) \right]^{1/2}
\]

where the option is sold at time \( t \) and the time until maturity is denoted by \( m \).

In this formula \( T_\tau (\tau > t) \) is stochastic and unknown at time \( t \). In order to avoid complication coming from the stochastic nature of \( T_\tau \), \( T_\tau \) is

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9 As pointed out by Hull (1997, p. 620), "For options that last less than a year, the pricing impact of a stochastic volatility is fairly small in absolute terms. It becomes progressively larger as the life of option increases."
approximated\(^\text{10}\) by \(T_t\). By applying this final approximation and by calculating the integrals, the option pricing formula is obtained:

\[
g^2(t, m, \sigma_{v,t}, \sigma_{w,t}, \rho) = \sigma_{v,t}^2 \left\{ m - 2c e^{-\frac{1}{2} (T_t - t - m)} + 2c e^{-\frac{1}{2} (T_t - t)} \right\} + \\
+ \sigma_{w,t}^2 \left\{ \frac{c}{2} e^{-\frac{1}{2} (T_t - t - m)} - \frac{1}{2} c e^{-\frac{1}{2} (T_t - t)} \right\} + 2c e^{-\frac{1}{2} (T_t - t - m)} \rho(dz_{v,t}, dz_{x,t}) \frac{\sigma_{v,t}}{\sigma_{w,t}} + \\
- 2c e^{-\frac{1}{2} (T_t - t)} \rho(dz_{v,t}, dz_{x,t}) \frac{\sigma_{v,t}}{\sigma_{w,t}} + \\
- ce^{-\frac{1}{2} (T_t - t - m)} \rho(dz_{v,t}, dz_{x,t}) \frac{\sigma_{v,t}}{\sigma_{w,t}} + ce^{-\frac{1}{2} (T_t - t)} \rho(dz_{v,t}, dz_{x,t}) \frac{\sigma_{v,t}}{\sigma_{w,t}}. \\
\]

(17)

This option pricing formula (17) is used to estimate the time-varying volatilities \(\sigma_{v,t}, \sigma_{w,t}\) of the filtered factors. By using formula (17) and cross-sectional data on options with different maturities but the same issuing date \(t\), the volatilities \(\sigma_{v,t}, \sigma_{w,t}\) can be estimated for each time \(t\). The intuition behind the identification is that longer options are more exposed to shocks occurring in the distant future than options with shorter maturities. Or in other words, \(\sigma_{w,t}\) has a higher relative weight in a longer option than in a shorter one, whereas the opposite holds for \(\sigma_{v,t}\).

4 Filtering Factors

The Kalman Filter technique is applied to extract the time series of the factors from the time series of the observable exchange rate. Filtering all three factors from only one series would be overambitious and unlikely to provide robust results. Luckily, Reuters polls on the prospective euro entry dates are an alternative and reliable source of information on market expectations for locking dates. Thus, the time of locking \(T_t\) is treated as being exogenously given. As \(T_t\) is not independent of the other two factors I use the conditional distributions of \(x_t\) and \(v_t\), where I condition on the realization of \(T_t\). The Kalman filter technique can be applied to filter factors only if the model is linear\(^\text{11}\) in all factors which are to be filtered. The log exchange rate \(x_t\) is linear in the remaining two factors, namely the latent exchange rate \(v_t\) and market expectations for the final conversion rate \(x_t\).

4.1 Filtering Problem

Since one of the factors \(T_t\) is exogenous and since \(T_t\) is not independent of the other two factors, I have to use the conditional distributions of \(x_t\) and \(v_t\),
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where I condition on the realization of $T_r$. The conditional expected innovations of $x_t$ and $v_t$ are $\rho(dz_{T,r},dz_{x,t})dz_{T,r}$ and $\rho(dz_{T,r},dz_{v,t})dz_{T,r}$ respectively, where $\rho$ denotes correlations. These expected changes of $dz_{x,t}$ and $dz_{v,t}$ are taken into account in the model by having a constant as a third state variable. The system covariance matrix $Q(t)$ is also conditional on $T_r$. The filtering problem can be written in the usual form:

$$\Lambda(t+1) = A(t)\Lambda(t) + w_1(t+1)$$

$$\Omega(t) = C(t)\Lambda(t) + w_2(t)$$

$$E\left[\begin{bmatrix} w_1(t+1) \\ w_2(t) \end{bmatrix}\left|\begin{bmatrix} w_1(t) \\ w_2(t) \end{bmatrix}\right.\right] = \begin{bmatrix} Q(t) & 0 \\ 0 & R \end{bmatrix}$$

In our problem, the transpose of the vector of states is $\Lambda(t) = (v_t \ x_t \ 1)$. The system matrix is

$$A(t) = \begin{bmatrix} 1 & 0 & \sigma_{x_t \ x_T} \rho(dz_{T,r},dz_{x,t}) \frac{dz_{T,r}}{\sigma_{x_t \ x_T}} \\ 0 & 1 & \sigma_{x_t \ x_T} \rho(dz_{T,r},dz_{x,t}) \frac{dz_{T,r}}{\sigma_{x_t \ x_T}} \\ 0 & 0 & 1 \end{bmatrix}.$$  

The vector $w_1(t)$ is assumed to be a Gaussian vector white noise. The observable variable is the log exchange rate $\Omega(t) = s_t$. Equation (10) implies that the observation matrix is

$$C(t) = \begin{bmatrix} 1 - e^{-\frac{t - T_r}{\tau}} & 0 \end{bmatrix}.$$  

The system covariance matrix can be written as

$$Q(t) = \begin{bmatrix} Q_{1,1}(t) & Q_{1,2}(t) & 0 \\ Q_{2,1}(t) & Q_{2,2}(t) & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

where the covariance is conditional on the observed $T_r$, therefore

$$Q_{1,1}(t) = \sigma_{x_t}^2 \left[ 1 - \rho^2(dz_{x,t},dz_{v,t}) \right],$$

$$Q_{1,2}(t) = \sigma_{x_t} \sigma_{v_t} \rho(dz_{x,t},dz_{v,t}) \rho(dz_{x,t},dz_{v,t}) \sigma_{x_t} \sigma_{v_t},$$

$$Q_{2,2}(t) = \sigma_{v_t}^2 \left[ 1 - \rho^2(dz_{x,t},dz_{v,t}) \right].$$

The error term $w_2(t)$ is assumed to be 0. In other words, we observe the exchange rate without error and model (10) perfectly describes the relationship between the factors and the exchange rate. Hence, the variance of the observation error term $R$ is set to 0. The Kalman filter remains valid even in this case.\(^{12}\)

In our problem, the observation matrix $C(t)$, the system matrix $A(t)$ and the system covariance $Q(t)$ are changing over time.

\(^{12}\) See Harvey (1990, p. 108) for a detailed discussion.
The parameters of the observation matrix $c$, $T$, and the parameters $\sigma_{x,t}$, $\sigma_{v,t}$, $\rho(dz_{v,t},dz_{x,t})$, $\rho(dz_{T,t},dz_{x,t})$, and $\rho(dz_{T,t},dz_{v,t})$ of the system covariance $Q(t)$ and of the system matrix $A(t)$ need to be either calibrated or estimated. Moreover, the initial values $x_{t_0}$ and $v_{t_0}$ of the factors belonging to the beginning of the sample period $t_0 = \text{Dec. 15, 2004}$, need to be set as well. The next section describes how these parameters are estimated and how $T$ is set.

### 4.2 Parameters

First, I describe how $T$ is set based on Reuters polls. Then, I show how the parameters $x_{t_0}$, $v_{t_0}$, $\rho(dz_{v,t},dz_{x,t})$, $\rho(dz_{T,t},dz_{x,t})$, $\rho(dz_{T,t},dz_{v,t})$ and $\sigma_{T,t}$ are calibrated. Finally, I describe how the parameters $\sigma_{v,t}$, $\sigma_{x,t}$ and $c$ are estimated from historical option prices and exchange rate data.

For calibrating the expected time of locking $T$, I take into consideration that the exchange rates of the countries that entered the ERM II system in recent years were almost fixed: The volatility of the Estonian kroon, the Lithuanian lita, the Slovenian tolar, the Cyprus pound and the Maltese lira dropped below 1% after entering ERM II.\(^{13}\) This finding implies that locking does not take place when a country enters EMU, but rather when it enters the ERM II regime. Therefore, the locking dates are modeled as ERM II entry dates and the parameter of locking dates is set equal to the average expectations of analysts derived from monthly and quarterly Reuters polls about ERM II entry dates for the Czech Republic, Poland and Hungary.\(^{14}\) Reuters queries analysts quarterly about their ERM entry expectations for Poland and the Czech Republic and monthly about their expectations for Hungary, i.e. after the monthly releases of most of the macro indexes. If analysts’ expectations are mainly based on the newest macrodata, their expectations are unlikely to change between two monthly Reuters polls. Therefore, expectations of ERM II entry dates reported on the monthly polling days are assumed to be formed on those very days. And the monthly observations are simply interpolated on $T$ of Hungary by translating them into daily constant data. The same interpolation is applied to the quarterly Reuters poll data for Poland and the Czech Republic.

Chart 1 shows the average expected ERM II entry dates reported for the Czech Republic, Hungary and Poland in the period from December 15, 2004, to August 3, 2006. From this chart we can see that market expectations were relatively stable until the autumn of 2005 and subsequently changed between the quarterly polls of August and November for the Czech Republic and Poland, and between September and October 2005 for Hungary. Until the autumn of

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\(^{13}\) The Estonian kroon, the Lithuanian lita and the Slovenian tolar joined ERM II on June 27, 2004. On May 2, 2005, three other EU Member States joined ERM II: Cyprus, Latvia and Malta.

\(^{14}\) In order to check the robustness of our results, I model the time of locking at the time of EMU entry in an alternative specification, in which the parameter of the time of locking is set equal to the average of the reported expectations of the individual analysts regarding the time of EMU entry. Since the results of the ERM II entry date specification do not differ qualitatively from those of the EMU entry date specification, only the former are presented here.
2005 the three countries had been expected to enter ERM II in the course of 2007. Thereafter, the expectations changed dramatically, as reflected by the monthly and quarterly Reuters polls, pointing to a postponement of ERM II entry to 2008 for the Czech Republic, to 2009 for Poland and to 2010 for Hungary.

For a given value of \( c \), one can calibrate the initial values of the factors and the correlations. The estimation method for the parameter \( c \) will be discussed in detail at a later point. For now, let us assume that we know the parameter \( c \) and want to calibrate the parameters \( x_0, v_0, \rho(dz_{T,t},dz_{x,t}), \rho(dz_{T,t},dz_{v,t}) \) and \( \rho(dz_{x,t},dz_{v,t}) \).

What makes these calibrations somewhat difficult is that no direct information on the latent exchange rate is available. For a given value of \( c \) (see below), the initial values of the factors and the correlations — in our case the parameters \( x_0, v_0, \rho(dz_{T,t},dz_{x,t}), \rho(dz_{T,t},dz_{v,t}) \) and \( \rho(dz_{x,t},dz_{v,t}) \) — are calibrated as follows. For the calibration of the initial states, \( x_0 \) and \( v_0 \), and some time-invariant parameters, the Reuters polls are used. I assume that \( x_0 \) is equal to the log of averaged expectations on the central parity reported by the last Reuters polls of 2004. The initial value of \( v_0 \) is calculated by plugging \( s_0, c \) and \( T_0 \) into equations (10).

One possibility to calibrate the correlations \( \rho(dz_{T,t},dz_{x,t}), \rho(dz_{T,t},dz_{v,t}) \) and \( \rho(dz_{x,t},dz_{v,t}) \) is to use not only the end-2004 Reuters poll data but all averaged expected central parities reported by the polls. By following this strategy of calibration, first, the latent exchange rates corresponding to each of the monthly and quarterly observations need to be calculated, using again equations (10) and the corresponding observations of the historical exchange rate, of parameter \( c \) and the calibrated \( T \). Then the calibrated correlations can be calculated from these monthly and quarterly data on \( x, v \) and \( T \). This
strategy of calibration has the major drawback that only a few observations\textsuperscript{15} can be used to calculate the correlations. Moreover, by following this strategy one might obtain correlations with a sign that is not in line with the theoretical considerations\textsuperscript{16} presented in subsection 2.1. Indeed, six of the nine correlation parameters of the three countries have the wrong sign if their calibration is based on the above method. Consequently, I opt to simply set all nine correlations to 0.

The estimated $c$ maximizes the filtering likelihood, which is obviously a function of the calibrated initial state parameters. Consequently, the sequence of estimation and calibration should be the following: First these parameters should be calibrated for all candidates of $c$. Then the filtering likelihood can be calculated for the set of calibrated parameters and the candidate for $c$. Finally, by searching for the optimal $c$, the estimated $c$ parameter and the calibrated parameters depending on $c$ are determined simultaneously.

It is difficult to estimate the volatility of market expectations regarding the time of locking for the following reasons. First, this volatility is likely to fluctuate substantially over time; second, as only a few observations on $T'$ are available to estimate the time-varying $\sigma_{T,t}$, I have to rely more on intuition than on the data. The instantaneous volatility $\sigma_{T,t}$ is assumed to be very large whenever market expectations for locking dates jump. However, $\sigma_{T,t}$ is assumed to be negligible\textsuperscript{17} whenever market expectations for the time of locking are unchanged. This assumption renders the system matrix $A$ independent from the jumps in $T$.\textsuperscript{18} In contrast to the system matrix $A$, the system covariance matrix $Q$ is still affected by the changes in $T$ through its time-invariant correlations with the other factors.

For a given value of time-invariant parameters $c$ and $\rho(dz_{x,t}, dz_{v,t})$ it is possible to estimate the time-varying volatilities of the factors $v$ and $x$. Parameters $\sigma_{v,t}$ and $\sigma_{x,t}$ are estimated from six implied volatilities $\sigma_{imp}$ for every point in time $t$ by ordinary least squares (OLS). The basic idea of the estimation is to minimize the distance between the theoretical option prices given by the option pricing formula (17) and the historical option prices. The six options have different maturities $m(i)$. In case of the Czech koruna and the Polish złoty the maturities are one month $m(1)$, two months $m(2)$, three months $m(3)$, six months $m(4)$, nine months $m(5)$ and one year $m(6)$, whereas in the case of the Hungarian forint the currency options have one-week $m(1)$, one-month $m(2)$, two-month $m(3)$, three-month $m(4)$, six-month $m(5)$ and one-year $m(6)$ maturities. The OLS estimates of $\sigma_{v,t}$ and $\sigma_{x,t}$ satisfy

\begin{equation}
\rho(dz_{T,t}, dz_{x,t}) \geq 0, \quad \rho(dz_{T,t}, dz_{v,t}) \geq 0 \quad \text{and} \quad \rho(dz_{x,t}, dz_{v,t}) \leq 0.
\end{equation}

\textsuperscript{15} The number of observations is 7 in case of the Czech Republic and Poland and it is 20 in case of Hungary.

\textsuperscript{16} Based on the theoretical considerations the correlations have to meet the following sign restrictions:

\begin{equation}
\rho(dz_{T,t}, dz_{x,t}) \geq 0, \quad \rho(dz_{T,t}, dz_{v,t}) \geq 0 \quad \text{and} \quad \rho(dz_{x,t}, dz_{v,t}) \leq 0.
\end{equation}

\textsuperscript{17} Whenever $\sigma_{T,t}$ is negligible, the option pricing formula (17) is valid, because all the applied approximations of the derivation of (17) are justified.

\textsuperscript{18} An alternative assumption is that $\sigma_{T,t}$ is always finite and the $A$ matrix is affected by the changes of $T$. The time-varying parameter $\sigma_{T,t}$ could be chosen so that the process of $V$ is pulled back to its reported value in each month or quarter. This specification would be interesting only if the Reuters poll data on the expected central parity were more reliable and one aimed to filter $X$ between every two Reuters polls.
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\[ \min_{\sigma_x, \sigma_v} \sum_{i=1}^{6} \left[ g(t, m(i), \sigma_x, \sigma_v, \rho(z_{v,t}, dz_{x,t})) - \sigma_{t,i}^{imp} \right]^2. \quad (21) \]

The term \( \sigma_{t,i}^{imp} \) of equation (21) is either the implied volatility \( \sigma_{t,i}^{imp} \) or a transformation of it. The possible need for a transformation of the implied volatilities can be explained along the following lines. Obviously, if the option pricing model of section 3 were to perfectly capture the relationship between the volatility of the factors and the implied volatilities, then there would be no need for any transformation. Since the filtered factors are heavily dependent on their estimated volatilities, it is crucial to investigate what else can affect the implied volatilities apart from the volatilities of the factors. Moreover, if these other possible effects do not happen to be orthogonal to the volatilities of the factors in the option pricing formula, then we face the omitted-variable problem. Hence, the estimated volatilities of the factors will be biased.

One possibly omitted variable is the one that captures the effect of an implicit or explicit fluctuation band. Until this point, I have not taken into account that the fluctuation of the exchange rate of the Hungarian forint against the euro is limited by an exchange rate band. In addition, the Czech Republic and Poland might apply an implicit fluctuation band that can have significant but different effects on the historical option prices with different maturities. The closer the exchange rate is to the edges of the band, the more limited is its volatility.\(^{19}\) Moreover, the diminishing effect on the volatility is higher over longer periods. Consequently, option prices with longer maturities should be affected more heavily by the relative position of the exchange rate in the fluctuation band than by option prices with shorter maturities.

First, I transform the implied volatilities in order to purge the possible effect of an explicit or implicit band. When finding empirical evidence of significant effects of the band on the implied volatilities I use the transformed data to estimate \( \sigma_v \) and \( \sigma_x \) by (21); in the absence of evidence for the effect of a possible target zone on the volatilities, I use the untransformed implied volatility data to estimate \( \sigma_v \) and \( \sigma_x \).

The applied transformation is such that it does not affect the implied volatility of the option with the shortest maturity \( \sigma_{t,1}^{imp} \). All the other implied volatilities are transformed to \( \sigma_{t,i}^{imp} = \sigma_{t,i}^{imp} - \hat{\beta}_{i,0} - \hat{\beta}_{i,1} S_t - \hat{\beta}_{i,2} S_t^2 \), where \( \hat{\beta}_{i,j} \) are the estimated parameters of the following regression.

\[ \sigma_{t,i}^{imp} - \sigma_{t,1}^{imp} = \hat{\beta}_{i,0} + \hat{\beta}_{i,1} S_t + \hat{\beta}_{i,2} S_t^2 + \varepsilon_{t,i}^{imp} \quad i \in \{2,3,4,5,6\}. \quad (22) \]

In this regression the volatility wedge, defined as \( \sigma_{t,i}^{imp} - \sigma_{t,1}^{imp} \), is regressed on a constant and on the exchange rate and on its square.

Table 1 shows the common explanatory power of the constant, the exchange rate and the square of exchange rate for the five volatility wedges and for the three countries. As we can see, the \( R^2 \) values are only high for Hungary. This can be interpreted as finding evidence for the effect of the target zone on the

\(^{19}\) This finding is supported by the theoretical models on target zones e.g. by Krugman (1991) and Naszódi (2004).
volatility wedges, on the differences between the option prices with different maturities. In contrast, in the case of the Czech Republic and Poland, the exchange rate does not explain much of the variance of the volatility wedges. Consequently, the implied volatility data need to be transformed for Hungary before estimating $\sigma_{x,t}$ and $\sigma_{y,t}$ but not for the other two countries.

By estimating (21) we obtain the time-varying volatilities of the factors. Chart 2 shows the time series of the estimated volatilities of the factors. The estimated volatility of $x$ is often 0, but may jump to between 10% and 30% or even beyond during turbulent times. In Hungary, for instance, the estimated volatility of $x$ soared to around 70% in July 2006. The high estimated volatilities of $x$ can, incidentally, be associated with those times when the long implied volatility substantially exceeds the short implied volatility.

The estimates for $c$ are around 2 for all three countries: 1.80 (Czech Republic), 2.05 (Hungary) and 2.365 (Poland). Table 2 shows that these parameter estimates are highly significant. One can interpret a parameter value of around 2 as follows. If a country will lock its exchange rate to the euro in four years, then the elasticity of the exchange rate with respect to market expectations for the final conversion rate ($e^{-\frac{c}{2}} = e^{-\frac{2}{c}}$) is almost 14%. If locking is expected to occur in two years, then this elasticity or the relative weight of the log final conversion rate in the log exchange rate is more than 40%.

Chart 3 shows the relative weights of the expected log locking rate in the log exchange rate in the investigated period. Positive shocks in $T$ decrease the relative weight of $x$ whereas negative shocks increase it. The largest change in the relative weights took place after September 2005, when market expectations for the ERM II entry dates shifted substantially for all three countries. Yet the relative weight of $x$ remained significant for all three countries. Even the smallest relative weight of $x$ exceeded 10% in the case of the Czech Republic and Poland and 7% in the case of Hungary.
Chart 2

Estimated Volatilities of Expected Locking Rate $x$ and Latent Exchange Rate $v$

Czech Republic

Hungary

Poland

Source: Author's calculations.
4.3 Filtered Market Expectations

Chart 4 shows the historical exchange rates of the Czech koruna, the Hungarian forint and the Polish zloty against the euro, the filtered states and analysts’ average expectations for the central parity as polled by Reuters. Market expectations about the final conversion rate may be thought to be close to the expected central parity of the ERM II regime. In that case, the expected central parity is a good reference for the filtered expected final conversion rate to be compared with. Here, we compare the filtered market expectations with the average expectations reported by the Reuters polls, although we think that the polls have only limited information content with respect to the central parity. After all, respondents’ views on central ERM II parities vary a lot in each poll. There is at least 6% difference between the two extreme views of the analysts, with even differences of more than 20% being quite common. These differences indicate that uncertainty around the reported expectations is likely to be high; one needs to bear this in mind when taking the average reported expectations as the general view of the market on central parity.

As is evident from chart 4, the patterns of the filtered expected final conversion rate and of the reported expected central parity are similar for all three countries. Moreover, each pattern is similar to that of the corresponding historical exchange rate. However, the reported market expectations and the filtered $x$ values differ significantly most of the time. The reported expectations of the market are usually outside the 90% confidence interval of the filtered $x$ for all three countries.

We have important findings on both the level of the expected final conversion rate and on its volatility. If our previous view on the role of locking was based purely on the Reuters poll data on the averaged market expectations for central parities, then these new findings may modify our view in some aspects.
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Chart 4

Filtered Market Expectations Regarding the Final Conversion Rate

Czech Republic

Hungary

Poland

Source: Author's calculations.
The filtered market expectations for euro locking rates are below the averaged market expectations for central parity for a long period for all three countries. In the case of the Czech Republic, the filtered expected locking rate is almost always lower than the reported averaged expectations regarding central parity. The only exception is the most recent observation of August 2006. The same holds for Hungary and Poland for their first subperiods. The filtered expected locking rate is smaller than the reported averaged expectations regarding central parity in Hungary until March 2006 and for Poland before October 2005. If one considers the filtered data to be more reliable than the Reuters poll data, then this paper contributes substantially to our knowledge on market expectations: In the first part of the investigated period the market expected the Czech koruna, the Hungarian forint and the Polish złoty to be locked at a stronger final conversion rate than suggested by the Reuters polls. In the case of the Czech koruna and the Polish złoty, the difference between the filtered market expectations and the reported averaged expectations regarding the central parity decreased substantially in the second part of the sample. As for the Hungarian forint, the market expected an even higher final conversion rate than suggested by the Reuters polls in the second part of the sample. Finally, at the end of the sample the two seem to coincide for the Hungarian forint as well.

The volatility levels of market expectations for the final conversion rate are important, because relatively stable market expectations can stabilize the exchange rate. The locking rate is often referred to as the nominal anchor of the exchange rate that can stabilize the exchange rate through the expectation channel. Regarding the volatilities, one can see that the filtered $x$ is more volatile than the reported averaged expectations regarding central parity in all three countries. This finding might adversely modify our previous view based purely on the Reuters polls regarding the stabilizing feature of locking. Still, if the volatility of $x$ is lower than that of $s$, then market expectations for the final conversion rate might have a stabilizing effect on the exchange rate. What can be seen from chart 4 is that most of the time the volatility of $s$ exceeds the volatility of $x$ in all three countries. In the Czech Republic and Poland locking seems to have had a stabilizing effect on the exchange rate from March 2006 until August 2006. Moreover, in Poland this appears to be true also for the period from March to October 2005. In those periods the volatility of market expectations for locking rates was almost always 0 (as shown in chart 3), and the filtered $x$ values were more stable than the exchange rates. Moreover, the relative weights of $x$ in the Czech koruna and Polish złoty were significant at around 20%. In the case of Hungary we can detect by visual inspection two periods characterized by the stabilizing effect of the prospect of locking, namely the periods from October 2005 to January 2006 and from March 2006 to June 2006. What might make the stabilizing feature of locking smaller in the case of Hungary as compared with the

<table>
<thead>
<tr>
<th>Volatility of Exchange Rate $s$ and Latent Exchange Rate $v$ in %</th>
<th>CZ</th>
<th>HU</th>
<th>PL</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_s$</td>
<td>4.70</td>
<td>7.67</td>
<td>9.26</td>
</tr>
<tr>
<td>$\sigma_v$</td>
<td>5.02</td>
<td>8.26</td>
<td>11.98</td>
</tr>
<tr>
<td>$\sigma_s - \sigma_v$</td>
<td>-0.32</td>
<td>-0.59</td>
<td>-2.72</td>
</tr>
<tr>
<td>$(\sigma_s - \sigma_v) / \sigma_v$</td>
<td>-6.36</td>
<td>-7.15</td>
<td>-22.67</td>
</tr>
</tbody>
</table>

Source: Author's calculations.
other two countries is that the relative weight of \( x \) in \( s \) was only around 10% in these periods.

The big picture on the stabilizing feature of locking in the entire sample period is provided in table 3. The stabilizing effect of locking is calculated as the absolute and the relative difference between the volatilities of the historical and the latent exchange rates. Across the sample period, the stabilizing effect of locking is found to have been highest in Poland, second-highest in Hungary and least important in the Czech Republic. However, the Czech koruna would be the least volatile of the three currencies even if the Czech Republic were not aiming at joining the euro area.

5 Conclusions

This paper investigates market expectations for the final euro conversion rate of EMU candidate currencies based on a theoretical model for exchange rates subject to future locking. In this theoretical model, exchange rates subject to future locking converge to actual market expectations for the final conversion rate in the expected term; the nearer the (expected) time of locking, the higher is the speed of convergence. In the empirical part of the paper, the Kalman filter is used to extract the subjective expectation of market participants for the final conversion rate from historical exchange rate data for the Czech Republic, Hungary and Poland.

Our previous view on the role of locking, which was mainly based on Reuters poll data, has been modified in some aspects. First, the level of the filtered market expectations for the final conversion rate differs significantly from the averaged reported market expectations regarding the central parity in all three countries. Second, the stabilizing feature of market expectations for the final conversion rate on the exchange rate proved to be smaller in the case of filtered expectations than in the case of the averaged reported market expectations. The magnitude of the stabilizing effect depends on two determinants: the stability of market expectations for the locking rate and the importance of expectations in determining the exchange rate. In case of an earlier entry to the euro area, the stabilizing effect is likely to be more substantial because market expectations for the locking rate are likely to be more stable. Moreover, the relative weight of the expectations in the exchange rate is also higher. Based on this intuitive argument, the prospect of locking should contribute most to the stabilization of the Czech koruna and least to that of the Hungarian forint. Yet the results somewhat contradict this intuitive approach. Based on the investigation of the entire sample period, the stabilizing effect of locking is found to be highest in Poland, second-highest in Hungary and least important in the Czech Republic. However, the Czech koruna would be the least volatile of the three currencies even if the Czech Republic were not aiming at joining the euro area.

This paper presents the result of a work in progress. Future research will be directed at estimating the expected locking dates from interest rate data and estimating the correlation parameters instead of fixing them to 0. These modifications will hopefully cope with some of the counter-intuitive results that can be found in the current version.
References


